KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

MARKING GUIDELINE

COMMON TEST

APRIL 2021

MARKS: 100

This memorandum consists of 9 pages.

QUESTION 1

1.1	100 ; 124	AA✓✓answers	(2)
1.2	44 52 64 80 1D 8 12 16 2D 4 4 $\boldsymbol{2a = 4 \quad \therefore a = 2}$ $\boldsymbol{3a + b = 8 \quad \therefore b = 2}$ $\boldsymbol{a + b + c = 44 \quad \therefore c = 40}$ $\boldsymbol{T_n = 2n^2 = 2n + 40}$ **OR** $\boldsymbol{2a = 4 \quad \therefore a = 2}$ $\boldsymbol{3a + b = 8 \quad \therefore b = 2}$ $\boldsymbol{\therefore c = T_0 = 40}$ $\boldsymbol{T_n = 2n^2 + 2n + 40}$ **OR** $\boldsymbol{T_n = T_1 + (n-1)d_1 + (n-1)(n-2)d_2}$ **OR** $\boldsymbol{T_n = \frac{(n-1)}{2}[2a + (n-2)d] + T_1}$	A✓a value CA✓b value CA✓c value CA✓nth term **OR** A✓a value CA✓b value CA✓c value CA✓nth term **OR** **OR**	(4) (4)
1.3	$\boldsymbol{T_{30} = 2(30)^2 + 2(30) + 40 = 1900}$	CA✓substitution CA✓answer	(2)
1.4	$\boldsymbol{T_n = 2n^2 + 2n + 40}$ $\boldsymbol{T_n = 2(n^2 + n + 20)}$ $\boldsymbol{2(n^2 + n + 20)}$ is even for all $\boldsymbol{n \in Z}$	A✓Taking out common factor of 2 A✓Rewriting nth term A✓ is even for all $\boldsymbol{n \in Z}$ **Note: Mark CA provided T_n (from1.2) is a factor of 2**	(3)
			[11]

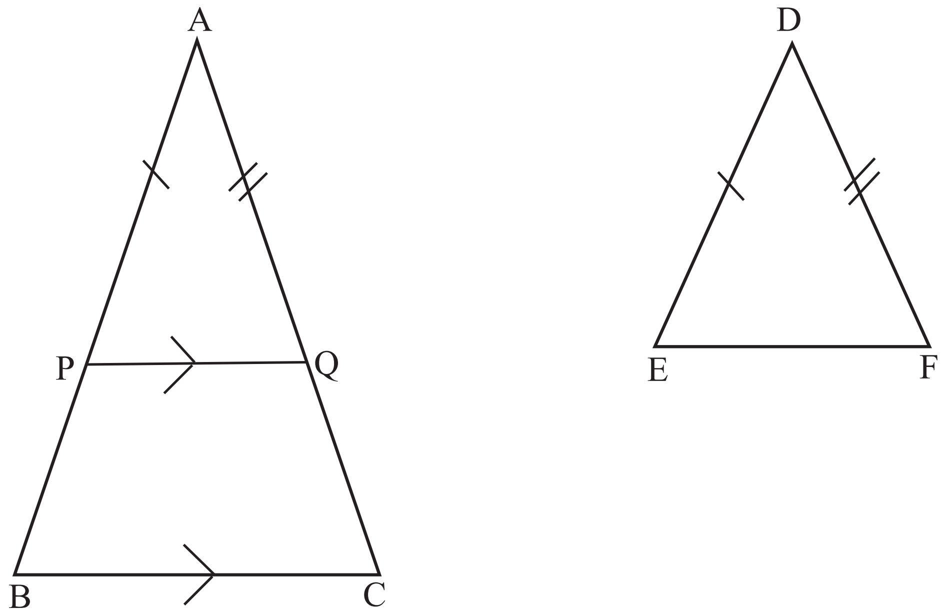
QUESTION 2

2.	$a+7d=31 \quad \rightarrow (1)$ $15(2a+29d)=1830$ $2a+29d=122 \quad \rightarrow (2)$ $a=31-7d \quad \rightarrow (3)$ $2(31-7d)+29d=122$ $62-14d+29d=122$ $15d=60$ $d=4$ $a=3$ $3\,;7\;;\;11\;;\;\ldots$	A✓equation (1) A✓equation (2) CA✓making a the subject CA✓correct substitution of a CA✓d value CA✓a value CA✓sequence	[7]

QUESTION 3

3.1	$ar=\frac{5}{128} \quad \rightarrow (1)$ $ar^8=5 \quad \rightarrow (2)$ $r^7=128$ $r^7=2^7$ $r=2$	A✓equation (1) A✓equation (2) CA✓ $r^7=128$ CA✓exponential form CA✓answer	(5)
3.2	$(-8)+(-8)(0.5)+(-8)(0.5)^2+\cdots$ $\frac{-8(0.5^m-1)}{0.5-1}=-\frac{255}{16}$ $0.5^m-1=-\frac{255}{256}$ $0.5^m=\frac{1}{256}=0.5^8$ $m=8$	A✓generating series CA✓correct substitution into correct formula CA✓writing in exponential form or using logs CA✓answer	(4)
3.3.1	$-1<r<1$ $-1<\frac{x}{2}<1$ $-2<x<2$	A✓condition for convergence A✓r value CA✓answer	(3)
3.3.2	$x<-2 \text{ or } x>2$	CACA✓✓answer	(2)
			[14]

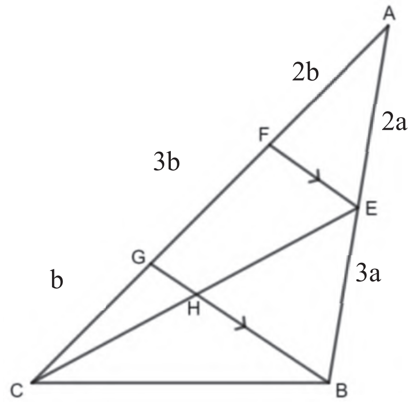
QUESTION 4

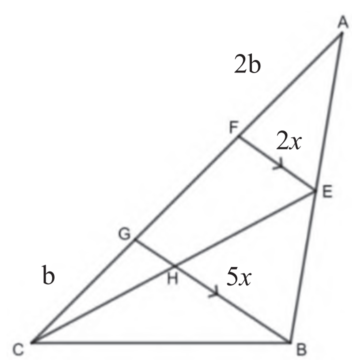


4.1	Draw AP = DE and AQ = DF In $\Delta\mathbf{ABC}$ **and** $\Delta\mathbf{DEF}$ **1.** AP = DE (Construction) **2.** AQ = DF (Construction) **3.** $\hat{\mathbf{A}} = \hat{\mathbf{D}}$ (Given) $\therefore \Delta\mathbf{APQ} \equiv \Delta\mathbf{DEF}$ (SAS) Now $\mathbf{A\hat{P}Q} = \mathbf{D\hat{E}F}$ But $\mathbf{D\hat{E}F} = \hat{\mathbf{B}}$ (Given) $\therefore \mathbf{A\hat{P}Q} = \hat{\mathbf{B}}$ PQ || BC (Corresponding angles =) $\frac{\mathbf{AB}}{\mathbf{AP}} = \frac{\mathbf{AC}}{\mathbf{AY}}$ (Prop. Thm. PQ||BC) $\frac{\mathbf{AB}}{\mathbf{DE}} = \frac{\mathbf{AC}}{\mathbf{DF}}$ (Construction AP = DE and AQ = DF)	✓S Construction (or could be shown on diagram) ✓S/R ✓S ✓S ✓S/R ✓S/R ✓R	(7)
4.2.1	In $\Delta\mathbf{DAH}$ **and** $\Delta\mathbf{OCH}$ 1. $D\hat{A}H = O\hat{C}H = 90°$ (Radius ⊥Tangent) 2. $\hat{\mathbf{H}}_2$ is common 3. $\mathbf{A\hat{D}H} = \mathbf{C\hat{O}}H$ (Remaining angles) $\therefore \Delta\mathbf{DAH} \parallel\!\mid \Delta\mathbf{OCH}$ (AAA)	✓ S ✓ R ✓S ✓R(AAA)	(4)

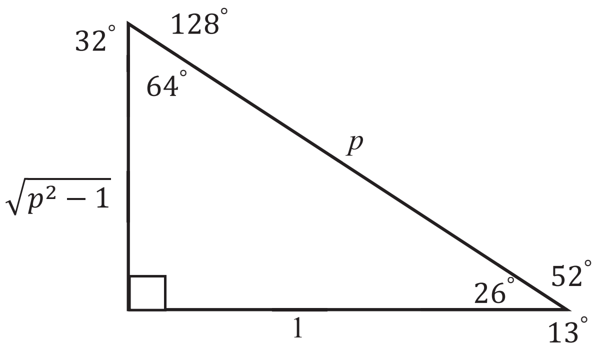
4.2.2	$\frac{\mathbf{DA}}{\mathbf{OC}} = \frac{\mathbf{DH}}{\mathbf{OH}} = \frac{\mathbf{AH}}{\mathbf{CH}}$ ($\Delta\mathbf{DAH} \mathbin{\vert\vert\vert} \Delta\mathbf{OCH}$) $\mathbf{OH} = \frac{\mathbf{DH} \times \mathbf{OC}}{\mathbf{DA}}$ DA = DC (Tangents drawn from common point equal) AO = OC (Radii of a circle) Therefore $\mathbf{OH} = \frac{\mathbf{AO.DH}}{\mathbf{DC}}$	✓S/R ✓S ✓S✓R ✓S✓R	(6)
4.2.3	In $\Delta\mathbf{ABF}$ **and** $\Delta\mathbf{BJF}$ 1. $\mathbf{B\hat{A}}$F = J$\hat{\mathbf{B}}$F (Tangent – Chord Theorem) 2. $\hat{\mathbf{F}}$ is common) 3. $\mathbf{A\hat{B}F = B\hat{J}F}$ (Remaining angles) ∴ $\Delta\mathbf{ABF} \mathbin{\vert\vert\vert} \Delta\mathbf{BJF}$ (AAA) ∴ $\frac{\mathbf{AB}}{\mathbf{BJ}} = \frac{\mathbf{BF}}{\mathbf{JF}} = \frac{\mathbf{AF}}{\mathbf{BF}}$ ($\Delta\mathbf{ABF} \mathbin{\vert\vert\vert} \Delta\mathbf{BJF}$) $\mathbf{BF^2 = JF\,.\,AF}$	✓S✓R ✓S ✓S✓R ✓S	(6)
			[23]

QUESTION 5



5.1	Let AE = 2a therefore EB = 3a $\frac{\text{AF}}{\text{FG}}=\frac{2}{3}$ (Prop. Thm.; FE//GB) or (Line // one side of Δ)	✓S ✓R	(2)
5.2	Let AF = 2b and FG = 3b Then CG = b (Given AF = 2CG) $\frac{\text{CH}}{\text{HE}}=\frac{\text{CG}}{\text{GF}}=\frac{b}{3b}$ (Prop. Thm.; GH//FE) or (Line // one side of Δ) $\therefore \frac{\textbf{CH}}{\textbf{HE}}=\frac{\mathbf{1}}{\mathbf{3}}$	✓S ✓S ✓R ✓S	(4)
5.3	$\frac{\text{AE}}{\text{AB}}=\frac{\text{AF}}{\text{AG}}=\frac{\text{FE}}{\text{GB}}=\frac{2}{5}$ …. (Prop. Thm; FE ‖ GB) or (Line // one side of Δ) $\mathbf{C\hat{G}B = G\hat{F}E}$ ….. (Corresp Angles ; FE ‖ GB) Let FE = $2x$ and GB = $5x$ Then $\frac{\textbf{Area of } \Delta \textbf{ BCG}}{\textbf{Area of } \Delta \textbf{ AFE}}=\frac{\frac{1}{2}(b)(5x)\sin \text{C}\hat{\text{G}}\text{B}}{\frac{1}{2}(2b)(2x)\sin \text{A}\hat{\text{F}}\text{E}}$ $=\frac{\frac{1}{2}(b)(5x)\sin \text{C}\hat{\text{G}}\text{B}}{\frac{1}{2}(2b)(2x)\sin\left(180^\circ-\text{C}\hat{\text{G}}\text{B}\right)}$ $=\frac{5}{4}$  **OR** **Area of Δ BCG** $=\frac{1}{6}$ **Area of ΔABC** …(Equal Heights) **Area of Δ AFE** $=\frac{1}{3}$ **Area of ΔAEC** …(Equal Heights) **Area of Δ AEC** $=\frac{2}{5}$ **Area of ΔABC** …(Equal Heights) **Area of Δ AFE** $=\frac{2}{15}$ **Area of ΔABC** $\frac{\textbf{Area of } \Delta \textbf{ BCG}}{\textbf{Area of } \Delta \textbf{ AFE}}=\frac{15}{12}=\frac{5}{4}$	✓S/R ✓S ✓S ✓S **OR** ✓S/R ✓S/R ✓S ✓S	(4) (4)
			[10]

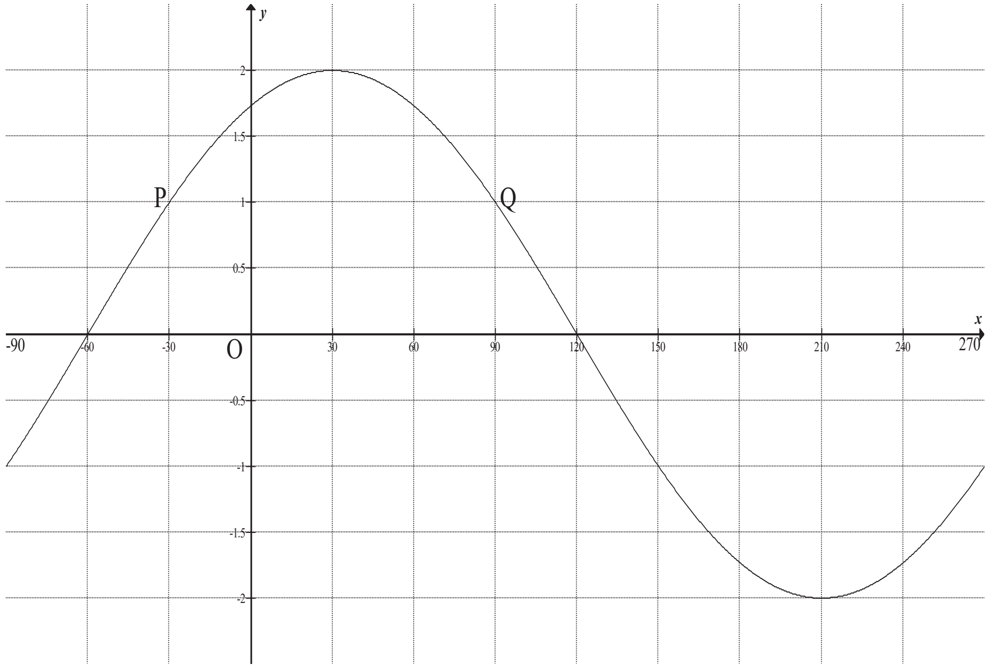
QUESTION 6

6.1.1	 $\cos 52^\circ = \cos[2(26^\circ)]$ $= 2\cos^2 26^\circ - 1$ $= 2(\frac{1}{p})^2 - 1$	A✓diagram A✓writing as double angle A✓expansion CA✓answer	(4)
6.1.2	$\sin 71^\circ = \sin(45^\circ + 26^\circ)$ $= \sin 45^\circ \cos 26^\circ + \cos 45^\circ \sin 26^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{1}{p} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{p^2-1}}{p}$	A✓$\sin(45^\circ + 26^\circ)$ A✓compound angle expansion CA CA ✓✓each term	(4)
6.2	$\frac{\cos(-180^\circ).\tan\theta.\cos 690^\circ.\sin(\theta - 180^\circ)}{\cos^2(\theta - 90^\circ)}$ $= \frac{\cos(180^\circ) \times \frac{\sin\theta}{\cos\theta}.\cos 30^\circ.(-\sin\theta)}{\sin^2\theta}$ $= \frac{(-1) \times \frac{\sin\theta}{\cos\theta}.\left(\frac{\sqrt{3}}{2}\right)(-\sin\theta)}{\sin^2\theta}$ $= \frac{\frac{\sqrt{3}\sin^2\theta}{2\cos\theta}}{\sin^2\theta}$ $= \frac{\sqrt{3}}{2\cos\theta}$	A✓$\frac{\sin\theta}{\cos\theta}$ A✓$\cos 30^\circ$. A✓$-\sin\theta$ CA✓$\frac{\sqrt{3}}{2}$ or 0,866 CA✓answer	(5)
6.3	LHS = $\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \cdots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ + 6\sin 90^\circ$ $= \cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \ldots - \cos 2^\circ - \cos 1^\circ - \cos 0^\circ + 6\sin 90^\circ$ $= 6 =$ RHS **OR** LHS = $(\cos 0^\circ + \cos 180^\circ) + (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ) \ldots\ldots\ldots + 6\sin 90^\circ$ LHS = $(0) + (0) + (0) \ldots\ldots\ldots + 6\sin 90^\circ$ LHS = 6	A✓$-\cos 2^\circ$ A✓$-\cos 1^\circ$ A✓$-\cos 0^\circ$ A✓All terms cancel except 6 A✓$(\cos 0^\circ + \cos 180^\circ)$ A✓$(\cos 1^\circ + \cos 179^\circ)$ A✓$(\cos 2^\circ + \cos 178^\circ$ A✓All terms cancel except 6	(4)
			[17]

QUESTION 7

7.1	$\frac{1-\sin 2x}{\sin x-\cos x}$ $= \frac{\sin^2 x-2\sin x\cos x+\cos^2 x}{\sin x-\cos x}$ $= \frac{(\sin x-\cos x)^2}{\sin x-\cos x}$ $= \sin x-\cos x$ $=$ RHS	A✓ $\sin^2 x+\cos^2 x=1$ A✓ $2\sin x\cos x$ A✓ $(\sin x-\cos x)^2$	(3)
7.2	$\tan 3x \cdot \frac{1}{\tan 24^\circ}-1=0$ $\tan 3x=\tan 24^\circ$ $3x=24^\circ+180k\,;k\in Z$ $x=8^\circ+60k\,;k\in Z$	A✓transposing and forming equation A✓$3x=24^\circ$ A✓$180k$ A✓ $k\in Z$ CA✓ $x=8^\circ+60k$ **Note: If calculator is used maximum marks may be allocated**	(5)
7.3	$2\left[\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right]$ $= 2[\sin 60^\circ\sin x+\cos 60^\circ\cos x]$ $= 2[\cos(x-60^\circ)]$ $\therefore$ Maximum value is 2, since maximum value of $\cos(x-60^\circ)=1$ **OR** $2\left[\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right]$ $= 2[\cos 30^\circ\sin x+\sin 30^\circ\cos x]$ $= 2[\sin(x+30^\circ)]$ $\therefore$ Maximum value is 2, since maximum value of $\sin(x+30^\circ)=1$	A✓ $2\left[\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right]$ A✓ $[\sin 60^\circ\sin x+\cos 60^\circ\cos x]$ A✓ $\cos(x-60^\circ)$ CA✓ answer **OR** A✓ $2\left[\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x\right]$ A✓ $2[\cos 30^\circ\sin x+\sin 30^\circ\cos x]$ A✓ $\sin(x+30^\circ)$ CA✓ answer	(4) (4)

7.4.1



		A✓for both x – intercepts A✓for both turning points	(2)
7.4.2	$\cos(x-30^\circ)=0,5$ $2\cos(x-30^\circ)=1$ $x-30^\circ=60^\circ \quad or \quad x-30^\circ=-60^\circ$ $x=90^\circ \ at\ Q \quad or \quad x=-30^\circ \ at\ P$	A✓$x-30^\circ=60^\circ$ and $x-30^\circ=-60^\circ$ CA✓ 90° and -30° CACA✓✓for P and Q	(4)
			[18]