KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

APRIL 2021

MARKS: 100

TIME: 2 hours

N.B. This question paper consists of 6 pages, an answer sheet, 1 diagram sheet and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: $44;\ 52;\ 64;\ 80;\ \ldots$

1.1 Write down the next two terms of the sequence. (2)

1.2 Determine the n^{th} term of the quadratic sequence. (4)

1.3 Calculate the 30^{th} term of the sequence. (2)

1.4 Prove that the quadratic sequence will always have even terms. (3)

[11]

QUESTION 2

The 8^{th} term of an arithmetic sequence is 31 and the sum of the first 30 terms is 1830. Determine the first three terms of the sequence. **[7]**

QUESTION 3

3.1 The second term of a geometric sequence $\frac{5}{128}$ and the ninth term is 5. Determine the value of the common ratio. (5)

3.2 Calculate the value of m if

$$\sum_{k=1}^{m}(-8).(0.5)^{k-1} = -\frac{255}{16}$$

(4)

3.3 Given: $\frac{24}{x} + 12 + 6x + 3x^2 + \cdots\ ;\ x \neq 0.$

3.3.1 Determine the value of x for which the series converges. (3)

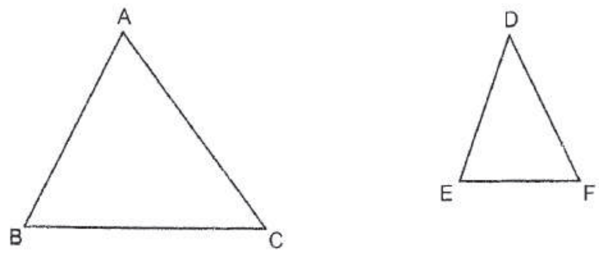
3.3.2 Write down the value of x for which the series is increasing. (2)

[14]

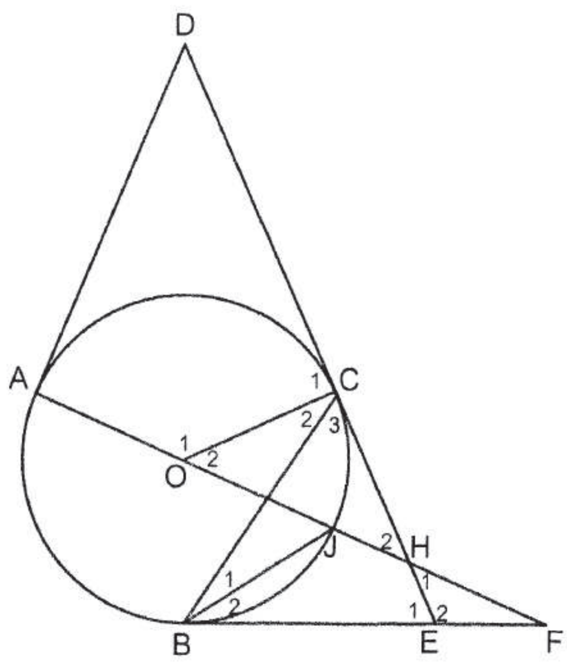
QUESTION 4

4.1 Given ΔABC and ΔDEF with $\hat{A} = \hat{D}, \hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove that $\frac{AB}{DE} = \frac{AC}{DF}$ (7)



4.2 In the figure AD, DC and BE are tangents to the circle at A, C and B respectively. O is the centre of the circle. DE and AF intersect at H. AH produced meets BE produced in F. AJ, BC and BJ are chords.



Prove that:

4.2.1 ΔDAH ||| ΔOCH. (4)

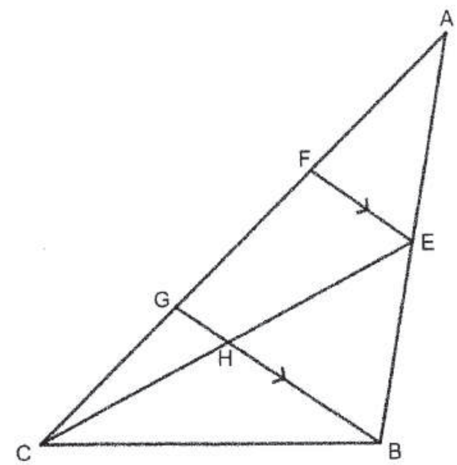
4.2.2 $OH = \frac{AO.DH}{DC}$ (6)

4.2.3 If BA is drawn, then $BF^2 = JF.AF$ (6)

[23]

QUESTION 5

In the figure AF = 2CG and FE || GB. $\frac{AE}{AB} = \frac{2}{5}$.



Determine (with reasons):

5.1 $\frac{AF}{FG}$ (2)

5.2 $\frac{CH}{HE}$ (4)

5.3 $\frac{Area\ of\ \Delta BCG}{Area\ of\ \Delta AFE}$ (4)

[10]

QUESTION 6

6.1 Given $\cos 26^\circ = \frac{1}{p}$

Without using a calculator, calculate the value of the following in terms of p.

6.1.1 $\cos 52^\circ$ (4)

6.1.2 $\sin 71^\circ$ (4)

6.2 Simplify without using into a single trigonometric ratio.

$$\frac{\cos(-180^\circ).\tan\theta.\cos 690^\circ.\sin(\theta-180^\circ)}{\cos^2(\theta-90^\circ)}$$ (5)

6.3 Show that

$\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \cdots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ + 6\sin 90^\circ = 6$ (4)

[17]

QUESTION 7

7.1 Prove the following identity:

$$\frac{1 - \sin 2x}{\sin x - \cos x} = \sin x - \cos x \qquad (3)$$

7.2 Determine the general solution of:

$$\tan 3x \,.\, \frac{1}{\tan 24^\circ} - 1 = 0 \qquad (5)$$

7.3 Determine the maximum value of $\sqrt{3}\,\sin x + \cos x$, without the use of a calculator. (4)

7.4 Given: $f(x) = 2\cos(x - 30^\circ)$

7.3.1 Sketch the graph of f for the domain $x \in [-90^\circ; 270^\circ]$ on the axes provided. (2)

7.3.2 Use the letters P and Q to indicate on the graph the solution of the equation $\cos(x - 30^\circ) = 0{,}5$ and the x – coordinates of P and Q. (4)

[18]

NAME: ________________________ **GRADE:** __________

ANSWER SHEET

Question 7.3.1

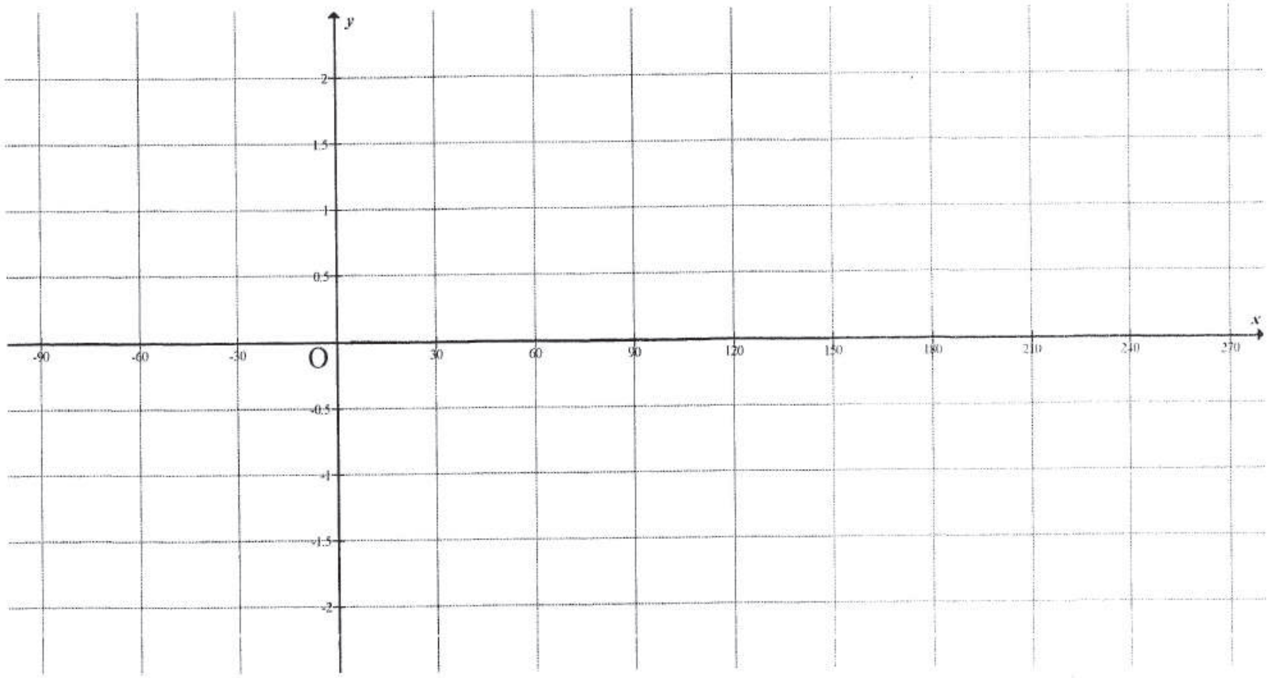


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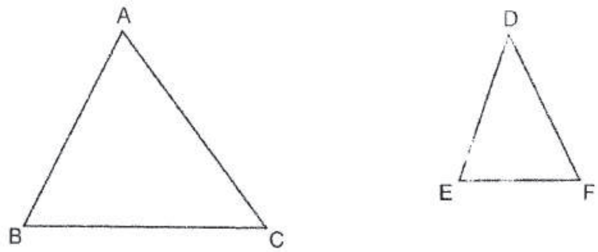
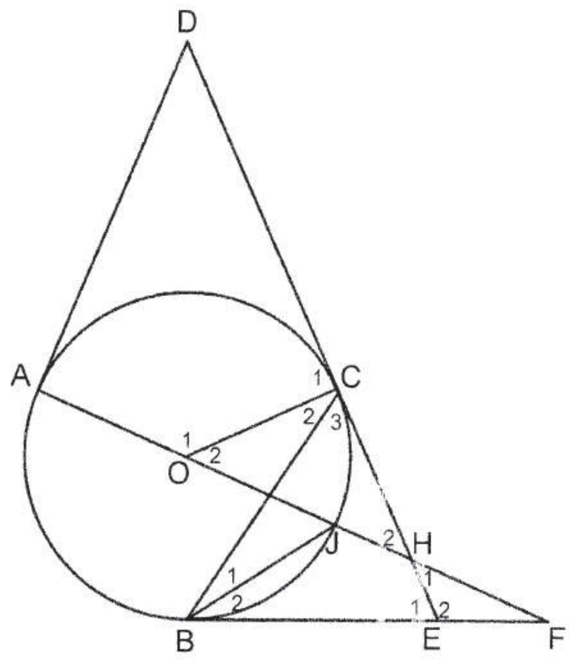
DIAGRAM SHEET

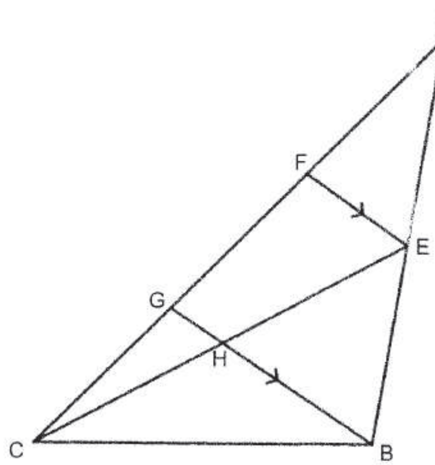
QUESTION 4.1



QUESTION 4.2



QUESTION 5



INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum f.x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$