Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2020

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 15 pages, including a 1-page information sheet and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The following table shows a comparison of a school's Grade 12 final marks in 2019 and the learners' School Based Assessment (SBA) marks for the year.

LEARNERS	1	2	3	4	5	6	7	8	9	10
SBA MARK	99	93	77	74	63	62	63	63	47	37
FINAL MARK	94	81	73	65	59	58	55	49	43	31

1.1 Determine the equation of the least squares regression line for the data. (Round off your answer correct to 4 decimal places.) (3)

1.2 Determine the correlation coefficient between the SBA mark and the final mark. (1)

1.3 Comment on the correlation between the SBA mark and the final mark. (1)

1.4 Learner 11 scored 51% for SBA. Predict the final mark he should get, correct to the nearest unit. (2)

1.5 Given that the mean for the final mark is 60,8, calculate how many learners were within one deviation of the mean. (3)

[10]

QUESTION 2

The speeds, in kilometres per hour, of cyclists that passed a point on the route of the Ironman Race were recorded and summarised in the table below:

Speed (km/h)	Frequency (*f*)	Cumulative Frequency
$0 < x \leq 10$	10	10
$10 < x \leq 20$		30
$20 < x \leq 30$	45	
$30 < x \leq 40$	72	
$40 < x \leq 50$		170

2.1 Complete the above table in the ANSWER BOOK provided. (2)

2.2 Make use of the axes provided in the ANSWER BOOK to draw a cumulative frequency curve for the above data. (3)

2.3 Indicate clearly on your graph where the estimates of the lower quartile (Q_1) and median (M) speeds can be read off. Write down these estimates. (2)

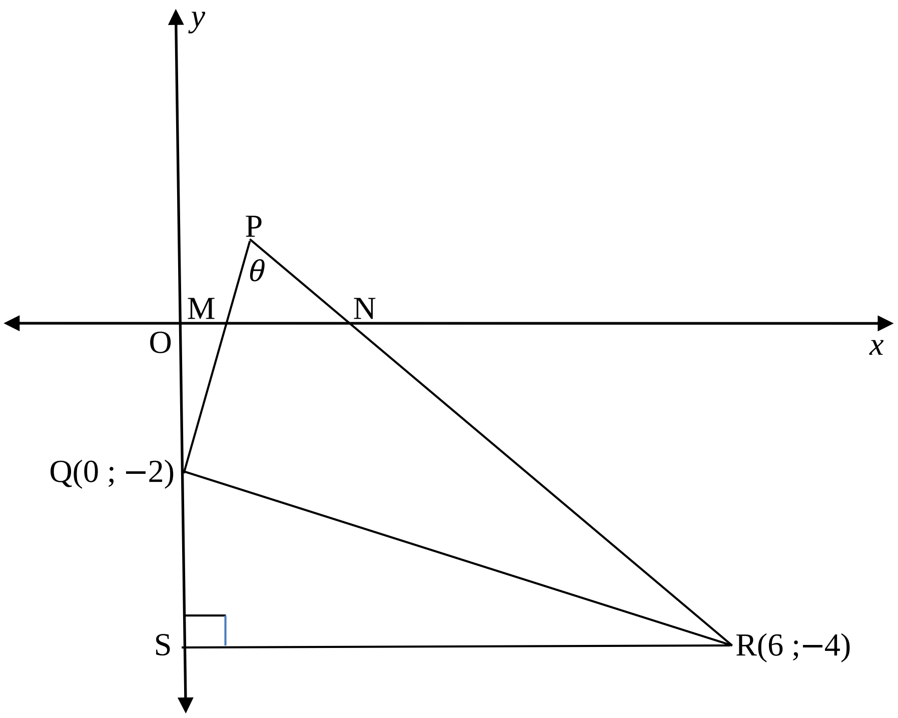
2.4 Draw a box and whisker diagram for the data. Use the number line in the ANSWER BOOK. (2)

2.5 Use your graph to estimate the number of cyclists that passed the point with speeds greater than 35 km/h. (1)

[10]

QUESTION 3

In the diagram, P, Q $(0 ; -2)$ and R $(6 ; -4)$ are the vertices of triangle PQR. The equation of PQ is $3x - y - 2 = 0$. The equation of PR is $y = -x + 2$. RS is the perpendicular from R to the y-axis. $\mathrm{Q\hat{P}R} = \theta$.



3.1 Calculate the gradient of QR. (2)

3.2 Prove that $\mathrm{P\hat{Q}R} = 90°$. (2)

3.3 Calculate the coordinates of P. (3)

3.4 Calculate the length of QR. Leave your answer in surd form. (2)

3.5 Determine the equation of the circle through Q, P and R. Give the answer in the form: $(x-a)^2 + (y-b)^2 = r^2$. (5)

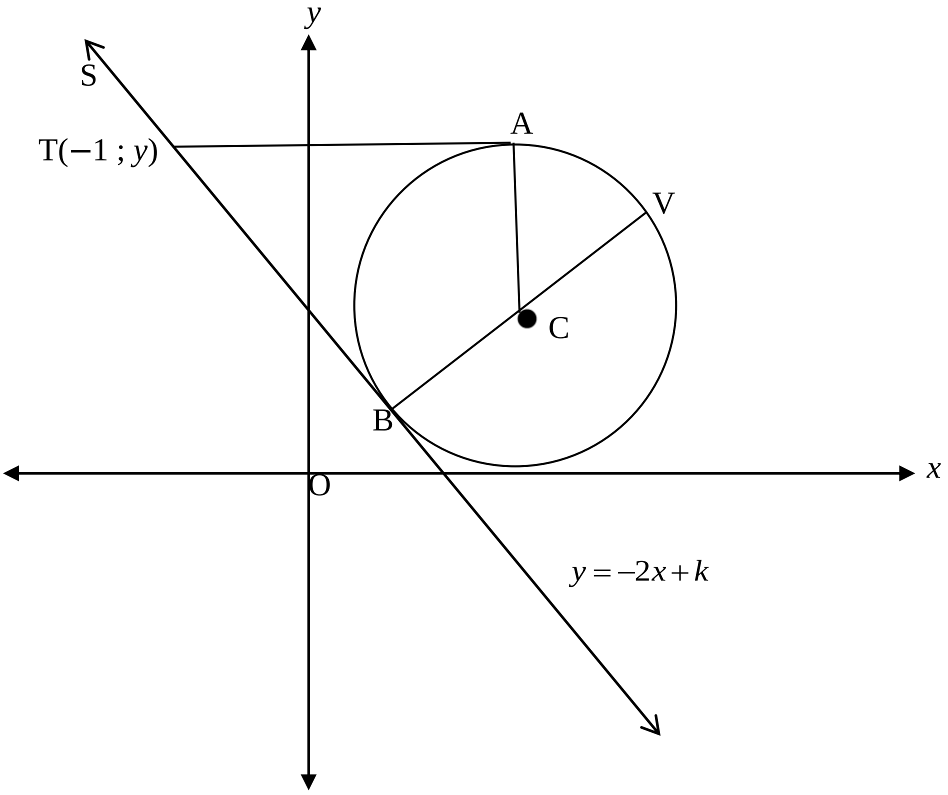
3.6 Calculate the size of angle θ. (5)

3.7 Calculate the area of ΔPQR. (3)

[22]

QUESTION 4

In the diagram below, C is the centre of the circle defined by $x^2 - 6x + y^2 - 4y + 9 = 0$.
T $(-1\ ;\ y)$ is a point outside the circle. Two tangents are drawn to the circle from T. STB is tangent to the circle at B and has equation $y = -2x + k$. TA is tangent to the circle at A and is parallel to the x-axis. BV is a diameter of the circle.



4.1	Determine the coordinates of C.	(4)
4.2	Determine the equation of BV.	(3)
4.3	Determine the equation of line TA.	(1)
4.4	Calculate the length of TB. Give reason(s).	(4)
4.5	Determine the value of k.	(2)
4.6	Calculate the size of $A\hat{C}B$. Give reason(s).	(4)
		[18]

QUESTION 5

5.1 If $\cos 22° = p$; determine the following in terms of p:

5.1.1 $\cos 158°$ (2)

5.1.2 $\sin 112°$ (2)

5.1.3 $\sin 38°$ (4)

5.2 Determine all the values of P in the interval $[0°; 360°]$ which satisfy the equation: $\sin \text{P} = \sin 2\text{P}$. (4)

5.3 If ΔABC is a scalene triangle, show that: $\cos(A + B) = -\cos \text{C}$. (2)

5.4 Prove the following identity:

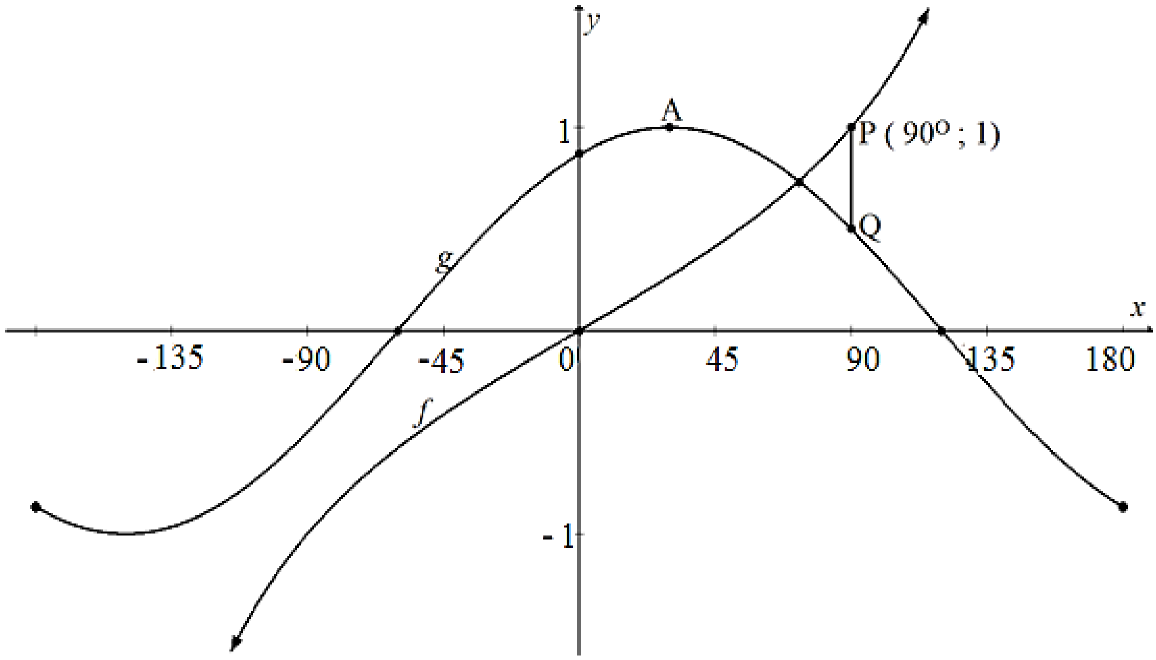
$$\frac{\cos^2 x - \cos x - \sin^2 x}{2\sin x \,.\, \cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x}$$ (5)

5.5 Determine the general solution of: $4 + 7\cos\theta + \cos 2\theta = 0$. (6)

[25]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same set of axes for $-180° \leq x \leq 180°$. The points P(90°;1) and Q lie on *f* and *g* respectively. Use the diagram to answer the following questions.



6.1 Determine the value of *b*. (1)

6.2 Write down the coordinates of A, the turning point of *g*. (2)

6.3 If PQ is parallel to the *y*-axis, determine the coordinates of Q. (2)

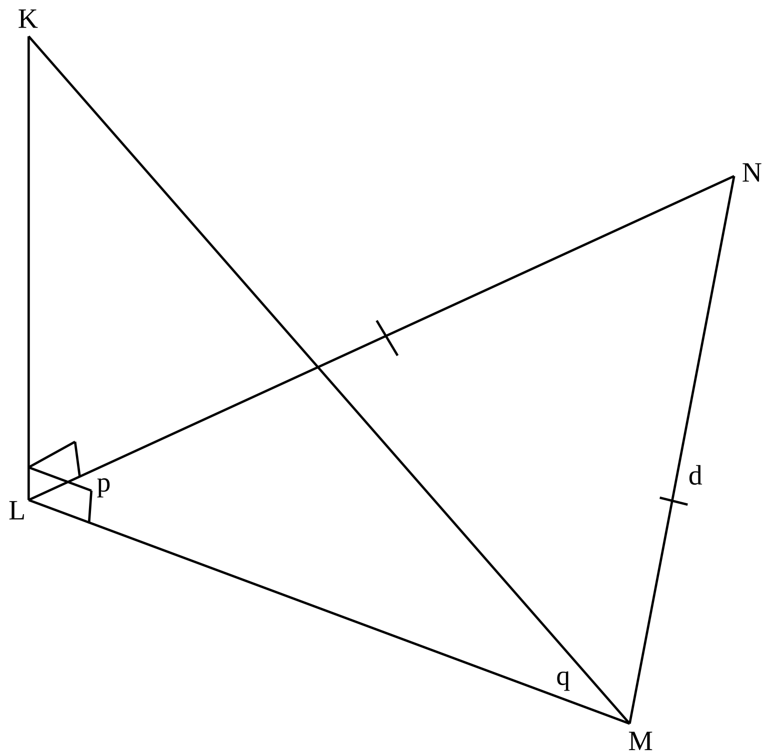
6.4 Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$. (1)

6.5 Determine the range of *h* if $h(x) = 2g(x) + 1$. (2)

[8]

QUESTION 7

Points L, M and N are in the same horizontal plane. KL is a vertical tower. The angle of elevation of K from M is $q°$. $N\hat{L}M = p°$; $NL = NM = d$ and $KL = h$.



7.1 Determine the size of $L\hat{N}M$ in terms of p. (2)

7.2 Prove that $LM = \dfrac{d \sin 2p}{\sin p}$. (2)

7.3 Hence, show that $h = 2d \cos p \tan q$. (3)

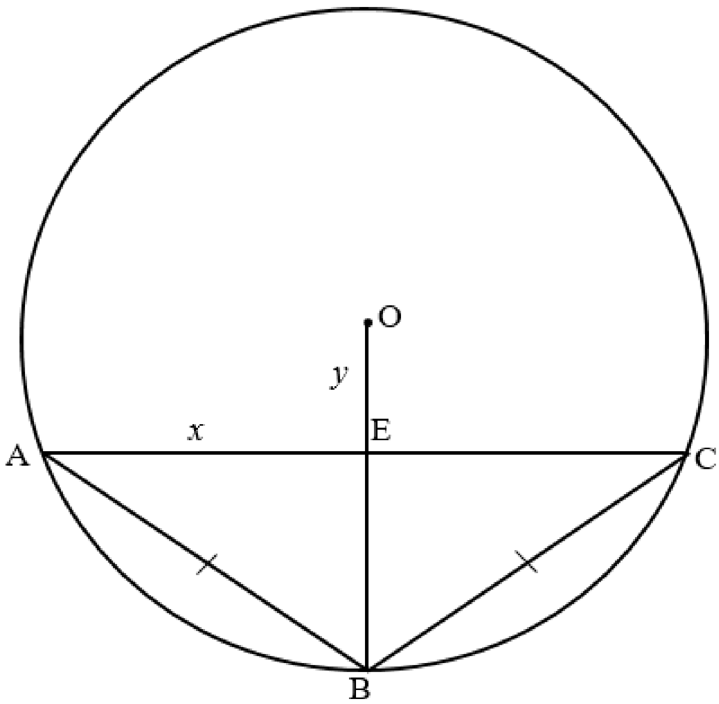
[7]

QUESTION 8

8.1 Complete the following theorem statement:

The line drawn from the centre of a circle perpendicular to a chord … (1)

8.2 In the diagram below, circle ABC with centre O is given. OB = 8 units and AB = BC = 10 units. E is the midpoint of AC. Let OE = *y* and AE = *x*.

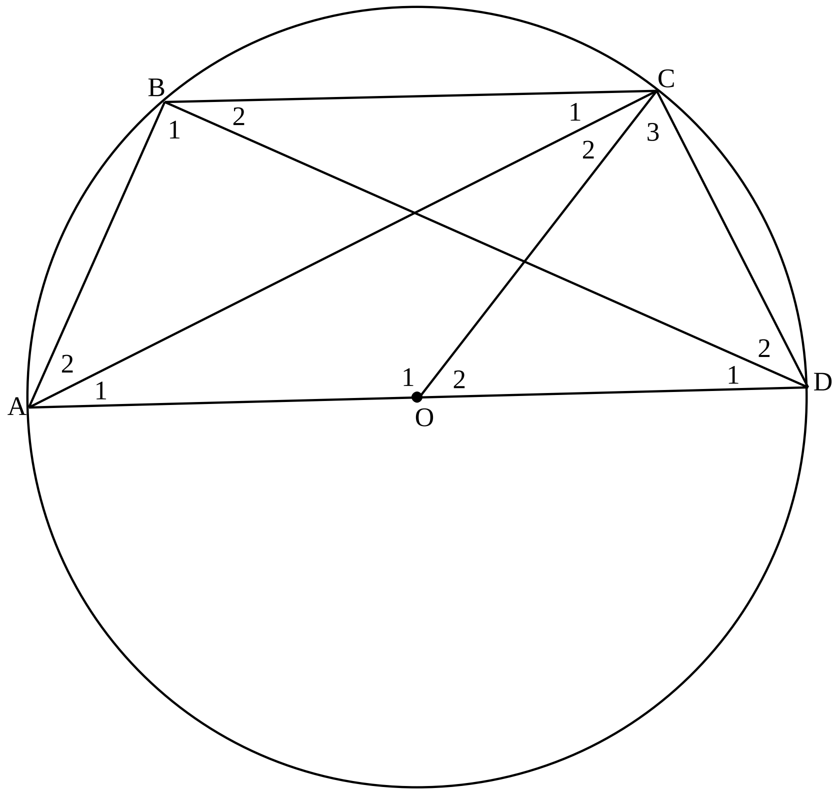


Calculate, with reasons, the length of OE. (5)

8.3 Complete the following theorem statement:

The angle subtended by an arc at the centre of a circle is … at the circle (on the same side of the chord as the centre). (1)

8.4 In the diagram, O is the centre of a circle ABCD. AOD is the diameter and OC is a radius. AB, BC, CD, AC and BD are straight lines.



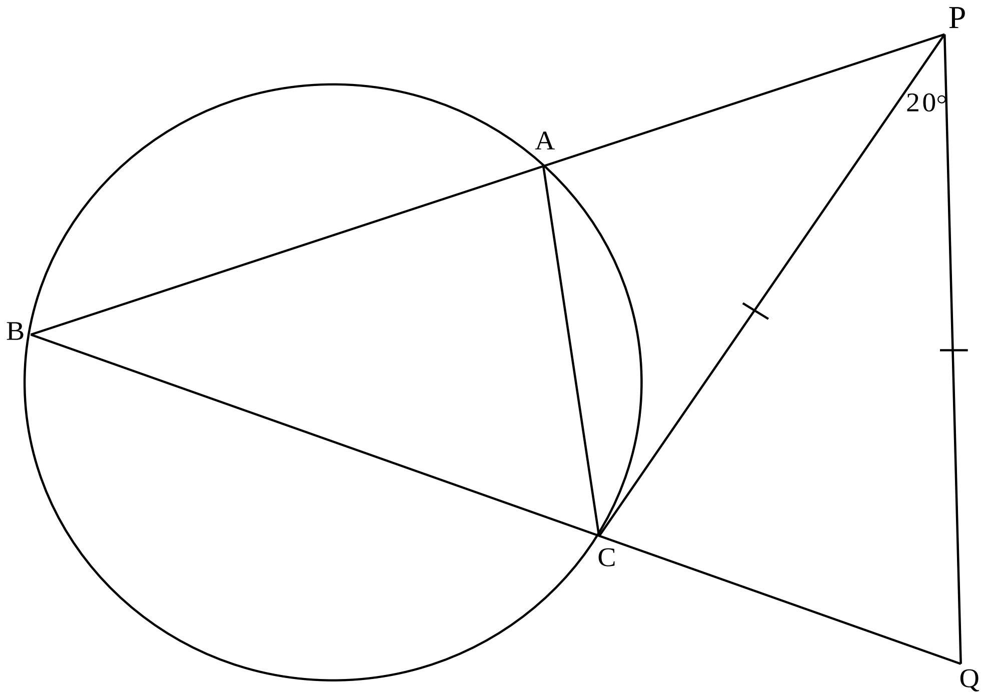
Write down, with reasons, an equation that expresses the relationship between each of the given groups of angles.

	ANGLES	EQUATION / RELATIONSHIP	REASON
e.g.	$\hat{M}_3; \hat{P}$	$\hat{M}_3 = 2 \times \hat{P}$	$\angle$ at centre = 2 x $\angle$ at circum.
8.4.1	$\hat{O}_2; \hat{B}_2$		
8.4.2	$\hat{D}_1; \hat{C}_3; \hat{D}_2$		
8.4.3	$\hat{B}_1; \hat{B}_2; \hat{D}_1; \hat{D}_2$		
8.4.4	$\hat{D}_1; \hat{C}_1$		

(8)
[15]

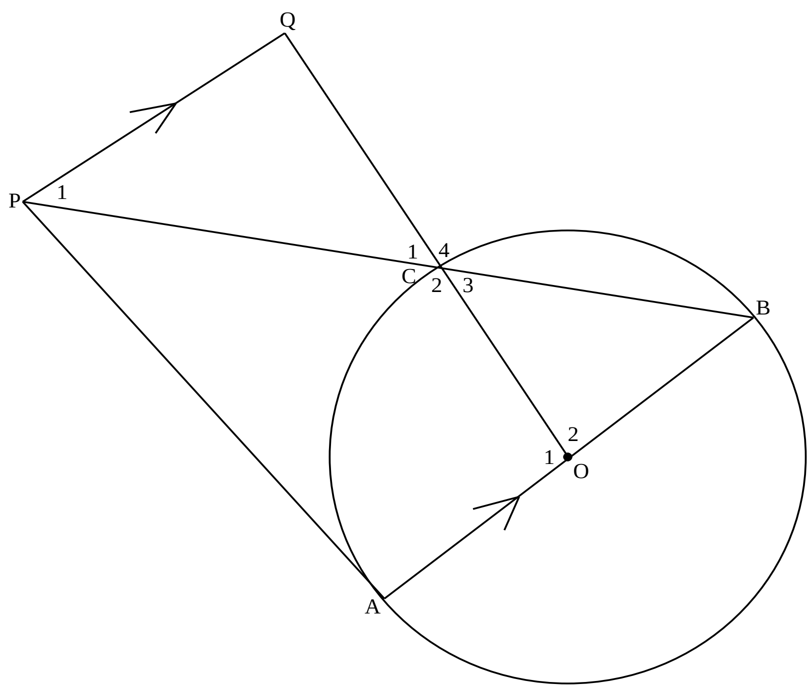
QUESTION 9

9.1 Given that PC is a tangent to the circle ACB; BAP and BCQ are straight lines. PC = PQ and $C\hat{P}Q = 20°$.



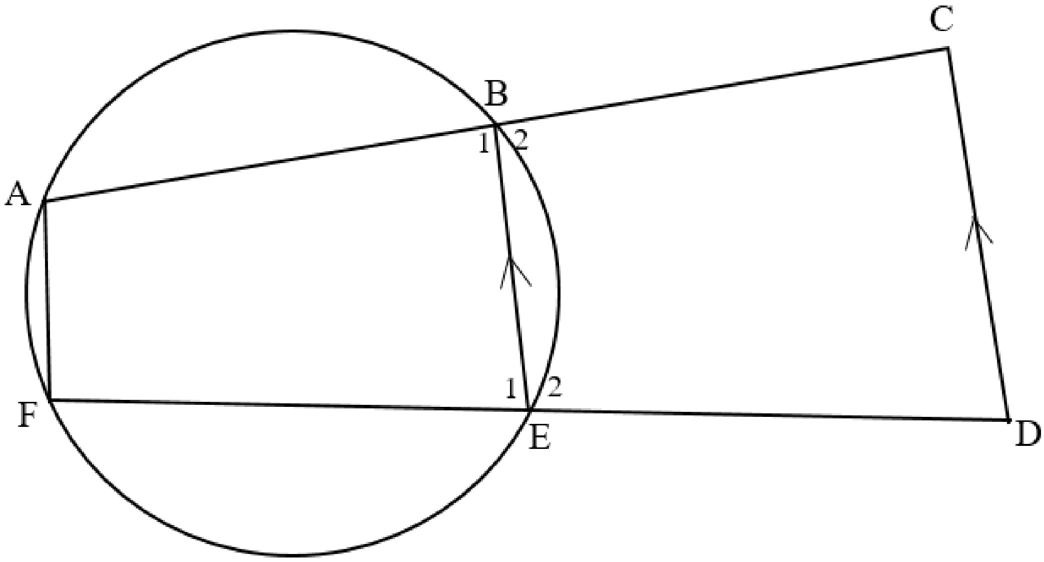
Prove, stating reasons, that BC is NOT a diameter. (5)

9.2 In the diagram below, O is the centre of circle ABC. The tangent PA to the circle and diameter AB meets at A. OCQ and BCP are straight lines. PQ∥AB.



Prove, stating reasons, that PQ = QC. (6)

9.3 In the diagram below, chords AB and FE of circles with centre O are produced to points C and D. BE∥CD.

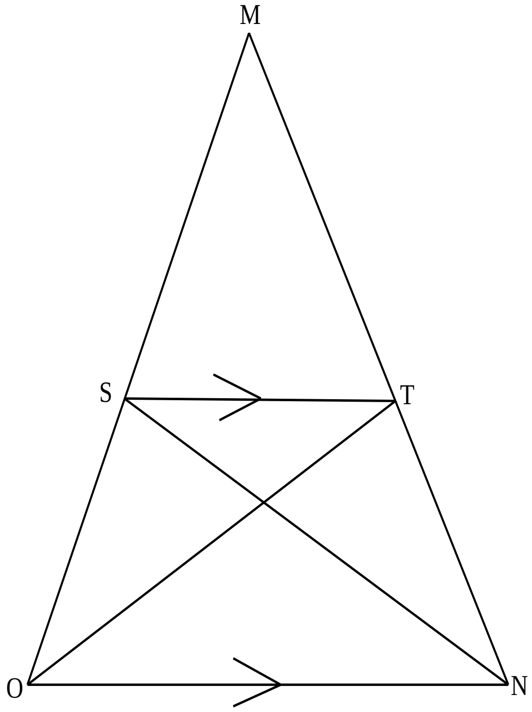


Prove that ACDF is a cyclic quadrilateral. (5)

[16]

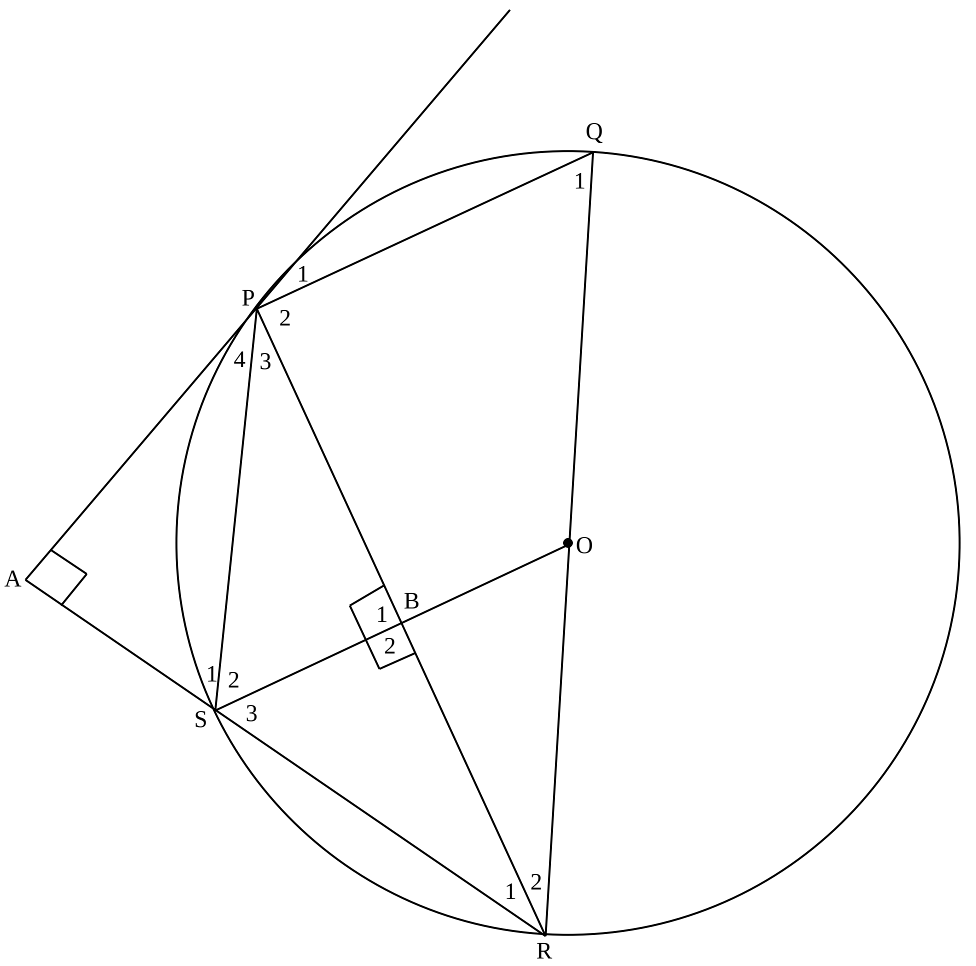
QUESTION 10

10.1 In the diagram, ΔMON is drawn. S is a point on MO and T is a point on MN such that ST$\parallel$ON. SN and OT are drawn.



Use the diagram to prove the theorem which states that a line parallel to one side of a triangle divides the other two sides proportionally. In other words, prove that: $\frac{MS}{SO}=\frac{MT}{TN}$. (5)

10.2 In the diagram, O is the centre of the circle. PQRS is a cyclic quadrilateral. The tangent through P intersects RS produced at A. OB⊥PR and PA⊥AS.



Prove that:

10.2.1 $\Delta APS \mathbin{|||} \Delta BRS$ (3)

10.2.2 AP . RS = BR . PS (1)

10.2.3 $\hat{P}_4 = \hat{R}_2$ (4)

10.2.4 BR . RQ = RS . RP (6)

[19]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$