basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P1

NOVEMBER 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 1 answer sheet and an information sheet consisting of 2 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.

2. Answer ALL the questions.

3. Answer QUESTION 4.1.3 and QUESTION 7.5 on the ANSWER SHEET provided. Hand in the ANSWER SHEET with your ANSWER BOOK.

4. Number the answers correctly according to the numbering system used in this question paper.

5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

6. Answers only will not necessarily be awarded full marks.

7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. If necessary, round off answers to TWO decimal places, unless stated otherwise.

9. Diagrams are NOT necessarily drawn to scale.

10. Write neatly and legibly.

QUESTION 1

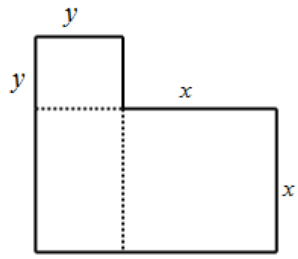
1.1 Solve for x:

1.1.1 $-2x(x+a)(3-x)=0$ (3)

1.1.2 $2x = 6 - x^2$ (correct to TWO decimal places) (4)

1.1.3 $5x(x-3) \leq 0$ and then represent the solution on a number line (3)

1.2 The total area represented by the L-shaped diagram below is 21 units^2. The equation $y-2x=-7$ represents the relationship of the sides of the two squares.



Solve for x and y (dimensions of the two squares) if:

$y-2x=-7$ and $x^2+xy+y^2=21$ (7)

1.3 The formula below represents the moment of inertia (E), with mass (M) and length (L):

$$E = \frac{1}{12}ML^2$$

1.3.1 Make L the subject of the formula. (2)

1.3.2 Calculate the value of L, if $E = 8{,}3 \times 10^{-2}$ kg.m^2 and $M = 1{,}6 \times 10^3$ kg. (2)

1.4 Express 36 as a binary number. (2)

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QUESTION 2

2.1 Given the roots: $x = \dfrac{-8 \pm \sqrt{q-3}}{2}$

Describe the nature of the roots if:

2.1.1 $q = 5$ (1)

2.1.2 $q = 3$ (1)

2.1.3 $q < 0$ (1)

2.2 Determine for which value(s) of p will the equation $3x^2 + 7x = 2x + p$ have non-real roots. (4)

[7]

QUESTION 3

3.1 Simplify (showing ALL calculations) the following without the use of a calculator:

3.1.1 $\left(2a^{\frac{7}{3}}\right)^3$ (2)

3.1.2 $\log_p p + \log_m 1$ (2)

3.1.3 $\dfrac{\sqrt{48} - \sqrt{12}}{2\sqrt{75}}$ (3)

3.2 Solve for x: $\log_2(x+62) - \log_2 x = 5$ (4)

3.3 Express the complex number $z = -\sqrt{2} + \sqrt{2}\ i$ in the polar form $z = a\ cis\ \theta$ (6)

3.4 Solve for p and q if $p + qi = (2-3i)^2$. (4)

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QUESTION 4

4.1 Given: $g(x) = 2^{-x} - 1$ and $h(x) = -\dfrac{6}{x} - 1$

4.1.1 Write down the equations of the asymptotes of h. (2)

4.1.2 Determine the coordinates of the x-intercept of h. (2)

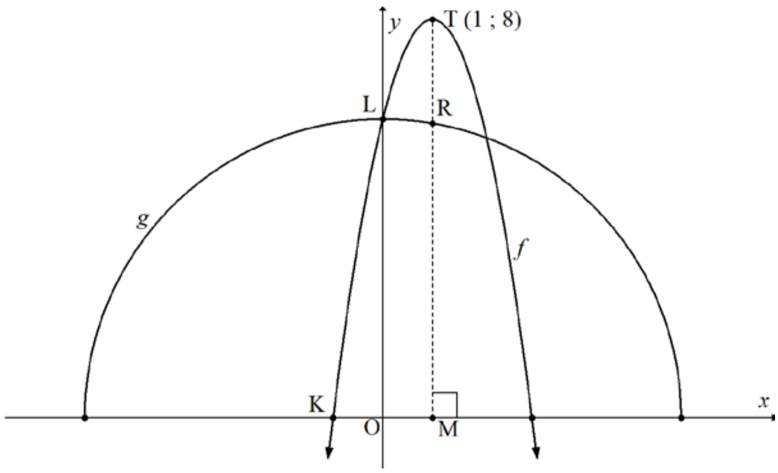
4.1.3 Sketch the graphs of g and h on the same set of axes on the ANSWER SHEET provided. Clearly show the asymptotes and the intercepts with the axes. (5)

4.1.4 Show that $(-2\,;3)$ is a point on the graph of g. (1)

4.1.5 Write down the range of g. (1)

4.1.6 Write down the domain of h. (1)

4.2 Sketched below are the graphs defined by $f(x) = a(x+p)^2 + q$ and $g(x) = \sqrt{36 - x^2}$ with $\mathrm{T}(1\,;8)$ the turning point of f. Line TM is drawn such that TM is perpendicular to the x-axis. Points L and K are the intercepts of f. Point L is a point of intersection of f and g. Point R lies on both line TM and the graph of g.



4.2.1 Write down the coordinates of M. (1)

4.2.2 Determine the length of TR (leave your answer in surd form). (3)

4.2.3 Show that $(0\,;6)$ are the coordinates of L. (1)

4.2.4 Hence, show that the graph of f is defined by $f(x) = -2(x+1)(x-3)$. (4)

4.2.5 Hence, give the coordinates of K. (1)

4.2.6 Determine the values of x for which $f(x) \times g(x) > 0$ and $x < 0$ (2)

[24]

QUESTION 5

5.1 The annual effective interest rate charged by a financial institution is 6,7%. Calculate the nominal interest rate charged per annum if compounded monthly. (4)

5.2 A company bought a new 3D wheel-alignment machine for R240 000. The machine depreciated at a rate of 16% per annum to half its original value over a certain period.

5.2.1 Give the depreciated value of the machine at the end of the period. (1)

5.2.2 Determine how long it will take for the machine to depreciate to half its original value. Give your answer to the nearest year. (5)

5.3 Mr Bohlale invested R40 000 at a bank for 7 years. The interest rate for the first 4 years was 11,2% per annum, compounded quarterly. The interest rate then changed to 13% per annum compounded annually for the remaining years. Calculate the total amount of money that Mr Bohlale will receive at the end of the investment period. (5)

[15]

QUESTION 6

6.1 Determine $f^{/}(x)$ using FIRST PRINCIPLES if $f(x)=7x-2$ (5)

6.2 Determine:

6.2.1 $\frac{d}{dx}\left(\pi^{2}\right)$ (1)

6.2.2 $D_{x}\left(x^{4}-\sqrt[3]{x}\right)$ (3)

6.2.3 $\frac{dy}{dx}$ if $y=\frac{x^{5}+2}{x^{2}}$ (4)

6.3 The tangent to the curve of the function defined by $p(x)=x^{3}+1$ passes through point $A(2;k)$.

6.3.1 Calculate the numerical value of k. (2)

6.3.2 Determine $p^{/}(x)$ (1)

6.3.3 Hence, determine the equation of the tangent to the curve of the function at point A. (3)

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QUESTION 7

Given: $f(x)=-x(x-3)(x-3)$

7.1 Write down the coordinates of the x-intercepts of f. (2)

7.2 Write down the y-intercept of f. (1)

7.3 Show that $f(x)=-x^{3}+6x^{2}-9x$ (2)

7.4 Determine the coordinates of the turning points of f. (5)

7.5 Sketch the graph of f on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axes and the turning points. (4)

7.6 Determine the values of x for which the graph of f is increasing. (2)

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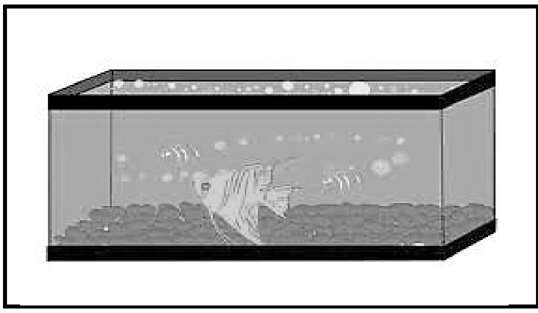
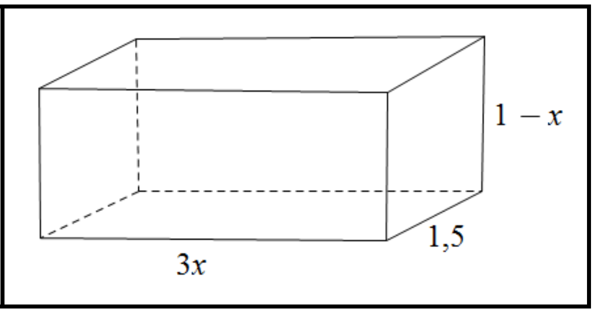
QUESTION 8

8.1 Mr Alexander built a rectangular fish tank. The length, breadth and height of the tank are $3x$ metres, 1,5 metres and $(1-x)$ metres respectively, as shown in the diagram below.





8.1.1 Determine a formula for the volume of the tank in terms of x. (3)

8.1.2 Hence, determine the value of x that will maximise the volume of the tank. (3)

8.2 During an experiment, learners must record the velocity (v) of an electronic toy car over a distance (m), t seconds after the experiment has begun. The velocity of the electronic toy car is given by $v(t) = 8 + 4t - t^2$

Determine:

8.2.1 The initial velocity of the toy car (1)

8.2.2 The velocity of the toy car when $t = 0{,}2$ seconds (2)

8.2.3 The rate at which the velocity changes with respect to time when $t = 1{,}2$ seconds (4)

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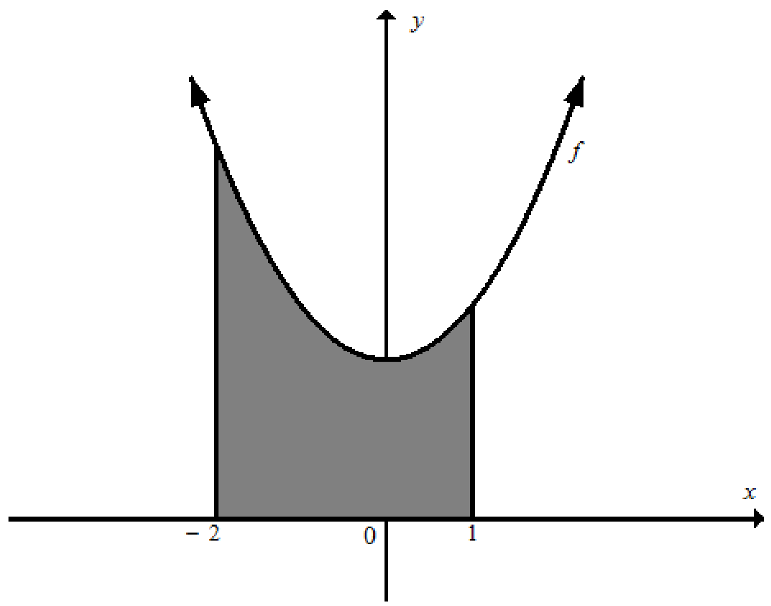
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int\left(-\frac{6}{x}\right)dx$ (2)

9.1.2 $\int(x-1)^2\,dx$ (4)

9.2 The sketch below represents the bounded area of the curve of the function defined by $f(x) = x^2 + 3$



Determine the shaded area bounded by the curve and the x-axis between the points where $x = -2$ and $x = 1$ (6)

[12]

TOTAL: 150

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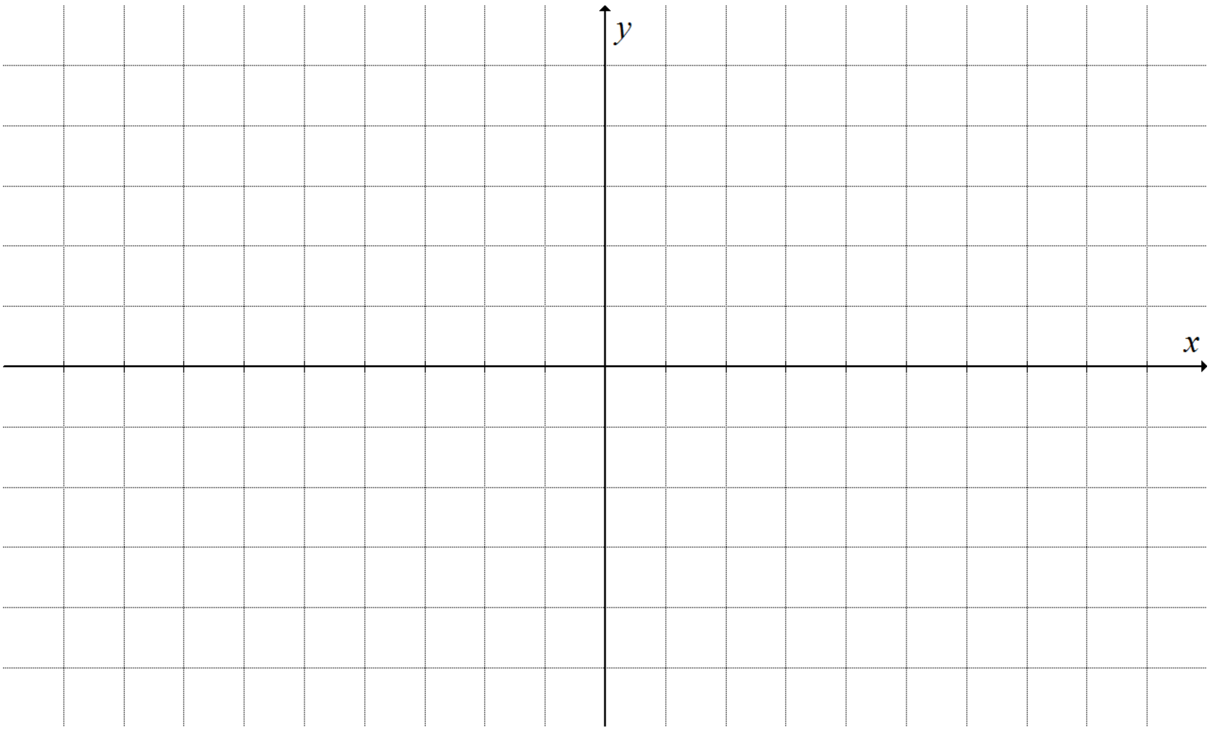
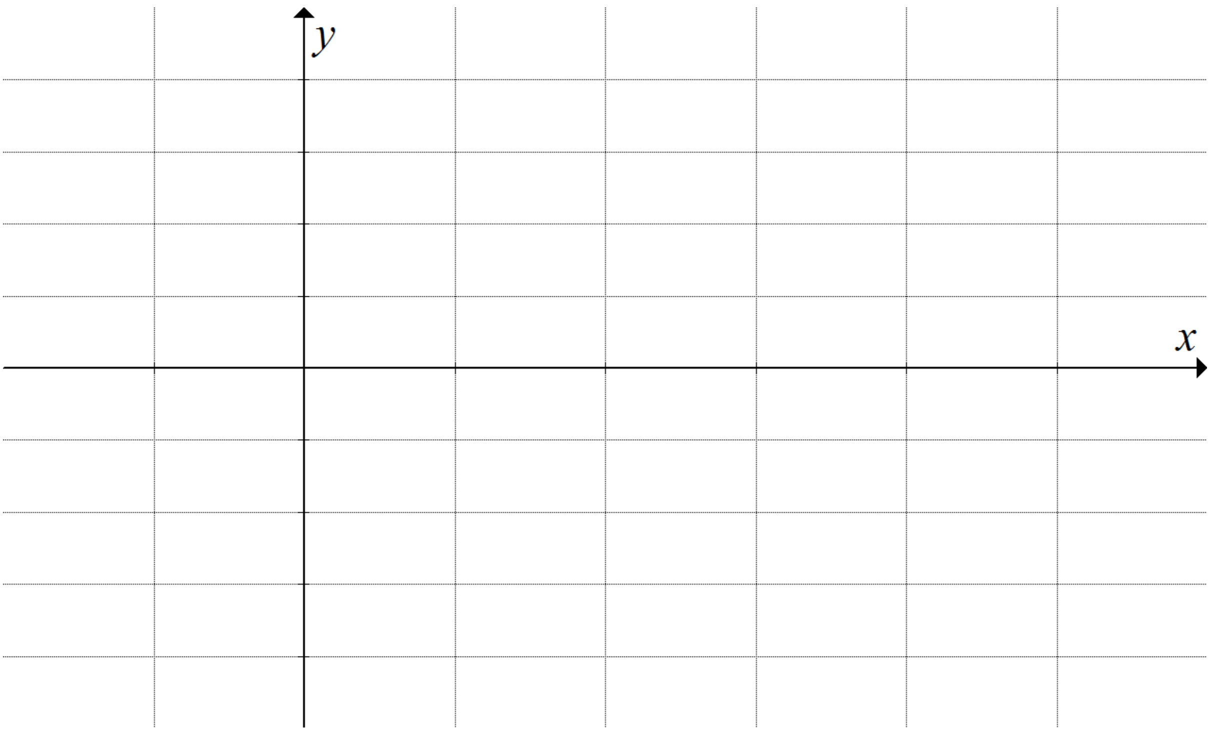
QUESTION 4.1.3



QUESTION 7.5



INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\dfrac{b}{2a}$ $\qquad y = \dfrac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b \ , \quad a > 0 \ , \ a \neq 1 \text{ and } b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$ $\qquad A = P(1 - i)^n$ $\qquad A = P(1 + i)^n$

$i_{eff} = \left(1 + \dfrac{i}{m}\right)^m - 1$

$f^{/}(x) = \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C \quad , \ n \neq -1$

$\int \dfrac{1}{x} dx = \ln x + C, \qquad x > 0$ $\qquad \int a^x dx = \dfrac{a^x}{\ln a} + C \quad , \quad a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad \text{M}\left(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad m = \tan\theta$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ of\ \ \Delta ABC = \dfrac{1}{2} ab \cdot \sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad \cot^2\theta + 1 = \text{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2} = \dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates