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**NATIONAL SENIOR
CERTIFICATE/*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

SEPTEMBER 2021

**TECHNICAL MATHEMATICS P2/*TEGNIJSE WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 15 pages./
Hierdie nasienriglyn bestaan uit 15 bladsye.

NOTE:

- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

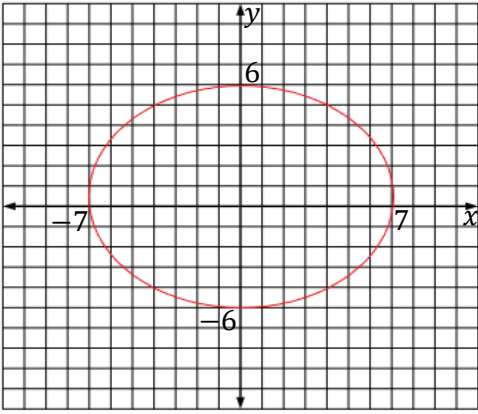
- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method / Metode
A	Accuracy / Akkuraatheid
AO	Answer only / Slegs antwoord
CA	Consistent accuracy / Deurlopende akkuraatheid
F	Formula / Formule
I	Identity / Identiteit
R	Rounding / Afronding
S	Simplification / Vereenvoudiging
ST	Statement / Bewering
RE	Reason / Rede
ST RE	Statement and correct reason / Bewering en korrekte rede
SF	Substitution correctly in correct formula / Korrekte vervanging in die korrekte formule
NPU	No penalty for omitting units / Geen penalisering vir eenhede uitgelaat

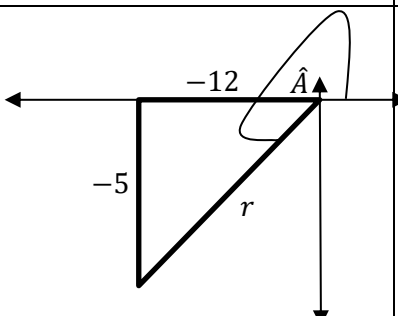
QUESTION/VRAAG 1

Q		M
1.1	$q = -4$	✓ A (1)
1.2	$AB^2 = (-2 + 5)^2 + (-4 - 0)^2$ $= 9 + 16$ $AB = 5 \text{ units / eenhede}$	✓ M ✓ 5 CA (2)
1.3	$m_{CD} = \frac{-4-0}{3-0}$ $= -\frac{4}{3}$ $\tan \theta = -\frac{4}{3}$ $\text{Ref / verw } \angle = \tan^{-1} \left(\frac{4}{3} \right)$ $= 53,13^\circ$ $\therefore \theta = 126,87^\circ$	✓ $-\frac{4}{3} A$ ✓ $53,13^\circ CA$ ✓ $126,87^\circ CA$ (3)
1.4	$BC = 3 - (-2) = 3 + 2 = 5 \text{ units / eenhede}$ $AO = 0 - (-5) = 5 \text{ units / eenhede}$ $\therefore ABCO$ is a parallelogram (One pair of opp. side = and $\parallel$ / Een paar teenoorstaande sye = en $\parallel$) But / maar, $AO = AB$ (from / vanuit 1.2) $\therefore ABCO$ is a rhombus / is 'n ruit (parallelogram with all sides = / parallelogram met all sye =)	✓ BC & AO M ✓ parm and / en reason / rede ✓ rhombus / ruit ✓ reason / rede (4)
		[10]

QUESTION/VRAAG 2

Q	M	
2.1.1	$x^2 + y^2 = 25 \dots 1$ $y - x - 1 = 0 \dots 2$ $y = x + 1 \dots 3$ sub 3 in 1: $x^2 + (x + 1)^2 = 25$ $x^2 + x^2 + 2x + 1 = 25$ $2x^2 + 2x - 24 = 0$ $x^2 + x - 12 = 0$ $(x + 4)(x - 3) = 0$ $x = -4$ or / of $x = 3$ $\therefore y = -3$ or/of $y = 4$	$\checkmark y = x + 1$ $\checkmark x^2 + (x + 1)^2 = 25$ $\checkmark x^2 + x - 12 = 0$ $\checkmark x = -4$ $\checkmark x = 3$ $\checkmark y = -3$ CA $\checkmark y = 4$ CA (7)
2.1.2	$(3; 2): (3)^2 + (2)^2 = 13$ $\therefore x^2 + y^2 < r^2$ $\therefore$ lies inside the circle / lê binne die sirkel	$\checkmark 13$ $\checkmark x^2 + y^2 < r^2$ CA $\checkmark$ conclusion / afleiding (3)
2.1.3	$m_{rad} = \frac{0-3}{0-(-4)}$ $= -\frac{3}{4}$ $\therefore m_{tan} \times m_{rad} = -1$ $\therefore m_{tan} \times -\frac{3}{4} = -1$ $\therefore m_{tan} = -1 \div -\frac{3}{4}$ $\therefore m_{tan} = \frac{4}{3}$ $y = mx + c$ $\therefore y = \frac{4}{3}x + c$ $(-4; 3): 3 = \frac{4}{3}(-4) + c$ $3 = \frac{-16}{3} + c$ $\frac{25}{3} = c$ $\therefore y = \frac{4}{3}x + \frac{25}{3}$	$\checkmark -\frac{3}{4}$ $\checkmark \frac{4}{3}$ $\checkmark \frac{25}{3} = c$ $\checkmark y = \frac{4}{3}x + \frac{25}{3}$ CA (4)
2.2	$36x^2 + 49y^2 = 1764$ $\frac{36x^2}{1764} + \frac{49y^2}{1764} = 1$ $\frac{x^2}{49} + \frac{y^2}{36} = 1$	 $\checkmark \frac{x^2}{49} + \frac{y^2}{36} = 1$ $\checkmark$ Shape / vorm $\checkmark$ x-intercept / x-afsnit $\checkmark$ y-intercept / y-afsnit (4)
		[18]

QUESTION/VRAAG 3

Q	M	
3.1.1	$\tan 4(122.3^\circ) + \frac{2}{3} \cos \left(\frac{210.5^\circ}{4} \right)$ $= -0,8$	✓ SF ✓ -0,8 (2)
3.1.2	$\frac{1}{\sin \left(\frac{\hat{A}}{3} + 2B \right)}$ $= \frac{1}{\sin \left(\frac{210.5^\circ}{3} + 2(122.3^\circ) \right)}$ $= -1,4$	✓ $\frac{1}{\sin \left(\frac{\hat{A}}{3} + 2B \right)}$ ✓ SF ✓ -1,4 (3)
3.2.1	$\tan \hat{A} = \frac{5}{12} = \frac{-5}{-12}$ $\therefore \operatorname{cosec}^2 \hat{A} = \frac{1}{\sin^2 \hat{A}}$ $\operatorname{cosec}^2 \hat{A} = \frac{169}{25}$ OR/OF $\tan \hat{A} = \frac{5}{12} = \frac{-5}{-12}$ $\therefore \operatorname{cosec}^2 \hat{A} = \left(\frac{13}{-5} \right)^2$ $\operatorname{cosec}^2 \hat{A} = \frac{169}{25}$	 $r^2 = (-12)^2 + (-5)^2$ $\therefore r = 13$ ✓ $\tan \hat{A} = \frac{5}{12} = \frac{-5}{-12}$ A ✓ correct quadrant / regte kwadrant A ✓ $r = 13$ A ✓ $\frac{1}{\sin^2 \hat{A}}$ or / of $\operatorname{cosec}^2 \hat{A} = \left(\frac{13}{-5} \right)^2$ CA ✓ $\frac{169}{25}$ CA (5)
3.2.2	$\sec \hat{A} - \sin \hat{A} = \frac{1}{\cos \hat{A}} - \sin \hat{A}$ $= \frac{1}{\frac{-12}{13}} - \frac{-5}{13}$ $= \frac{13}{-12} - \frac{-5}{13}$ $= \frac{-12}{-109} - \frac{-5}{13}$ $= \frac{-109}{156}$	OR/OF $\sec \hat{A} - \sin \hat{A} = \frac{1}{\cos \hat{A}} - \sin \hat{A}$ $= \frac{1}{\frac{13}{-12}} - \frac{-5}{13}$ $= \frac{-12}{-109} - \frac{-5}{13}$ $= \frac{-109}{156}$ ✓ $\frac{1}{\cos \hat{A}}$ or / of ✓ ✓ $\frac{13}{-12}$ ✓ $\frac{1}{\frac{-12}{13}}$ ✓ $\frac{-109}{156}$ CA (3)
3.2.3	$\tan \hat{A} = \frac{5}{12}$ $\text{ref / verw} \angle = \tan^{-1} \left(\frac{5}{12} \right)$ $= 22,62^\circ$ $\therefore \hat{A} = 180^\circ + 22,62^\circ$ $\hat{A} = 202,62^\circ$	✓ method / metode ✓ $22,62^\circ$ ✓ $202,62^\circ$ (3)
		[16]

QUESTION/VRAAG 4

Q		M
4.1	$\frac{\sin(\pi - x) \cdot \operatorname{cosec}(2\pi - x) \cdot \tan(\pi + x)}{\sec(2\pi - x) \cdot \cos(2\pi - x)}$ $= \frac{\sin(180^\circ - x) \cdot \operatorname{cosec}(360^\circ - x) \cdot \tan(180^\circ + x)}{\sec(360^\circ - x) \cdot \cos(360^\circ - x)}$ $= \frac{\sin(x) \cdot -\frac{1}{\sin(x)} \cdot \tan(x)}{\frac{1}{\cos(x)} \cdot \cos(x)}$ $= \frac{-1 \cdot \tan(x)}{1}$ $= -\tan(x)$ OR/OF $\frac{\sin(\pi - x) \cdot \operatorname{cosec}(2\pi - x) \cdot \tan(\pi + x)}{\sec(2\pi - x) \cdot \cos(2\pi - x)}$ $= \frac{\sin(180^\circ - x) \cdot \operatorname{cosec}(360^\circ - x) \cdot \tan(180^\circ + x)}{\sec(360^\circ - x) \cdot \cos(360^\circ - x)}$ $= \frac{\sin(x) \cdot -\operatorname{cosec}(x) \cdot \tan(x)}{\sec(x) \cdot \cos(x)}$ $= \frac{-1 \cdot \tan(x)}{1}$ $= -\tan(x)$	✓ correct conversion of rad to degrees / korrekte herleiding van rad na grade ✓ $\sin(x)$ ✓ $-\frac{1}{\sin(x)}$ ✓ $\tan(x)$ ✓ $\frac{1}{\cos(x)}$ ✓ $\cos(x)$ ✓ $\frac{-1 \cdot \tan(x)}{1}$ ✓ $-\tan(x)$ (8) OR/OF ✓ correct conversion of rad to degrees / korrekte herleiding van rad na grade ✓ $\sin(x)$ ✓ $-\operatorname{cosec}(x)$ ✓ $\tan(x)$ ✓ $\sec(x)$ ✓ $\cos(x)$ ✓ $\frac{-1 \cdot \tan(x)}{1}$ ✓ $-\tan(x)$ (8)
4.2	$LHS/LK = \frac{\cos\theta}{1-\sin\theta} - \tan\theta$ $= \frac{\cos\theta}{1-\sin\theta} - \frac{\sin\theta}{\cos\theta}$ $= \frac{\cos\theta \cdot \cos\theta - \sin\theta(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$ $= \frac{\cos^2\theta - \sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)}$ $= \frac{1-\sin\theta}{\cos\theta(1-\sin\theta)}$ $= \frac{1}{\cos\theta}$ $= \sec\theta = RHS/RK$	✓ $\frac{\sin\theta}{\cos\theta}$ ✓ $\cos\theta(1-\sin\theta)$ ✓ $\cos^2\theta - \sin\theta + \sin^2\theta$ ✓ $\frac{1-\sin\theta}{\cos\theta(1-\sin\theta)}$ ✓ $\frac{1}{\cos\theta}$ (5)
		[13]

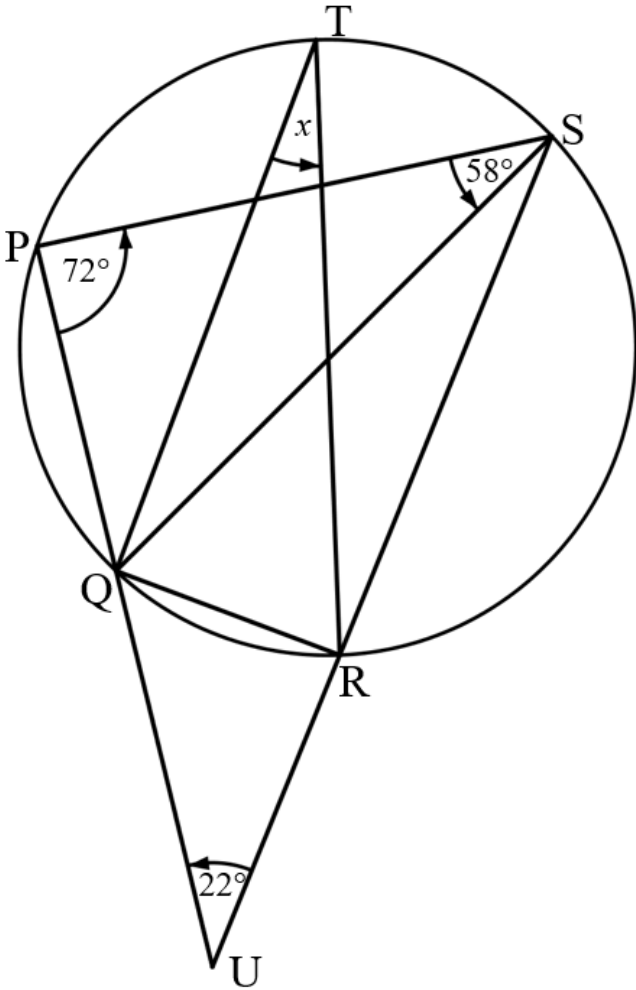
QUESTION/VRAAG 5

Q		M
5.1		✓ cos start and end point / cos begin- en eindpunte ✓ cos turningpoints / cos draaipunte ✓ cos x -intercepts/cos x -afsnitte ✓ sin start and end point / sin begin- en eindpunte ✓ sin turningpoints / sin draaipunte ✓ sin x -intercepts/sin x -afsnitte (6)
5.2.1	120°	✓ 120° (1)
5.2.2	(a) $x = 30^\circ$ and / en $x = 120^\circ$	✓ $x = 30^\circ$ ✓ $x = 120^\circ$ (2)
	(b) $90^\circ \leq x \leq 150^\circ$	✓ $90^\circ \leq$ ✓ $\leq 150^\circ$ (2)
		[11]

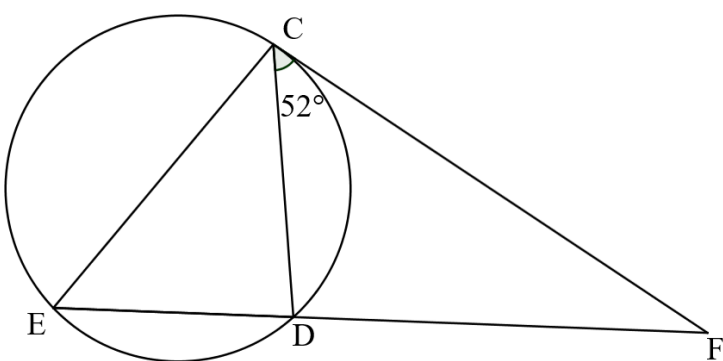
QUESTION/VRAAG 6

Q		M
6.1	AB = 8 cm (opp sides of rec = / teenoorst. sye van reghoek =)	✓ 8 cm ST ✓ RE (2)
6.2	$\frac{BE}{8} = \tan 30^\circ$ $BE = 4,62 \text{ cm} = 5 \text{ cm}$	✓ $\frac{BE}{8} = \tan 30^\circ$ M ✓ 5 cm (2)
6.3	$\frac{\sin \hat{EBC}}{9} = \frac{\sin 30^\circ}{4,62}$ $\sin \hat{EBC} = \frac{\sin 30^\circ}{4,62} \times 9$ $\hat{EBC} = 76,91^\circ = 77^\circ$	✓ SF ✓ $\sin \hat{EBC} = \frac{\sin 30^\circ}{4,62} \times 9$ ✓ 77° (3)
6.4	$\hat{BEC} = 180^\circ - 30^\circ - 77^\circ$ $= 73^\circ$ (Int. $\angle$'s of $\triangle BEC$ / Binne. $\angle$ 'e van $\triangle BEC$) $\text{Area } \triangle BCE = \frac{1}{2} a. b. \sin \hat{C}$ $= \frac{1}{2} (5)(9) \sin 73^\circ$ $= 21,52 \text{ cm}^2$	✓ ST RE ✓ $\frac{1}{2} (5)(9) \sin 73^\circ$ SF ✓ $21,52 \text{ cm}^2$ CA (3)
6.5	$CF^2 = CE^2 + FE^2 - 2(CE)(FE) \cos \hat{CEF}$ $CF^2 = (9)^2 + (10)^2 - 2(9)(10) \cos(25^\circ)$ $= 17,86 \dots$ $CF = \sqrt{17,86 \dots}$ $CF = 4,22 \dots = 4 \text{ cm}$	✓ $(9)^2 + (10)^2 - 2(9)(10) \cos(25^\circ)$ SF ✓ $17,86 \dots$ S ✓ 4 cm CA (3)
		[13]

QUESTION/VRAAG 8

Q		M
8.1	supplementary / <i>supplementêr</i>	✓ (1)
8.2		
8.2.1 (a)	$\widehat{QSR} = \widehat{T} = x$ ($\angle$'s subt.chord QR / ($\angle$ 'e onderspan deur krd QR)	✓ ST ✓ RE (2)
8.2.2 (b)	$\widehat{PQR} = 180^\circ - (58^\circ + x)$ (opp. $\angle$'s quad/teenoorts. $\angle$ 'e kvh) $= 122^\circ - x$	✓ ST ✓ RE ✓ $122^\circ - x$ ST (3)
8.2.2	$\widehat{U} + \widehat{UQR} + \widehat{URQ} = 180^\circ$ (int. $\angle$'s Δ /binne $\angle$ 'e Δ) $22^\circ + 72^\circ + x + 58^\circ = 180^\circ$ $x = 28^\circ$ OR/OF $\widehat{P} + \widehat{S} + \widehat{U} = 180^\circ$ (int. $\angle$'s Δ / binne $\angle$ 'e Δ) $72^\circ + 58^\circ + x + 22^\circ = 180^\circ$ $x = 28^\circ$	✓ ST RE ✓ $x = 28^\circ$ ST (2) OR/OF ✓ ST RE ✓ $x = 28^\circ$ ST (2)
		[8]

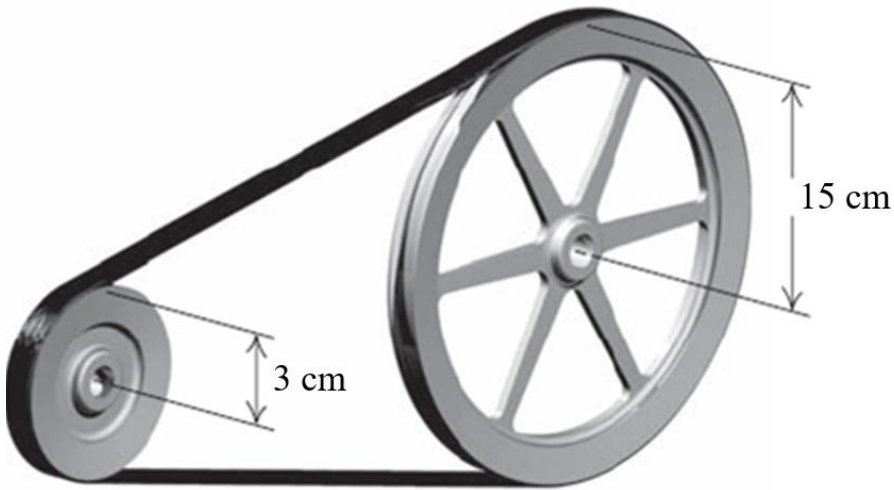
QUESTION/VRAAG 9

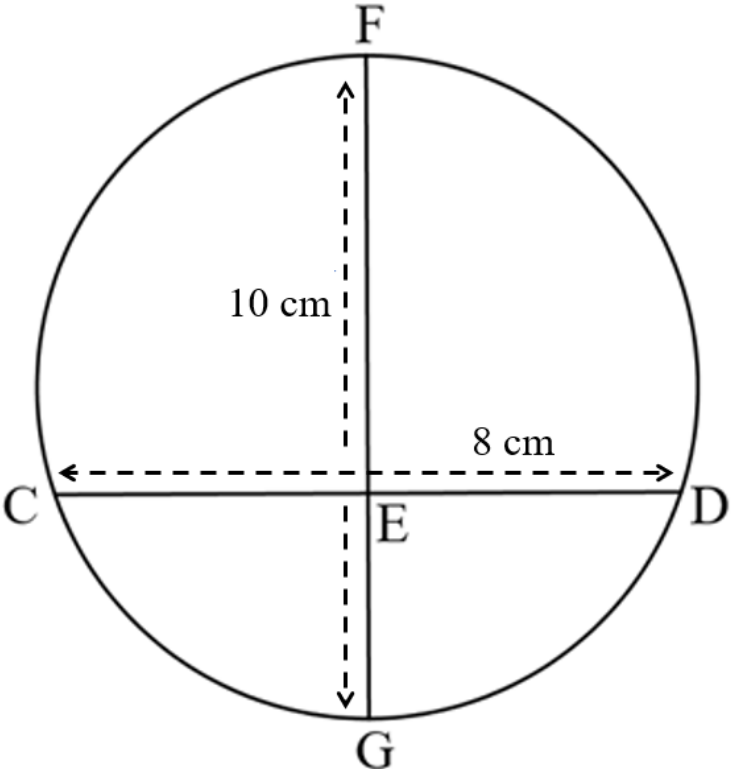
Q		M
9.1	All angles of the one triangle is equal to the angles in the other triangle / <i>Alle hoeke van die een driehoek is gelyk aan die hoeke van die ander driehoek</i> Sides of triangles are in proportion / <i>Sye van die driehoek is in verhouding</i>	✓ ST ✓ ST (2)
9.2		
9.2.1	In $\triangle CDF$ and/en $\triangle ECF$: $\widehat{FCD} = \widehat{CED} = 52^\circ$ (tanchord / <i>raaklyn koord</i>) $\widehat{CFD} = \widehat{CEF}$ (common $\angle$ / <i>gemene $\angle$</i>) $\widehat{CDF} = \widehat{CEC}$ (int. $\angle$'s Δ / <i>binne $\angle$'e Δ</i>) $\therefore \triangle CDF \parallel \triangle ECF$ ($\angle\angle\angle$)	✓ ST RE ✓ ST RE ✓ ST $\triangle CDF \parallel \triangle ECF$ OR ✓ RE ($\angle\angle\angle$) (4)
9.2.2	$\frac{CD}{EC} = \frac{DF}{CF} = \frac{CF}{EF}$ ($\triangle CDF \parallel \triangle ECF$) $\therefore CF^2 = EF \cdot FD$	✓ $\frac{CD}{EC} = \frac{DF}{CF} = \frac{CF}{EF}$ ST ✓ $CF^2 = EF \cdot FD$ ST (2)
9.2.3	$DF = 15 - 6 = 9$ $\frac{CD}{EC} = \frac{DF}{CF} = \frac{CF}{EF}$ (from 9.2.2 / <i>vanuit 9.2.2</i>) $\therefore \frac{9}{CF} = \frac{CF}{15}$ $\therefore CF^2 = 135$ $\therefore CF \approx 12 \text{ cm}$ OR/OF $CF^2 = 15 \times 9$ (from 9.2.2 / <i>vanuit 9.2.2</i>) $CF = \sqrt{135}$ $CF = 11,619 \dots$ $\therefore CF = 12 \text{ cm}$	✓ $DF = 9$ A ✓ $\frac{9}{CF} = \frac{CF}{15}$ SF ✓ $CF^2 = 135$ CA ✓ 12 cm CA R (4) OR/OF ✓ $CF^2 = 15 \times 9$ A ✓ $CF = \sqrt{135}$ S ✓ $CF = 11,619 \dots$ CA ✓ 12 cm CA R (4)
9.2.4	$\frac{CD}{EC} = \frac{12}{15} = \frac{4}{5}$	✓ $\frac{12}{15}$ SF A ✓ $\frac{4}{5}$ A (2)
9.2.5	$\widehat{ECF} = 44^\circ + 52^\circ = 96^\circ \neq 90^\circ$ $\therefore CE$ not a diameter / <i>CE nie 'n middellyn nie</i> (Diameter not perpendicular to tangent/<i>middellyn nie loodreg met raaklyn nie</i>)	✓ ST ✓ ST RE (2)

	OR/OF $\widehat{CDE} = 180^\circ - 44^\circ - 52^\circ = 86^\circ \neq 90^\circ$ $\therefore CE$ not a diameter / CE nie 'n middellyn nie (not Converse $\angle$'s in semi-circle / nie-omgekeerde $\angle$ 'e in semi-sirkel)	OR/OF ✓ ST ✓ ST RE (2)
		[16]

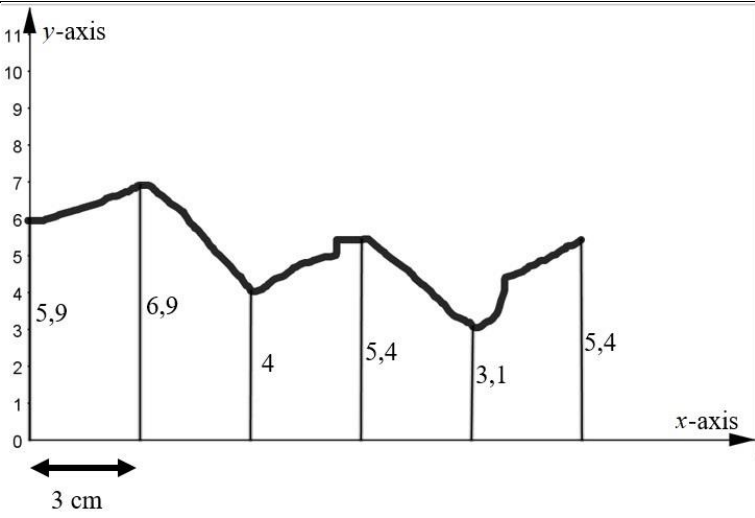
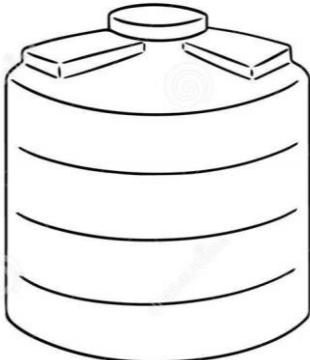
QUESTION/VRAAG 10

Q		M
10.1		
10.1.1	$\hat{A} = 30^\circ \times \frac{\pi}{180^\circ}$ $\hat{A} = \frac{\pi}{6}$ radians / radiale	✓ $\times \frac{\pi}{180^\circ}$ ✓ $\frac{\pi}{6}$ radians/radiale (2)
10.1.2	$s = r\theta$ $BG = (4) \left(\frac{\pi}{6} \right)$ $BG = \frac{2\pi}{3}$ cm	✓ F ✓ SF $\checkmark \frac{2\pi}{3}$ cm (3)
10.1.3	Area of a sector / Opp van 'n sektor = $\frac{r^2\theta}{2}$ $\therefore$ Area of sector AEC / Opp van sektor AEC = $\frac{(5)^2 \frac{\pi}{6}}{2}$ $= \frac{25\pi}{12}$ cm ²	✓ F ✓ SF $\checkmark \frac{25\pi}{12}$ cm ² (3)
10.1.4	Area of a sector / Opp van 'n sektor = $\frac{r^2\theta}{2}$ $\therefore$ Area of sector ABG / Opp van sektor ABG = $\frac{(4)^2 \frac{\pi}{6}}{2}$ $= \frac{4\pi}{3}$ cm ² $\therefore$ Area of BECG / Opp van BECG = $\frac{25\pi}{12} - \frac{4\pi}{3}$ $\therefore$ Area of BECG / Opp van BECG = $\frac{65\pi}{12}$ cm ²	✓ SF ✓ $\frac{4\pi}{3}$ ✓ M $\checkmark \frac{65\pi}{12}$ (4)

10.2		
10.2.1	$\omega = 2\pi n$ $\omega = 2\pi(120)$ $\omega = 240\pi \text{ rad/min}$	✓ F ✓ SF ✓ $240\pi \text{ rad/min}$ (3)
10.2.2	$v = \pi Dn$ $v = \pi(3 \times 2)(120)$ $v = 720\pi \text{ cm/min}$ OR/OF $v = r\omega$ $v = (3)(240\pi)$ $v = 720\pi \text{ cm/min}$	✓ F ✓ SF ✓ $720\pi \text{ cm/min}$ OR/OF ✓ F ✓ SF ✓ $720\pi \text{ cm/min}$ (3)
10.2.3	Linear speed of large pulley / <i>reglynige spoed van groter katrol</i> = $720\pi \text{ cm / min}$ $v = \omega r$ $720\pi = \omega(15)$ $48\pi \text{ rad / min} = \omega$	✓ F ✓ SF ✓ $48\pi \text{ rad/min}$ (3)

10.3		
	$4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (8)^2 = 0$ $4h^2 - 40h + 64 = 0$ $h^2 - 10h + 16 = 0$ $(h - 8)(h - 2) = 0$ $\therefore h = 8 \text{ cm or/of } h = 2 \text{ cm}$ $\therefore h = 2 \text{ cm}$ OR/OF $CE = 4$ (line from centre perpendicular to chord / lyn vanaf middelpunt loodreg op koord) $FO = OG$ (radii) $OC^2 = OE^2 + CE^2$ (Pyth) $\therefore OE^2 = OC^2 - CE^2$ $\therefore OE^2 = (5)^2 - (4)^2$ $\therefore OE^2 = 25 - 16$ $\therefore OE^2 = 9$ $\therefore OE = \pm\sqrt{9}$ $\therefore OE = \pm 3$ $\therefore OE = 3 \text{ cm}$ $\therefore EG = 5 - 3$ $\therefore EG = 2 \text{ cm}$	$\checkmark$ F $\checkmark$ SF $\checkmark$ Standard form/standaard vorm $\checkmark$ M $\checkmark h = 2 \text{ cm}$ OR/OF $\checkmark$ ST RE $\checkmark$ ST RE $\checkmark OE^2 = (5)^2 - (4)^2$ SF $\checkmark OE = 3 \text{ cm}$ CA $\checkmark EG = 2 \text{ cm}$ CA (5)
		[26]

QUESTION/VRAAG 11

Q		M
11.1		
	$A_T = a(m_1 + m_2 + m_3 + \dots + m_n)$ $A_T = 3(6,4 + 5,45 + 4,7 + 4,25 + 4,25)$ $A_T = 3(25,05)$ $A_T = 75,15 \text{ cm}^2$ OR/OF $A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1}\right)$ $A_T = 3\left(\frac{5,9 + 5,4}{2} + 6,9 + 4 + 5,4 + 3,1\right)$ $A_T = 3(25,05)$ $A_T = 75,15 \text{ cm}^2$	✓ F ✓ SF ✓ S ✓ 75,15 cm² OR/OF ✓ F ✓ SF ✓ S ✓ 75,15 cm² (4)
11.2		
	$TSA = 2\pi r^2 + 2\pi rh - \pi r^2 - \pi r^2$ $TSA = 2\pi \left(\frac{1,85}{2}\right)^2 + 2\pi \left(\frac{1,85}{2}\right)(2,5) - \pi \left(\frac{1,85}{2}\right)^2 - \pi \left(\frac{1}{2}\right)^2$ $TSA = 16,43 \text{ m}^2$	✓ F ✓ SF ✓ 17,22 m² (3)
		[7]
	TOTAL/TOTAAL:	150