

SA's Leading Past Year

Exam Paper Portal



You have Downloaded, yet Another Great
Resource to assist you with your Studies 😊

Thank You for Supporting SA Exam Papers

Your Leading Past Year Exam Paper Resource Portal

Visit us @ www.saexampapers.co.za



SA EXAM
PAPERS

**NATIONAL
SENIOR CERTIFICATE /
NASIONALE
SENIOR SERTIKAAT**

GRADE/GRAAD 12

JUNE/JUNIE 2021

**TECHNICAL MATHEMATICS P1/TEGNIJSE WISKUNDE V1
MARKING GUIDELINE/NASIENRIGLYN
(EXEMPLAR/EKSEMPLAAR)**

MARKS/PUNTE: 150

MARKING CODES/NASIENKODES	
A	Accuracy/Akkuraatheid
AO	Answer only/Slegs antwoord
CA	Consistent accuracy/Volgehoue akkuraatheid
M	Method/Metode
R	Rounding/Afronding
NPR	No penalty for rounding/Geen penalisering vir afronding nie
NPU	No penalty for units omitted/Geen penalisering vir eenhede weggelaat nie
S	Simplification/Vereenvoudiging
F	Correct formula/Korrekte formule
SF	Substitution in correct formula/Vervanging in korrekte formule

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

QUESTION/ VRAAG 1				
1.1	1.1.1	$x^2 - 8x - 33 = 0$ $(x-11)(x+3) = 0$ or / of $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)}$ $\therefore x = 11$ or / of $x = -3$	✓ Factors/ <i>Faktore</i> SF A ✓ $x = 11$ ✓ $x = -3$ CA	(3)
	1.1.2	$x^2 - 7x = 10(-3x - 1)$ $x^2 - 7x = -30x - 10$ $x^2 + 23x + 10 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(23) \pm \sqrt{(23)^2 - 4(1)(10)}}{2(1)}$ $x = \frac{-23 \pm \sqrt{489}}{2}$ $x = -0,44$ or / of $x = -22,56$	✓ S A ✓ SF CA ✓ both values of x / <i>beide waardes van x</i> CA	(3)
	1.1.3	$-2x^2 + 9x + 5 < 0$ $(-2x-1)(x-5) < 0$ C.V / K.W : $-\frac{1}{2}$ and / en 5 Solution / <i>oplossing</i> : $x < -\frac{1}{2}$ or / of $x > 5$	✓ Factors/ <i>Faktore</i> SF A ✓ Critical Values / <i>Kritiese waardes</i> CA ✓ $x < -\frac{1}{2}$ CA ✓ $x > 5$ CA	(4)

1.2	$P = 2(l + w)$ $90 = 2(1 + x)$ $45 - x = 1$ $41^2 = x^2 + (45 - x)^2$ $1681 = x^2 + 2025 - 90x + x^2$ $0 = 2x^2 - 90x + 344$ $0 = x^2 - 45x + 172$ $x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(1)(172)}}{2(1)}$ $x = 40,78 \text{ or/of } 4,22$ $\therefore \text{width / wydte is } 4,22 \text{ cm}$ OR/OF $l = \sqrt{41^2 - x^2} \quad \text{from/vanaf Pythagoras}$ $90^2 = 2\left(x + \sqrt{41^2 - x^2}\right)$ $45 = x + \sqrt{1681 - x^2}$ $45 - x = \sqrt{1681 - x^2}$ $(45 - x)^2 = 1681 - x^2$ $2025 - 90x + x^2 = 1681 - x^2$ $0 = 2x^2 - 90x + 344$ $0 = x^2 - 45x + 172$ $x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(1)(172)}}{2(1)}$ $x = 40,78 \text{ or/of } 4,22$ $\therefore \text{width/wydte is } 4,22 \text{ cm}$	$\checkmark$ length in terms of x / <i>lengte in terme van x</i> A $\checkmark$ M (Pyth.) CA $\checkmark$ S CA $\checkmark$ S CA $\checkmark$ SF CA $\checkmark$ width / <i>wydte</i> CA OR / OF $\checkmark$ length in terms of x / <i>lengte in terme van x</i> A $\checkmark$ M (Pyth.) CA $\checkmark$ S CA $\checkmark$ S CA $\checkmark$ SF CA $\checkmark$ width / <i>wydte</i> CA	(6)
1.3	$x = y + 3 \quad \text{and / en} \quad y - x^2 = -2x - 3$ substitute / <i>vervang</i> x into / <i>in</i> $y - x^2 = -2x - 3$ $y - (y + 3)^2 = -2(y + 3) - 3$ $y - y^2 - 6y - 9 = -2y - 6 - 3$ $-y^2 - 3y = 0$ $(-y)(y + 3) = 0$ $y = 0 \text{ or / of } y = -3$ $x = 0 \text{ or / of } x = 3$	$\checkmark$ Substitution/ <i>Vervanging</i> A $\checkmark$ S CA $\checkmark$ Factors/ <i>Faktore</i> SF CA $\checkmark$ Both y -values// <i>beide y-waardes</i> CA $\checkmark$ Both x -values / <i>beide x-waardes</i> CA	

		OR/OF $x = y + 3$ and / en $y - x^2 = -2x - 3$ $x - 3 = y$ and / en $y = x^2 - 2x - 3$ equate / gelykstelling y: $x - 3 = x^2 - 2x - 3$ $0 = x^2 - 3x$ $0 = x(x - 3)$ $x = 0$ or / of $x = 3$ $y = 0$ or / of $y = -3$	OR/OF ✓ equating/gelykstelling y A ✓ S CA ✓ Factors/Faktore CA ✓ x-values/waardes CA ✓ y-values/waardes CA	(5)
1.4	1.4.1	$K = 8 + 32 + 1 = 41$	✓ value of/waarde van K A	(1)
	1.4.2	$41 = 101001_2$	✓ 101001_2 CA From/vanaf 1.4.1	(1)
				[23]

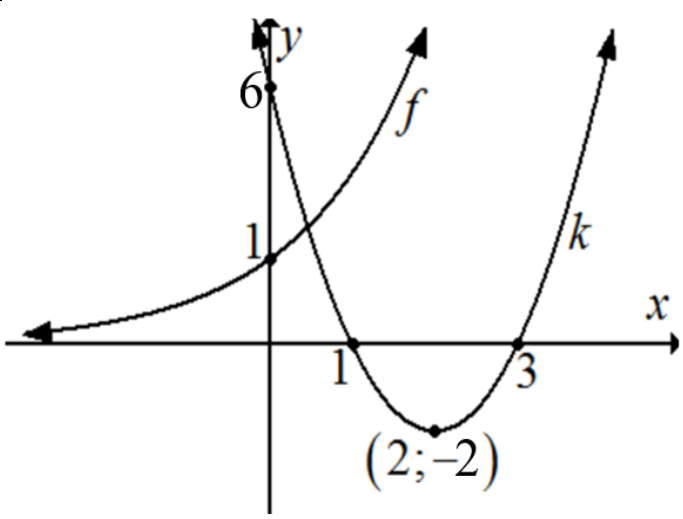
QUESTION/VRAAG 2			
2.1	$\Delta = b^2 - 4ac < 0$ $4 + 8m < 0$ $m < -\frac{1}{2}$	✓ Discriminant / <i>diskriminant</i> < 0 A ✓ S CA	(2)
2.2	$\Delta = b^2 - 4ac$ $= (-b)^2 - 4(a)\left(-\frac{1}{a}\right)$ $= 0 + 4 = 4$ Roots are real, rational and unequal <i>Wortels is reël, rasionaal en ongelyk</i>	✓ SF A ✓ S CA ✓ Real and unequal / <i>Reël en ongelyk</i> CA ✓ rational roots / <i>rasionale wortels</i> CA	(4)
			[6]

QUESTION/VRAAG 3				
3.1	3.1.1	$81^{\frac{3}{4}} - 8^{\frac{2}{3}} + \sqrt[4]{1296}$ $= 3^{4 \times \frac{3}{4}} - 2^{3 \times \frac{2}{3}} + 6^{4 \times \frac{1}{4}} \quad \text{or/of} \quad 3^{4 \times \frac{3}{4}} - 2^{3 \times \frac{2}{3}} + (2 \times 3)^{4 \times \frac{1}{4}}$ $= 27 - 4 + 6 = 29$	✓ Exponential form / <i>Eksponeensiële vorm</i> A ✓ S CA	(2)
	3.1.2	$\log \sqrt{65} + \log \sqrt{260} - \log 13$ $= \log \frac{(\sqrt{65} \times \sqrt{260})}{13}$ $= \log \frac{(\sqrt{5 \times 13 \times 5 \times 4 \times 13})}{13}$ $= \log \frac{(5^2 \cdot 13^2 \cdot 2^2)^{\frac{1}{2}}}{13}$ $= \log \frac{(5 \cdot 13 \cdot 2)}{13}$ $= \log 10 = 1$ OR/OF $\log \sqrt{65} + \log \sqrt{260} - \log 13$ $= \log \sqrt{13 \times 5} + \log \sqrt{13 \times 5 \times 2} - \log 13$ $= \log \sqrt{13} + \log \sqrt{5} + \log \sqrt{13} + \log \sqrt{5} + \log \sqrt{2} - \log 13$ $= 2 \log \sqrt{13} + 2 \log \sqrt{5} + \log 2 - \log 13$ $= \log 13 + \log 5 + \log 2 - \log 13$ $= \log(5 \times 2)$ $= \log 10 = 1$	✓ Log property / <i>eienskap</i> A ✓ Exponential form / <i>Eksponeensiële vorm</i> CA ✓ S CA ✓ Log property/ <i>eienskap</i> CA OR/OF ✓ Log property/ <i>eienskap</i> A ✓ S CA ✓ Log property/ <i>eienskap</i> CA ✓ Log property/ <i>eienskap</i> CA	(4)
3.2		$\frac{-2(\log 25 - \log 4)}{\log 2 - \log 5} = 4$ $\text{RHS} = \frac{-2(\log 5^2 - \log 2^2)}{\log 2 - \log 5}$ $= \frac{-2 \times 2(\log 5 - \log 2)}{-(\log 5 - \log 2)}$ $= 4 = \text{LHS}$	✓ Exponential form/ <i>Eksponeensiële vorm</i> A ✓ Log property/ <i>eienskap</i> CA ✓ Factors/ <i>Faktore</i> CA	(3)

3.3	3.3.1	$64x^{\frac{3}{2}} = 27x^{\frac{-3}{2}}$ $\frac{x^{\frac{3}{2}}}{x^{\frac{-3}{2}}} = \frac{27}{64}$ $x^3 = \left(\frac{3}{4}\right)^3$ $x = \frac{3}{4}$	✓M A ✓Exponential property/ Eksponensiële eienskap CA ✓Exponential property/ Eksponensiële eienskap CA	(3)
	3.3.2	$\log_3(x-3) - \log_3 5 = 1$ $\log_3 \frac{(x-3)}{5} = \log_3 3$ $\frac{(x-3)}{5} = 3$ $x-3 = 15$ $x = 18$ OR/OF $\log_3(x-3) - \log_3 5 = 1$ $\log_3(x-3) = \log_3 3 + \log_3 5$ $\log_3(x-3) = \log_3(3 \times 5)$ $x-3 = 15$ $x = 18$	✓Log property/eienskap A ✓Log property/ eienskap CA ✓S CA OR/OF ✓Log property/ eienskap A ✓Log property eienskap CA ✓S CA	(3)
3.4		$x - 3(5i + 2) = 4 - 3i + yi$ $x - yi = 3(5i + 2) + 4 - 3i$ $= 15i + 6 + 4 - 3i$ $= 10 + 12i$ $x = 10$ $y = -12$ OR/OF $x - 3(5i + 2) = 4 - 3i + yi$ $x - 15i - 6 = 4 + i(-3 + y)$ $x - 6 - 15i = 4 + i(-3 + y)$ $x - 6 = 4 \quad \text{and/en} \quad -15 = -3 + y$ $x = 10 \quad \text{and/en} \quad -12 = y$	✓S A ✓x-value/waarde CA ✓y-value/waarde CA OR/OF ✓S A ✓x-value /waarde CA ✓y-value/waarde CA	(3)

3.5	$V = 110,4 + 46,1i$ $r = \sqrt{(110,4)^2 + (46,1)^2}$ $= \sqrt{10062,95}$ $= 119,64$ $\tan \theta = \frac{46,1}{110,4}$ $\theta = 22,7^\circ$ or/of $\theta = 180^\circ + 22,7^\circ = 202,7^\circ$ $z = 119,64 \operatorname{cis}(22,7^\circ)$ or/of $z = 119,64 \operatorname{cis} 202,7^\circ$	✓ Substitution/ <i>vervanging</i> A ✓ S CA ✓ Ratio/ <i>verhouding</i> CA ✓ θ CA ✓ Polar form/ <i>Polêre vorm</i> CA	(5)
[23]			

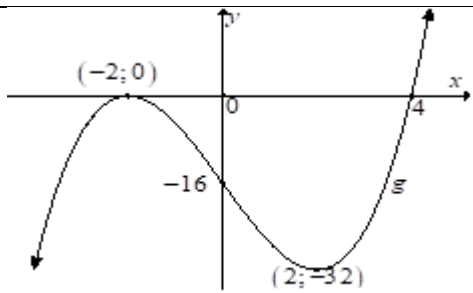
QUESTION/VRAAG 4				
4.1	4.1.1	$g(x) = \sqrt{r^2 - x^2}$ $3 = \sqrt{r^2 - 1^2}$ $9 + 1 = r^2$ $\sqrt{10} = r$ and / en $h(x) = \frac{a}{x}$ or / of $xy = a$ $3 = \frac{a}{1}$ $3 \times 1 = a$ $\therefore 3 = a$	✓ SF A ✓ value of / waarde van r CA ✓ value of / waarde van a A	(3)
	4.1.2	$0 \leq y \leq \sqrt{10}$	✓ 0 and / en $\sqrt{10}$ CA from/vanaf 4.1.2 ✓ Correct notation/ korrekte notasie A	(2)
	4.1.3	$B(-\sqrt{10}; 0)$	✓ Coordinates of / koördinate van B CA from/vanaf 4.1.1	(1)
	4.1.4	$y = 0$ $x = 0$	✓ $x = 0$ A ✓ $y = 0$ A	(2)
4.2	4.2.1	$k(x) = 2(x-2)^2 - 2$ $k(x) = 2(x-2)^2 - 2$ $k(0) = 2(0-2)^2 - 2$ $= 6$ y - int. = 6 OR / OF $k(x) = 2(x-2)^2 - 2$ $= 2(x^2 - 4x + 4) - 2$ $= 2x^2 - 8x + 6$ y - intercept / afsnit = 6	✓ Substitution/ vervanging A ✓ y-int/afsnit CA OR/OF ✓ S A ✓ y-int. CA	(2)
	4.2.2	TP(2; -2)	✓ x-coordinate/ x-koördinaat A ✓ y-coordinate / y-koördinaat A	(2)

4.2.3	$k(x) = 2(x-2)^2 - 2$ $= 2(x^2 - 4x + 4) - 2$ $= 2x^2 - 8x + 6$ $x\text{-int / afsnitte.}; k(x) = 0$ $0 = x^2 - 4x + 3$ $= (x-1)(x-3)$ $x = 1 \text{ or / of } x = 3$	✓S A ✓Equate to 0 / gelykstel aan 0 CA ✓Factors / faktore CA ✓Both x-values / beide x-waardes CA	(4)
4.2.4	$x \in \mathbf{R}$	✓ $x \in \mathbf{R}$ A	(1)
4.2.5		f: ✓Shape/vorm A ✓y-intercept/afsnit CA k: ✓Shape/vorm A ✓both x-intercepts / beide x-afsnitte CA ✓y- intercept afsnit CA ✓Turning point / draaipunt CA	(6)
4.2.6	$y \geq -2$	✓ $y \geq -2$ CA	(1)
[24]			

QUESTION/VRAAG 5			
5.1	$A = P(1 + ni)$ $= 35(1 + 9 \times 0,15)$ $= R82,25$	$\checkmark$ SF $\checkmark$ S	A CA (2)
5.2	$A = P(1 + i)^n$ $= 13\,565(1 + 0,065)^8$ $= 22\,450$	$\checkmark$ F $\checkmark$ SF $\checkmark$ S	A CA CA (3)
5.3	Year / Jaar 1 $A = P\left(1 + \frac{i}{m}\right)^n$ $= 120\,000\left(1 + \frac{0,09}{12}\right)^{12}$ $= R131\,256,8277$ $R131\,256,8277 + R50\,000 = R181\,256,8277$ Year / Jaar 2 $= R181\,256,8277\left(1 + \frac{0,085}{4}\right)^4$ $= R197\,161,7449$ $R197\,161,7449 - R35\,000 = R162\,161,7449$ The remaining 3 years Oorblywende 3 jare $= R162\,161,7449\left(1 + \frac{0,085}{4}\right)^{12}$ $= R208\,705,19$ OR/OF $A = P\left(1 + \frac{i}{m}\right)^n$ $= 120\,000\left(1 + \frac{0,09}{12}\right)^{12}\left(1 + \frac{0,085}{4}\right)^{16}$ $= R183\,753,2447$ $R50\,000\left(1 + \frac{0,085}{4}\right)^{16} = R69\,997,59473$ $R35\,000\left(1 + \frac{0,085}{4}\right)^{12} = R45\,045,65215$ Investment after 5 years / Belegging na 5 jaar $= R183\,753,2447 + R69\,997,59473 - R45\,045,65215$ $= R208\,705,19$	$\checkmark$ SF $\checkmark$ S $\checkmark$ Sum / Som $\checkmark$ SF $\checkmark$ S $\checkmark$ Difference / Verskil $\checkmark$ SF $\checkmark$ S $\checkmark$ SF $\checkmark$ S $\checkmark$ SF $\checkmark$ S $\checkmark$ S $\checkmark$ S	A CA CA A CA A CA A CA A CA CA CA (8)
			[13]

QUESTION/VRAAG 6				
6.1	$f(x) = -2x + \frac{1}{4}$ $f(x+h) = -2(x+h) + \frac{1}{4}$ $= -2x - 2h + \frac{1}{4}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2x - 2h + \frac{1}{4} - \left(-2x + \frac{1}{4}\right)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2x - 2h + \frac{1}{4} + 2x - \frac{1}{4}}{h}$ $= \lim_{h \rightarrow 0} \frac{-2h}{h}$ $= \lim_{h \rightarrow 0} -2$ $= -2$		✓F A ✓SF CA ✓S CA ✓S CA ✓ $f'(x) = -2$ CA	(5)
6.2	6.2.1	$D_a \left[\frac{3}{2} a^2 - a^{-5} \right]$ $= 3a + a^{-4}$	✓ $3a$ A ✓ a^{-4} CA	(2)
	6.2.2	$f(x) = \frac{x^3 - 3x^5}{4x}$ $= \frac{x^2}{4} - \frac{3x^4}{4}$ $f'(x) = \frac{x}{2} - 3x^3$	✓S A ✓ $\frac{x}{2}$ CA ✓ $-3x^3$ CA	(3)
	6.2.3	$s = \frac{1}{2} ft^2$ $\frac{ds}{dt} = ft$ $= \pi t$	✓ ft A ✓ πt CA	(2)

	6.3.3	$f(x) = 3x^2$ $f'(x) = 6x$ Av.grad. = $\frac{f(8) - f(2)}{8 - 2}$ $= \frac{192 - 12}{8 - 2}$ $6x = 30$ $x = 5$	$\checkmark 6x$ A $\checkmark S$ CA $\checkmark$ Equating derivative and av. gradient / <i>Gelykstelling van</i> <i>afgeleide en gemid. gradiënt</i> CA $\checkmark x = 5$ CA	(4)
				[16]

QUESTION/VRAAG 7			
7.1	$g(x) = x^3 - 12x - 16$ $g(-2) = (-2)^3 - 12(-2) - 16$ $= 0$ $\therefore (x - 2)$ is a factor of / is 'n faktor van $g(x)$	✓ substitution by -2 / vervanging deur -2 A ✓ S CA	(2)
7.2	$g(x) = x^3 - 12x - 16$ $0 = (x + 2)(x^2 - 2x - 8)$ $0 = (x + 2)(x + 2)(x - 4)$ $x = -2$ or/of $x = 4$	✓ Equating to/Gelykstelling aan 0 A ✓ Quadratic factor / Kwadratiese faktor A ✓ Factors of quadratic factor / faktore van kwadratiese factor CA ✓ values of x / waarde van x CA	(4)
7.3	(0; -16)	✓ y-intercept /y-afsnit A	(1)
7.4	$f(x) = x^3 - 12x - 16$ $f'(x) = 3x^2 - 12$ $0 = 3x^2 - 12$ $0 = x^2 - 4$ $0 = (x - 2)(x + 2)$ $x = 2$ or/of $x = -2$ $g(2) = (2)^3 - 12(2) - 16 = -32$ $g(-2) = (-2)^3 - (-2) - 16 = 0$ TP(-2; 0) and/en (2; -32)	✓ Derivative /Afgeleide A ✓ 0 CA ✓ Factors/ Faktore CA ✓ Both x values /beide x -waardes CA ✓ (-2; 0) CA ✓ (2; -32) CA	(6)
7.5		✓ Shape /vorm A ✓ y-intercept /y-afsnit CA ✓ x-intercepts /x-afsnitte CA ✓ Both turning points /Beide draaipunte CA	(4)
7.6	$h(x) = (x - 2)^3 - 12(x - 2) - 16$	✓ $h(x)$ A	(1)
7.7	$-2 > x$ or/of $x < 2$	✓ $-2 > x$ CA ✓ $x < 2$ CA	(2)
			[20]

QUESTION/VRAAG 8					
8.1	8.1.1	$q = 820 - p$	$\checkmark q = 820 - p$	A	(1)
	8.1.2	$Z = pq$ $= p(820 - p)$ $= 820p - p^2$	$\checkmark$ Substitution/Vervanging CA $\checkmark$ S	CA	(2)
	8.1.3	$Z = 820p - p^2$ $\frac{dZ}{dp} = 820 - 2p$ $0 = 820 - 2p$ $\therefore p = 410$	$\checkmark$ Derivative/Afgeleide = 0 CA $\checkmark$ S	CA	(2)
8.2	8.2.1	$R(x) = -50x^2 + 3200x - 1860$ $R(15) = -50(15)^2 + 3200(15) - 1860$ $= R34\,890$	$\checkmark$ Substitution/Vervanging A $\checkmark$ S	CA	(2)
	8.2.2	$R(x) = -50x^2 + 3200x - 1860$ $R'(x) = -100x + 3200$ $0 = -100x + 3200$ $100x = 3200$ $x = 32$ $R(x) = -50x^2 + 3200x - 1860$ $R(32) = -50(32)^2 + 3200(32) - 1860$ $= R49\,340$ $= \text{artisan's maximum earnings /}$ $= \text{vakman se maksimum verdienste}$	$\checkmark$ Derivative/Afgeleide = 0 CA $\checkmark$ S	CA $\checkmark$ Substitution/Vervanging CA $\checkmark$ S	(4)
					[11]

QUESTION / VRAAG 9				
9.1	9.1.1	$\int \left(\frac{2}{\sqrt{x}} + \pi \right) dx$ $= \int \left(2x^{-\frac{1}{2}} + \pi \right) dx$ $= 4x^{\frac{1}{2}} + \pi x + c$	$\checkmark 2x^{-\frac{1}{2}}$ A $\checkmark 4x^{\frac{1}{2}}$ CA $\checkmark \pi x$ A $\checkmark c$ A	(4)
	9.1.2	$\int (x+1)(3x-2) dx$ $= \int (3x^2 + x - 2) dx$ $= x^3 + \frac{x^2}{2} - 2x + c$	$\checkmark S$ A $\checkmark x^3$ CA $\checkmark \frac{x^2}{2}$ CA $\checkmark -2x + c$ CA	(4)
9.2		$A = \int_{1,5}^3 (4x - x^2) dx$ $= \left[2x^2 - \frac{x^3}{3} \right]_{1,5}^3$ $= \left(2(3)^2 + \frac{(3)^3}{3} \right) - \left(2(1,5)^2 + \frac{(1,5)^3}{3} \right)$ $A = 9 - \frac{27}{8}$ $= 5,625 \text{ square units / vierkante eenhede}$	$\checkmark$ A definite integral formula/ <i>in Bepaalde integrale formule</i> A $\checkmark 2x^2$ CA $\checkmark -\frac{x^3}{3}$ CA $\checkmark$ Substitution in A by 3 / <i>Vervanging in A deur 3</i> CA $\checkmark$ Substitution in A by 1,5 / <i>Vervanging in A deur 1,5</i> CA $\checkmark S$ CA	(6)
				[14]
			TOTAL/TOTAAL:	150