EASTERN CAPE
Department of Education

OR TAMBO INLAND DISTRICT

GRADE 12

MATHEMATICS CONTROLLED TEST

TERM 1 2021

Date: 16 April 2021

Marks: 50

Time: 1 hour

This question paper consists of 5 pages including cover page

INSTRUCTIONS

1. This question paper consists of **THREE** questions. Answer all the questions.
2. Clearly show all calculations you have used in determining your answers.
3. Write neatly and legibly.
4. Give all your answers to **TWO** decimal places, except stated otherwise.
5. Diagrams are NOT necessarily drawn to scale.

QUESTION 1

1.1 The following geometric sequence is given: $10 ; 5 ; 2{,}5 ; 1{,}25 ;...$

1.1.1 Calculate the value of the 5th term, T_5 , of this sequence (1)

1.1.2 Determine the n^{th} term, T_n, in terms of n. (2)

1.1.3 Explain why the infinite series $10+5+2{,}5+1{,}25+...$ exists (2)

1.2 The sum of the first p terms of a sequence of numbers is given by:

$$S_p = p(p+1)(p+2)$$

Calculate the value of T_{20} (3)

1.3 Evaluate: $\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^k - \sum_{k=2}^{40} 5$ (5)

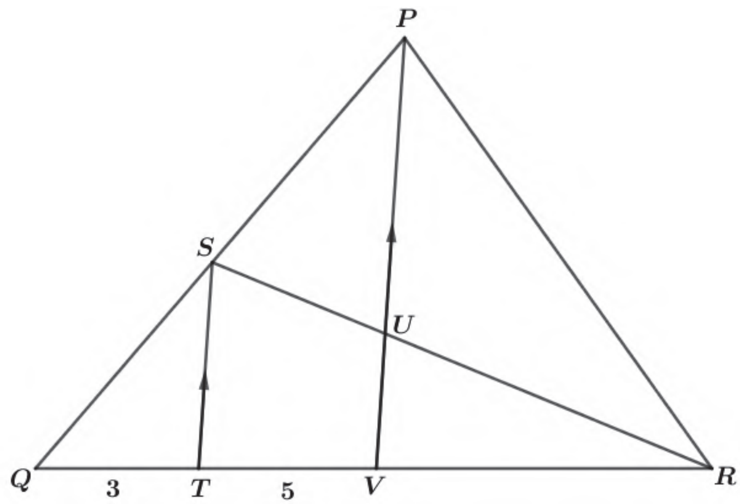
1.4 Atlehang generates a sequence, which he claims that the sequence can be arithmetic and also be geometric if and only if the first term in the sequence 1.

Is he correct? Motivate your answer by showing all your calculations (5)

(18)

QUESTION 2

2.1 In the diagram below, ΔPQR is given such that QT = 3 units, TV = 5 units, TR = 8 units and ST || PV



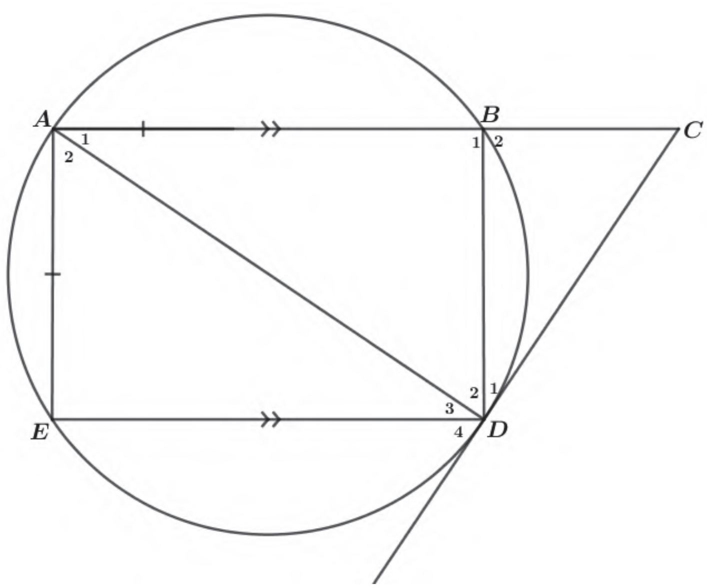
Determine the following with reasons:

2.1.1 $\frac{QS}{PS}$ (2)

2.1.2 $\frac{\text{Area } \Delta PQR}{\text{Area of quadrilateral } PSTR}$ (5)

2.2 In the diagram, a circle with a tangent CD is drawn. A, B, D, and E are points on the circumference of the circle. AE = AB and AB || ED.

$\hat{A}_1 = x$



2.2.1 Give, with reasons, three more angles equal to x. (3)

2.2.2 Prove that $\Delta DEA \parallel\!\mid \Delta DBC$ (2)

2.2.3 Prove that:

(a) $DC = \dfrac{BC \times AD}{DB}$ (3)

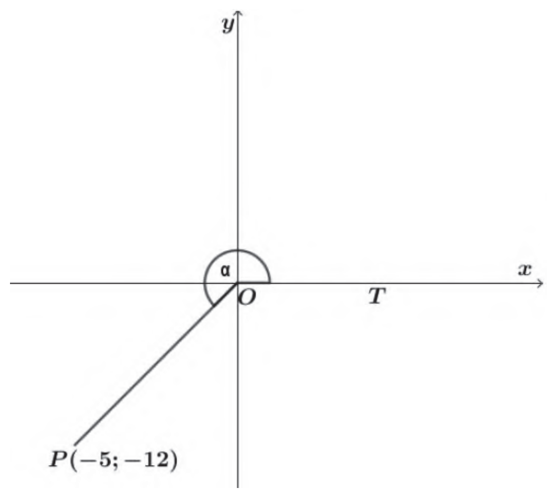
(b) $EA = \dfrac{DC \times DE}{AD}$ (3)

2.2.4 Hence, prove that $AE^2 = BC.ED$ (2)

(20)

QUESTION 3

In the diagram below, reflex $T\hat{O}P = \alpha$ and P has coordinates $(-5;-12)$.



3.1 Determine the value of each of the following trigonometric ratios WITHOUT using a calculator:

3.1.1 $\cos\alpha$ (1)

3.1.2 $\tan(180^\circ - \alpha)$ (2)

3.2 Simplify the following expression into a single trigonometric ratio:

$$\frac{\sin(x-180^\circ).\cos(x-90^\circ)}{\cos(-x-360^\circ).\sin(90^\circ+x)}$$ (6)

3.3 Given: $\cos(A-B) = cosAcosB + sinAsinB$

Use the identity for $\cos(A-B)$ to derive an identity for $\sin(A+B)$ (3)

(12)

TOTAL [50]

INFORMATION SHEET: MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$