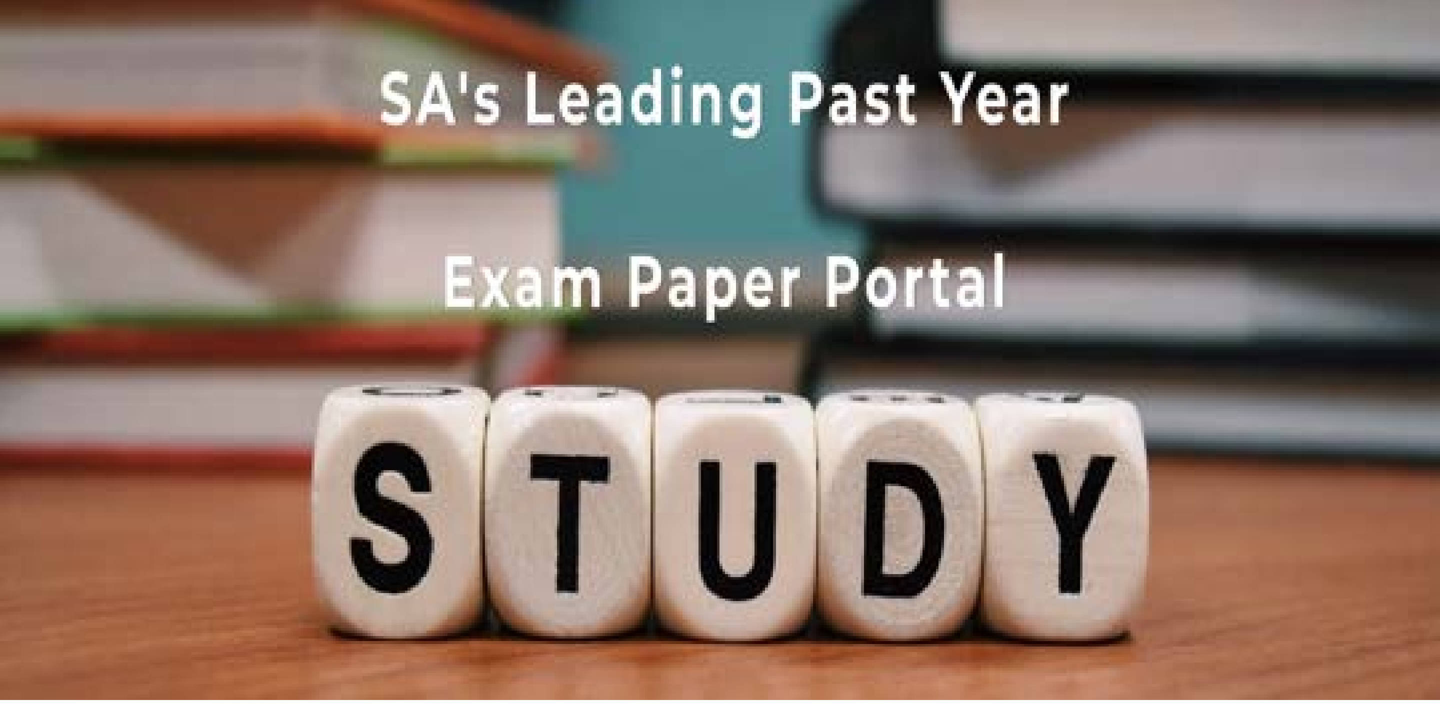

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SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P2

2019

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages and a 2-page information sheet.

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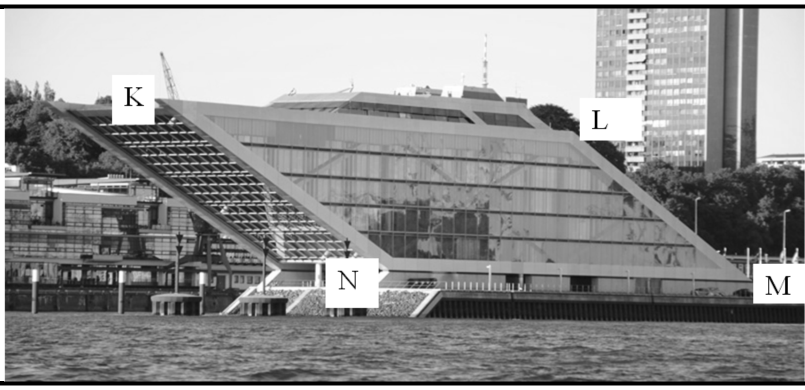
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

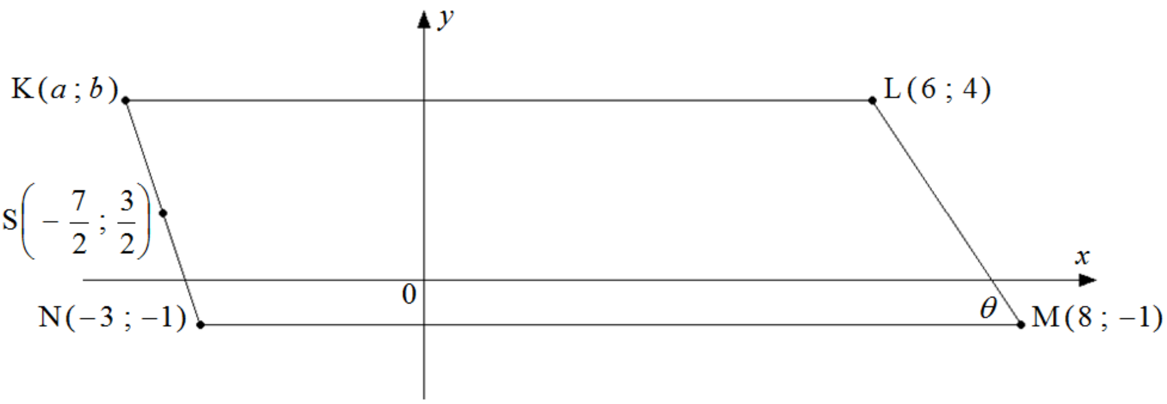
QUESTION 1

The slanted side KLMN of a building, as shown in the picture alongside, is modelled on a Cartesian plane.



The diagram below, not drawn to scale, shows quadrilateral KLMN, with vertices K(a ; b), L(6 ; 4), M(8 ; –1) and N(–3; –1).

Point $S\left(-\frac{7}{2};\frac{3}{2}\right)$ is the midpoint of KN and $L\hat{M}N=\theta$



Determine:

1.1 The length of NM (1)

1.2 The coordinates of K (3)

1.3 The size of θ (correct to the nearest degree) (3)

1.4 The coordinates of P, if NKLP is a parallelogram (2)

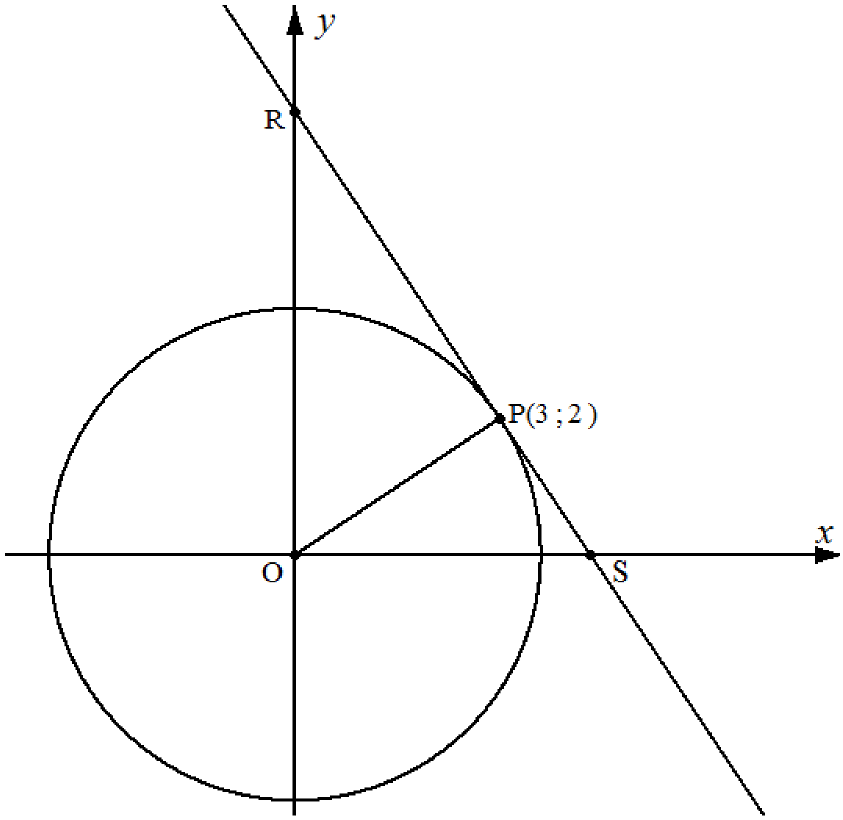
1.5 The coordinates of the midpoint of LM (1)

1.6 The equation of the straight line that passes through both point N and the midpoint of LM. Write the equation in the form $y = ...$ (4)

[14]

QUESTION 2

2.1 In the diagram below, O (0 ; 0) is the centre of the circle with equation $x^2 + y^2 = 13$
Straight line RS is a tangent to the circle at point P(3 ; 2).
R and S are the x- and y-intercepts of tangent RS respectively.



Determine (show ALL calculations):

2.1.1 Whether point $M(-2{,}5\ ;\ 2{,}5)$ lies inside or outside the circle (3)

2.1.2 The gradient of OP (1)

2.1.3 The equation of tangent RS in the form $y = \ldots$ (4)

2.2 Given: $7x^2 + 36y^2 = 252$

2.2.1 Write $7x^2 + 36y^2 = 252$ in the form $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$ (1)

2.2.2 Hence, sketch on the grid provided the graph defined by $7x^2 + 36y^2 = 252$

Clearly show ALL the intercepts with the axes. (3)
[12]

QUESTION 3

3.1 Given: $\cos \alpha = -\dfrac{\sqrt{7}}{4}$, for α□ $[0°;180°]$

Determine (without the use of a calculator) the value of each of the following:

3.1.1 $\cot \alpha$ (3)

3.1.2 $\tan^2 \alpha - \operatorname{cosec} \alpha$ (3)

3.2 Calculate the value of $\cos\left(x - y\right)$ if $x = 1{,}2\pi$ and $y = 0{,}4\pi$ (2)

3.3 Determine the value of $\theta \in [90^\circ ; 360^\circ]$ if $2\cot\theta - 5 = 0$ (5)

[13]

QUESTION 4

4.1 Given: $\dfrac{\cos(\pi + \theta) \cdot \tan(360° - \theta)}{\sin(180° - \theta)}$

4.1.1 Simplify: $\cos(\pi + \theta)$ (1)

4.1.2 Hence, simplify: $\dfrac{\cos(\pi + \theta) \cdot \tan(360° - \theta)}{\sin(180° - \theta)}$ (4)

4.2 Given the identity: $\cot^2 x \cdot \sec^2 x - (\sin^2 x + \cos^2 x) = \cot^2 x$

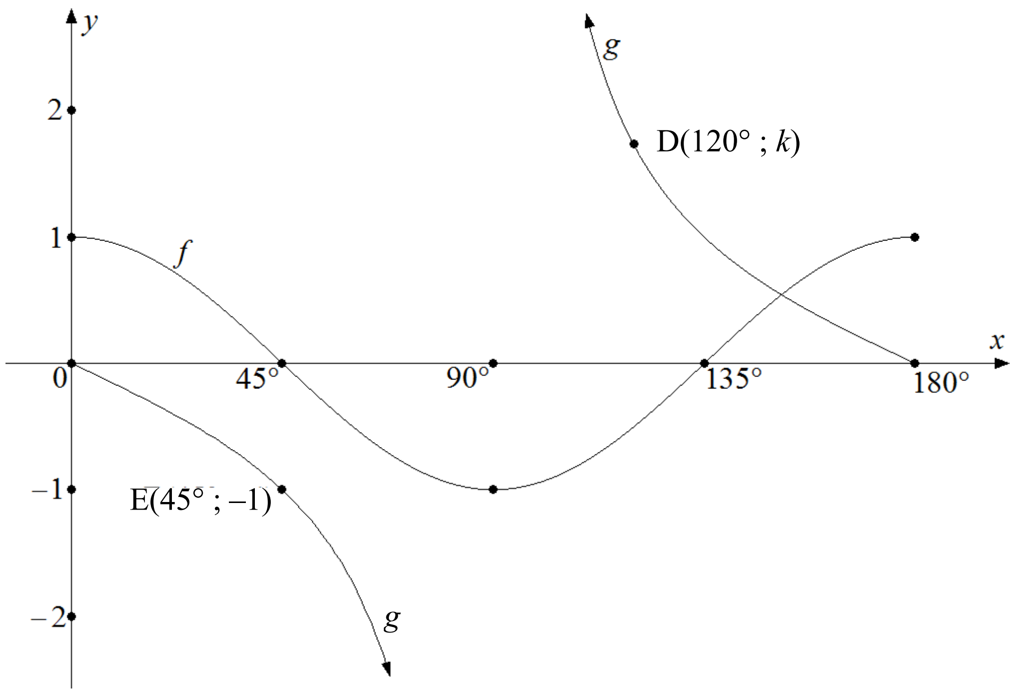
4.2.1 Prove the identity. (5)

4.2.2 Determine the value(s) of $x \in [0°;360°]$ for which the identity is undefined. (5)

[15]

QUESTION 5

The graph below represents the curves of functions f and g defined by $f(x) = a\cos bx$ and $g(x) = c\tan x$ for $x \in [0°\,;180°]$
Point $D(120°\,;k)$ and point $E(45°\,;-1)$ lie on g.



Use the graph above to answer the following questions.

5.1 Give the period of f. (1)

5.2 Determine the numerical values of a, b and c. (4)

5.3 Write down the values of x for which $f(x) - g(x) = 1$ (3)

5.4 Give the equation of the asymptote of g. (1)

5.5 Determine the numerical value of k. (2)

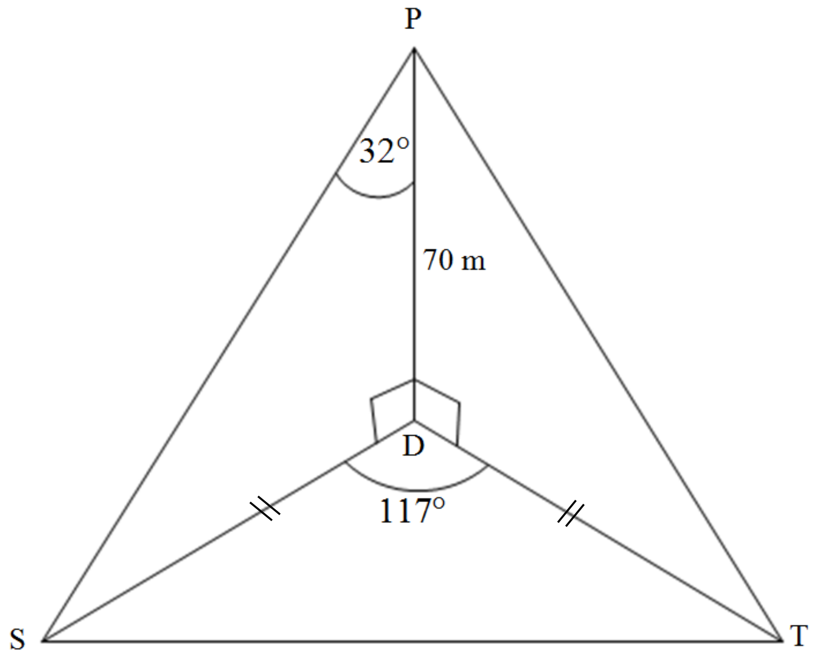
5.6 Determine the values of x for which $f(x) \times g(x) \leq 0$ for $x \in [\,0°\,;180°\,]$ (4)
[15]

QUESTION 6

The diagram below shows the position of a helicopter at point P, which is directly above point D on the ground.

Points S, D and T lie in the same horizontal plane, such that points S and T are equidistant from D.

PD = 70 m, $\hat{SDT} = 117°$ and $\hat{SPT} = 32°$



Calculate the following:

6.1 The distance SD (2)

6.2 The distance ST (3)

6.3 The area of ΔSDT (2)

[7]

QUESTION 7

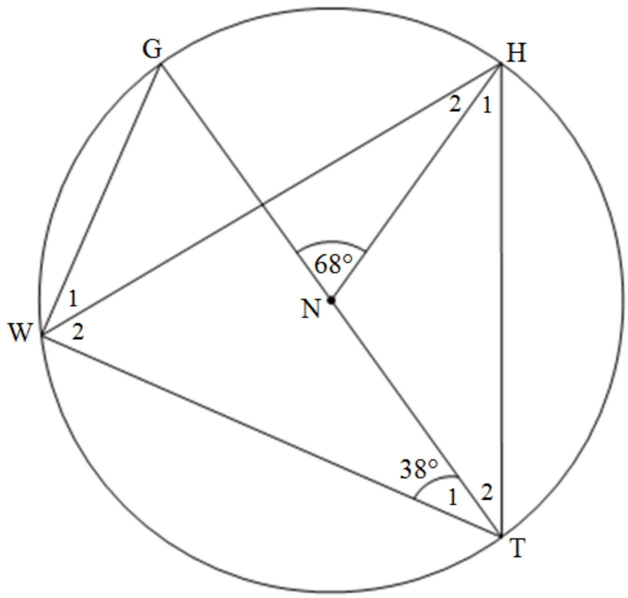
7.1 Complete the following theorem:

The angle subtended by the diameter at the circumference of the circle is … (1)

7.2 The diagram below shows circle GWTH with centre N.

GT is a diameter of the circle.

$G\hat{N}H = 68°$ and $\hat{T}_1 = 38°$



7.2.1 Determine, stating reason(s), the size of $\hat{W}_1$. (2)

7.2.2 Give a reason why $\hat{H}_1 = \hat{T}_2$ (1)

7.2.3 Hence, determine (stating reasons) the size of $\hat{H}_2$. (4)

[8]

QUESTION 8

8.1 Complete the following theorems:

8.1.1 The exterior angle of a cyclic quadrilateral is equal to … (1)

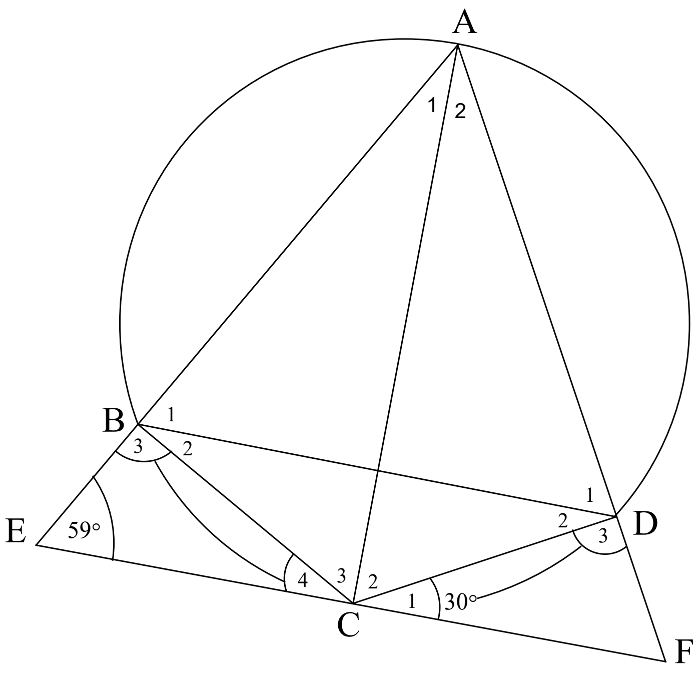
8.1.2 The angle between the tangent to a circle and the chord drawn from the point of contact is … (1)

8.2 The diagram below shows circle ABCD with AB produced to E and AD produced to F.

ECF is a tangent to the circle at C and CA bisects $E\hat{A}F$.

$\hat{C}_1 = 30°$

$\hat{E} = 59°$



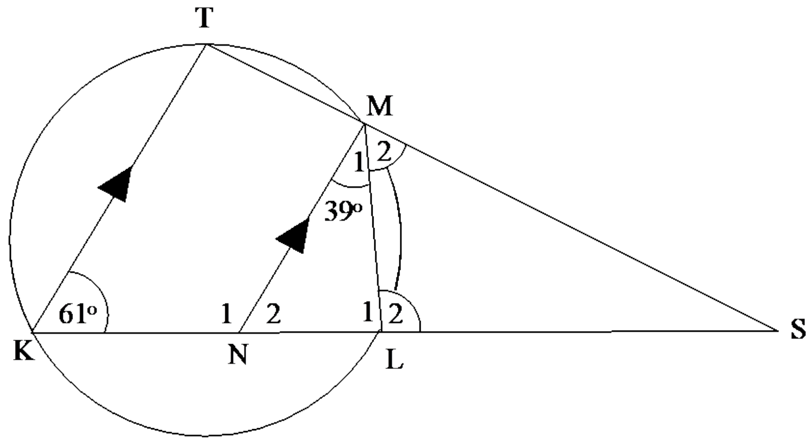
8.2.1 Give, with reasons, THREE other angles, each equal to 30°. (5)

8.2.2 Determine, with reason(s), the size of $\hat{B}_1$ if $\hat{D}_1 = 61°$ (2)

8.2.3 Give a reason why BD || EF. (1)

8.2.4 Determine, with reason(s), whether AC is a diameter of circle ABCD. (3)

8.3 The diagram below shows circle TKLM with chords TM and KL produced to meet at S.
TK || MN with N, a point on KL.
$\hat{K} = 61°$
$\hat{M}_1 = 39°$



8.3.1 Calculate, with reasons, the sizes of the following angles:

(a) $\hat{T}$ (3)

(b) $\hat{L}_1$ (2)

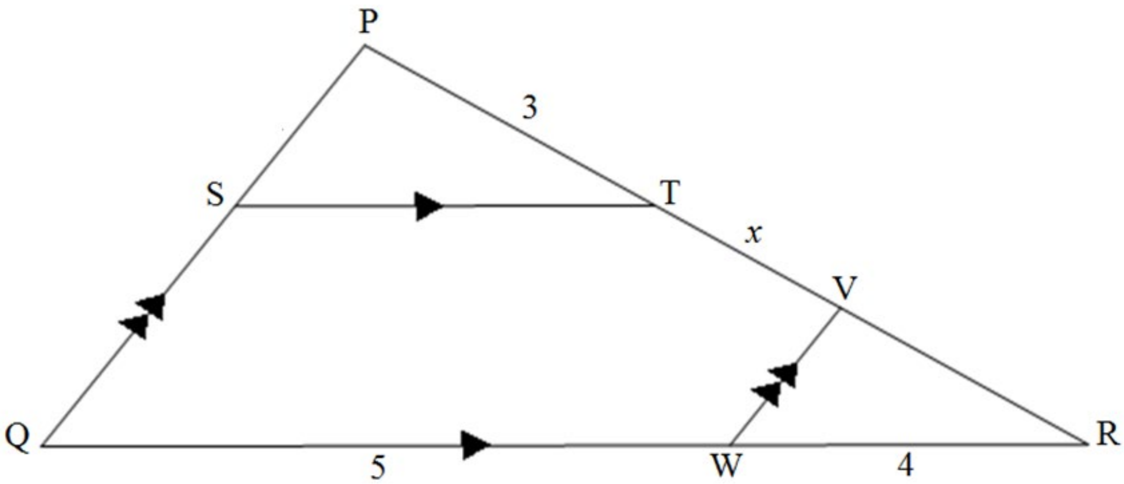
8.3.2 Show, with reasons, whether MS is a tangent to circle MNL. (3)
[21]

QUESTION 9

9.1 Complete the following theorem:

If a line divides two sides of a triangle in the same proportion, then the line is … (1)

9.2 In the diagram below, Δ PQR is drawn with S on PQ, T and V on PR and W on QR.
ST || QR and VW || PQ.
Furthermore, PS : SQ = 1 : 3
RW = 4 units, QW = 5 units, PT = 3 units and TV = x units.



9.2.1 Determine, with reason(s), the length of TR. (3)

9.2.2 (a) Express VR in terms of x. (1)

(b) Give the numerical value of $\frac{\text{RV}}{\text{VP}}$. (1)

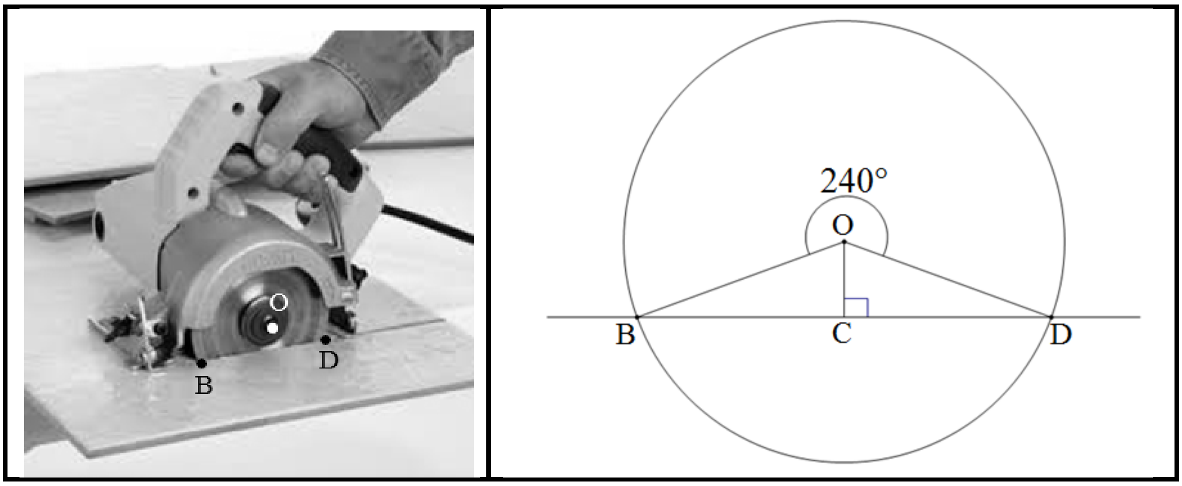
(c) Hence, determine the numerical value of x. (3)

[9]

QUESTION 10

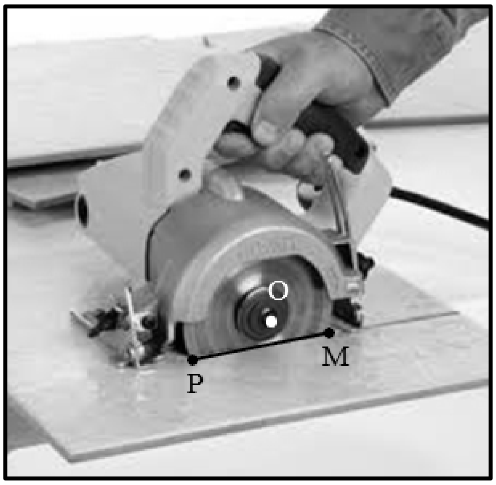
10.1 The picture and diagram below show an electric saw cutting a wooden board. Point O represents the centre of the circular blade of the saw.

Reflex $B\hat{O}D = 240°$ and $OC \perp BD$. The length of the radius of the blade is 6 cm.



10.1.1	Write down the size of obtuse $B\hat{O}D$.	(1)
10.1.2	Hence, convert the size of obtuse $B\hat{O}D$ to radians.	(2)
10.1.3	Calculate the length of minor arc BD.	(3)
10.1.4	Calculate the area of minor sector OBD.	(3)

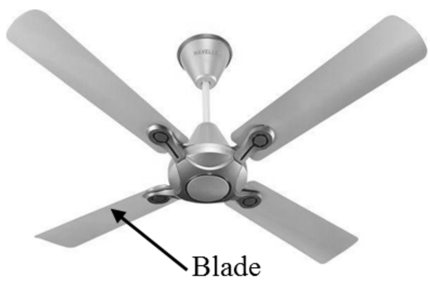
10.2 Another electric saw, as shown in the diagram below, with a larger blade having a diameter of 20 cm, is used to cut a wooden board. The height of the major segment of the blade is 75% the length of the diameter of the blade.

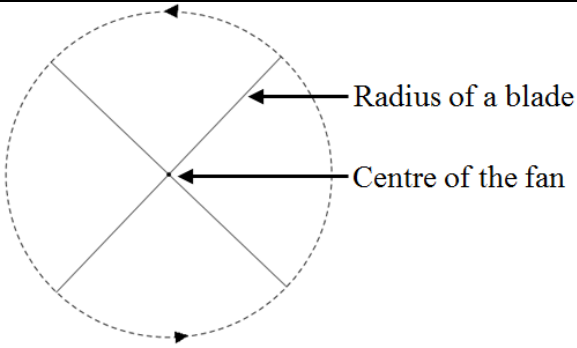


Determine:

10.2.1	The height of the major segment of the blade	(1)
10.2.2	The exact length of chord PM	(4)

10.3 The picture and diagram below show an electric fan that has four identical rotating blades that rotate at 160 revolutions per minute. The circumferential velocity of the tip of a blade is $\frac{32}{15}\pi$ m/s .





Determine:

10.3.1 The numerical value (in metres) of the radius of a blade (4)

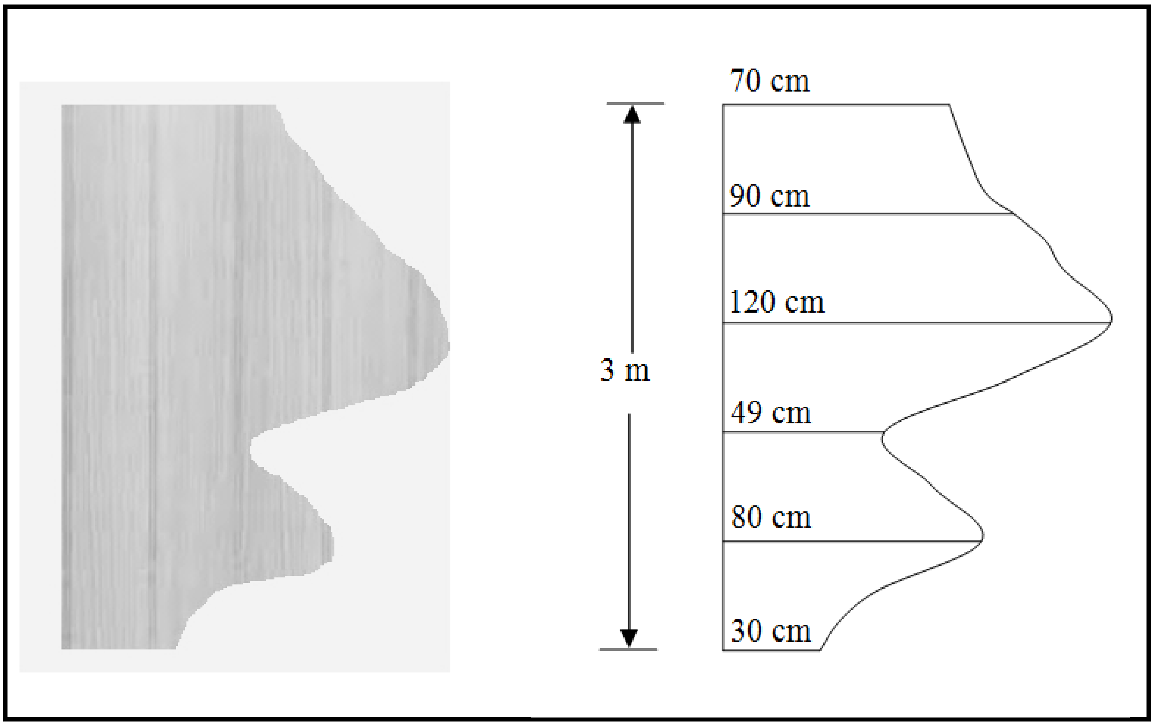
10.3.2 The angular velocity (in radians per second) of a rotating blade (3)
[21]

QUESTION 11

11.1 The irregular shape, as shown below, has one vertical straight side, 3 m long, which is divided into five equal parts.

The ordinates dividing the parts are:

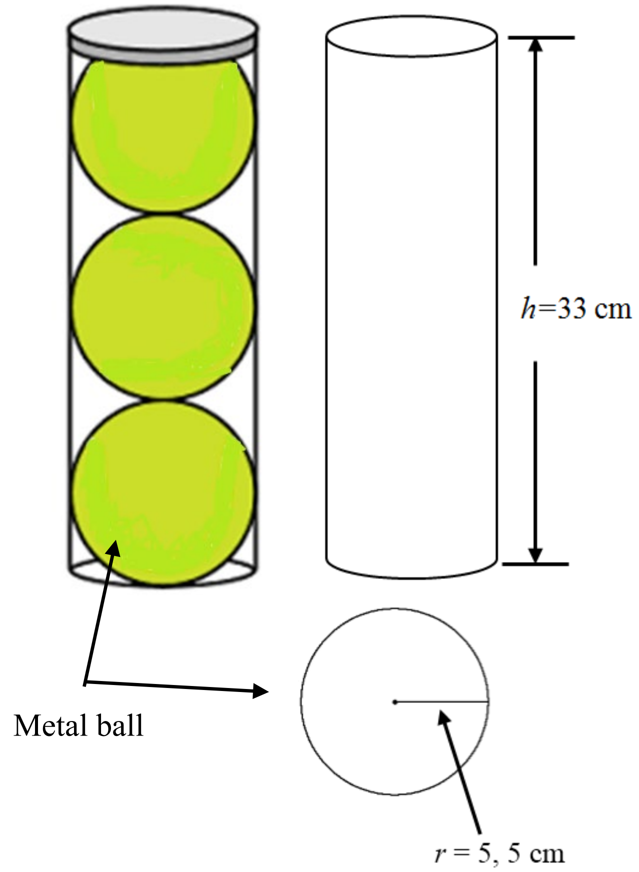
70 cm ; 90 cm ; 120 cm ; 49 cm ; 80 cm and 30 cm



11.1.1 Write down the length of the equal parts. (1)

11.1.2 Determine the area (in m^2) of the irregular shape by using the mid-ordinate rule. (4)

11.2 The closed right cylindrical container, shown in the diagram below, has been made such that exactly three identical spherical metal balls fit tightly inside. The interior height, h , of the cylinder is 33 cm. The interior diameter of the cylinder is 11 cm and the radius of the metal balls is 5,5 cm.



The following formulae may be used:

Surface area of a sphere = $4\pi r^2$
Total surface area of a cylinder = $2\pi r^2 + 2\pi r h$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

11.2.1 Calculate the total interior surface area of the closed cylindrical container. (3)

11.2.2 Calculate the volume of ONE spherical metal ball. (3)

11.2.3 Determine the volume of the empty space inside the cylindrical container with THREE metal balls. (4)
[15]

TOTAL: 150

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INFORMATION SHEET: TECHNICAL MATHEMATICS

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad x=-\frac{b}{2a}$ $\quad y=\frac{4ac-b^2}{4a}$

$a^x=b\Leftrightarrow x=\log_a b$, $\quad a>0$, $a\neq 1$ and $b>0$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$i_{eff}=\left(1+\frac{i}{m}\right)^m-1$

$f^{/}(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\int x^n dx=\frac{x^{n+1}}{n+1}+C$, $\quad n\neq -1$

$\int \frac{1}{x}dx=\ln x+C,\quad x>0$ $\qquad \int a^x dx=\frac{a^x}{\ln a}+C$, $\quad a>0$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\qquad \mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\qquad a^2=b^2+c^2-2bc\cdot\cos A$

area of $\Delta ABC=\frac{1}{2}ab\cdot\sin C$

$\sin^2\theta+\cos^2\theta=1$ $\quad 1+\tan^2\theta=\sec^2\theta$ $\quad \cot^2\theta+1=\mathrm{cosec}^2\theta$

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$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360° n$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2} = \dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, o_i = i^{th} ordinate and n = number of ordinates

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