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Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ NASIONALE SENIOR SERTIFIKAAT

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2021

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 24 pages.
*Hierdie nasienriglyne bestaan uit 24 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

QUESTION/VRAAG 1

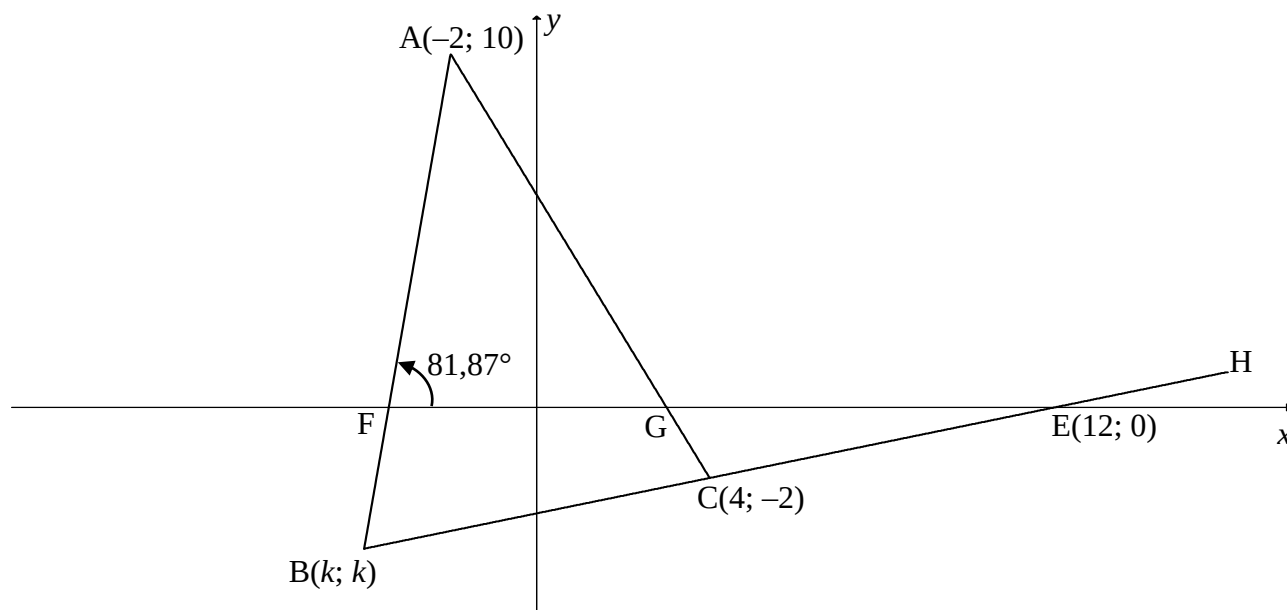
10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$	Answer only: Full marks <i>Slegs antw: Volpunte</i>	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10,1707 \approx 10,17$		✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32,17$ $\therefore 5$ days		✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/ <i>bestel</i> $20 \times 18 = 360$ sold/ <i>verkoop</i> Total not sold/ <i>Totaal nie verkoop nie</i> : 36 OR/OF $22 - 20 = 2$ $2 \times 18 = 36$		✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/ <i>Opsie B</i> <u>Any one of the following reasons/<i>Enige een van die vlg redes</i>:</u> • Median/<i>Mediaan</i> = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right		✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right <i>Data is positief skeef/skeef na regs</i>		✓ answer (1)
[10]			

QUESTION/VRAAG 2

Price of milk in rands per 5-litre container (x) <i>Prys van melk in rand, per 5 liter-houer (x)</i>	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) <i>Aantal 5 liter-houers melk verkoop (y)</i>	48	30	26	44	23	32	39	29	42	33

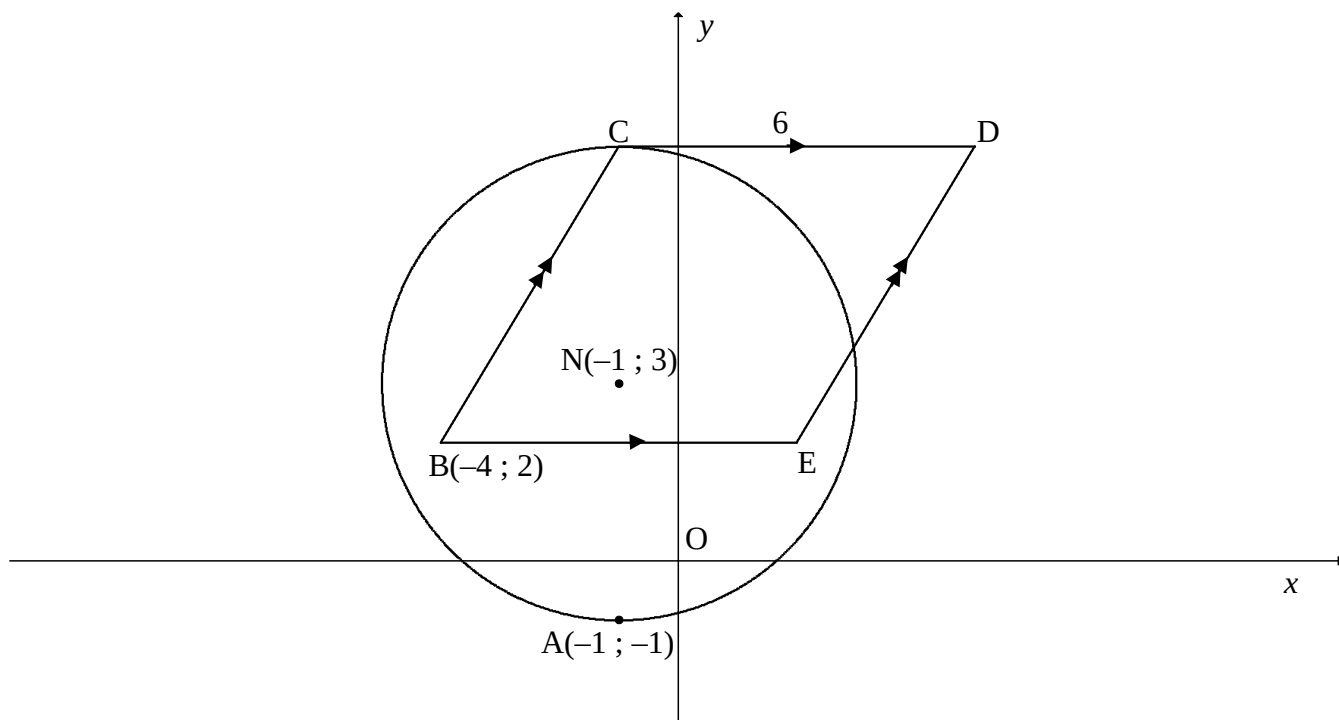
2.1	SCATTER PLOT	1 mark: 3 to 5 points plotted correctly 2 marks: 6 to 9 points plotted correctly 3 marks: all points plotted correctly
2.2	$a = 90,478... \approx 90,48$ $b = -1,773... \approx -1,77$ $\hat{y} = 90,48 - 1,77x$ Answer only: Full marks <i>Slegs antw: Volpunte</i>	✓ a ✓ b ✓ equation
2.3	$y = 23,069... \approx 23,07$ units/eenhede (calculator/sakrekenaar) OR/OF $y = 90,48 - 1,77(38)$ $y = 23,22$ units/eenhede	✓✓ answer ✓ substitution ✓ answer
2.4	$r = -0,94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold $\therefore$ there is a good chance of the prediction being accurate./ <i>Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan $\therefore$ daar is 'n goeie kans dat die voorspelling akkuraat is</i>	✓ value of r OR/OF strong relationship/ <i>sterk verwantskap</i> ✓ accurate/akkuraat
[10]		

QUESTION/VRAAG 3

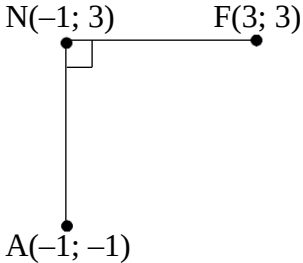
3.1.1	$m_{BE} = m_{CE} = \frac{0 - (-2)}{12 - 4} \quad \text{OR/OF} \quad m_{BE} = m_{CE} = \frac{-2 - 0}{4 - 12}$ $= \frac{1}{4} \qquad \qquad \qquad = \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$ Answer only: Full marks <i>Slegs antw: Volpunte</i>	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$ OR/OF $y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ $y - y_1 = m(x - x_1)$ $y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

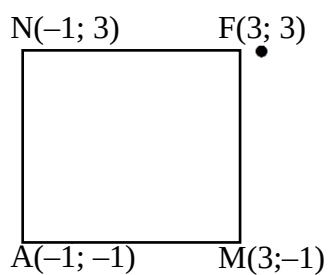
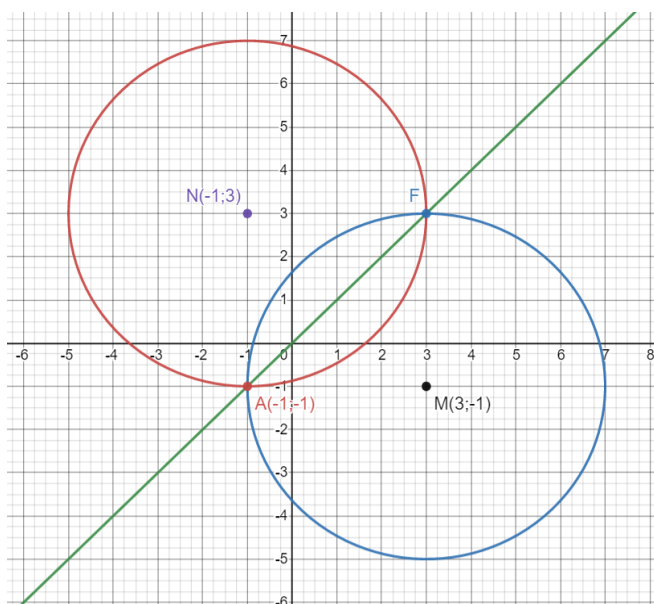
3.3.1	$y = \frac{1}{4}x - 3$ $k = \frac{1}{4}k - 3$ $\frac{3}{4}k = -3$ $k = -4$ $\therefore B(-4; -4)$ OR/OF $m_{BE} = \frac{1}{4}$ $\frac{0 - k}{12 - k} = \frac{1}{4}$ $-4k = 12 - k$ $k = -4$ $\therefore B(-4; -4)$ OR/OF $m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$ $m_{AB} = \frac{10 - k}{-2 - k}$ $7(-2 - k) = 10 - k$ $-14 - 7k = 10 - k$ $-6k = 24$ $k = -4$ $\therefore B(-4; -4)$ OR/OF EB: $y = \frac{1}{4}x - 3$ and AB: $y = 7x + 24$ $\frac{1}{4}x - 3 = 7x + 24$ $\frac{27}{4}x = -27$ $x = k = -4$ $\therefore B(-4; -4)$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ equating EB & AB ✓ answer (2)
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3.3.2	In $\triangle AFG$: $m_{AC} = \frac{10 - (-2)}{-2 - 4} = -2$ $\tan \theta = m_{AC} = -2$ $\theta = 180^\circ - 63,43\dots^\circ$ $\therefore \theta = 116,57^\circ$ $\therefore \hat{A} = 116,57^\circ - 81,87^\circ [\text{ext } \angle \text{ of } \Delta]$ $\therefore \hat{A} = 34,70^\circ$ OR/OF In $\triangle ABC$: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ $(2\sqrt{17})^2 = (6\sqrt{5})^2 + (10\sqrt{2})^2 - 2(6\sqrt{5})(10\sqrt{2}) \cdot \cos A$ $\cos A = \frac{(6\sqrt{5})^2 + (10\sqrt{2})^2 - (2\sqrt{17})^2}{2(6\sqrt{5})(10\sqrt{2})}$ $= 0,822\dots$ $\therefore A = 34,7^\circ$	✓ $m_{AC} = -2$ ✓ $\tan \theta = -2$ ✓ $\theta = 116,57^\circ$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ $\cos A$ subject ✓ answer (4)
3.3.3	$M\left(\frac{12 + (-2)}{2}; \frac{10 + (0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12 - p)^2 + (0 - p)^2}$ $(4\sqrt{17})^2 = (\sqrt{(12 - p)^2 + (0 - p)^2})^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p - 16)(p + 4) = 0$ $\therefore p = 16 \quad \text{or} \quad p = -4 \text{ (n.a.)}$ $\therefore T(16; 16)$	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x - 12)^2 + y^2 = (4\sqrt{17})^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{\text{radius}} = \frac{1}{4}$ $m_{\text{tangent}} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ OR/OF $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ m_{tangent} ✓ substitution of B ✓ equation (3)
		[24]

QUESTION/VRAAG 4

4.1	Radius = 4 units/eenhede	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1; 7)$	✓ x value ✓ y value (2)
4.2.2	$CD = 6$ units $\therefore D(5; 7)$	✓ x value ✓ y value (2)
4.2.3	$\perp h = 5$ units $DC = 6$ units $\text{Area } \triangle BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ OR/OF $\perp h = 5$ units $DC = 6$ units $\text{Area } \triangle BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

	OR/OF Let angle of inclination of BC = α $\tan \alpha = \frac{5}{3}$ $\alpha = 59,036...^\circ$ $\hat{BCD} = 180^\circ - \alpha$ $\hat{BCD} = 180^\circ - 59,036...^\circ$ $\hat{BCD} = 120,96^\circ$ Area $\triangle BCD = \frac{1}{2}(\sqrt{34})(6) \sin 120,96^\circ$ $= 15 \text{ units}^2$	✓ $\hat{BCD} = 120,96^\circ$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; -1) [reflection of N(-1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3 - (-1))^2 + (-1 - 3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5,66 \text{ units}$	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; -1) $m_{MN} = \frac{3 - (-1)}{-1 - 3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x + 1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1) OR/OF N(-1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore F(3 ; 3)$ midpoint $\left(\frac{-1 + 3}{2}; \frac{-1 + 3}{2} \right) = (1 ; 1)$ 	✓ equation of MN ✓ equating AF & MN ✓ x value ✓ y value (4) ✓ ✓ coordinates of F ✓ x value ✓ y value (4)

OR/OF

NAMF is a square ($NA=NF=AM=MF$ and $NA \perp AM$)

Midpoint NM = (1 ; 1)
= Midpoint of AF

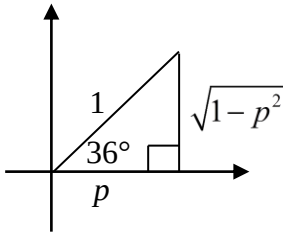
✓ NAMF = square

✓ x ✓ y of midpt NM
✓ midpt AF

(4)

[15]

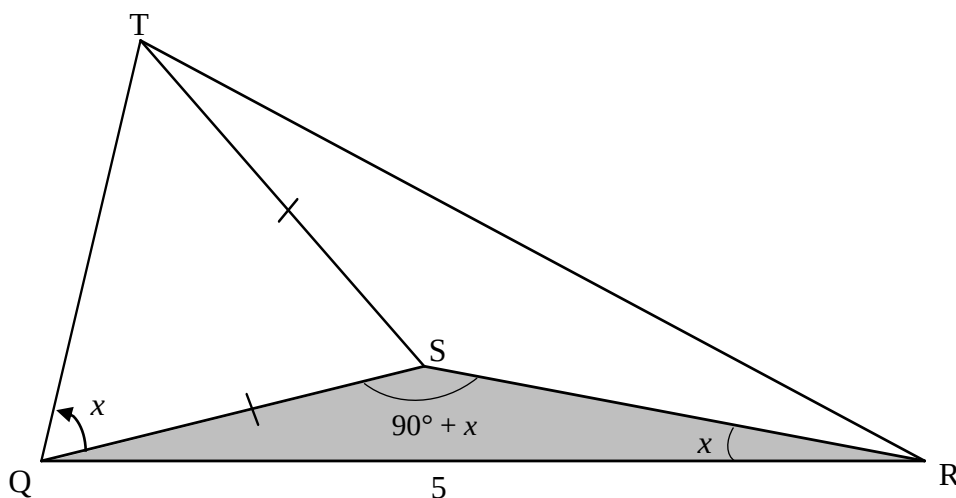
QUESTION/VRAAG 5

5.1	$\frac{\sin 140^\circ \cdot \sin(360^\circ - x)}{\cos 50^\circ \cdot \tan(-x)}$ $= \frac{\sin 40^\circ (-\sin x)}{\sin 40^\circ (-\tan x)}$ $= \frac{-\sin x}{-\tan x}$ $= \frac{\sin x}{\cos x}$ $= \cos x$	✓ $\sin 40^\circ$ ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ answer
5.2	$\text{LHS} = \frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)}$ $\text{LHS} = \frac{-2(1 - \cos^2 x) + \cos x + 1}{1 - (-\cos x)}$ $\text{LHS} = \frac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$ $\text{LHS} = \frac{2\cos^2 x + \cos x - 1}{1 + \cos x}$ $\text{LHS} = \frac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$ $\text{LHS} = 2\cos x - 1$ $\therefore \text{LHS} = \text{RHS}$	$\text{RHS} = 2\cos x - 1$ ✓ identity i. t. o. $\cos x$ ✓ $\cos(540^\circ - x) = -\cos x$ ✓ standard form ✓ factors
5.3.1	$\sin 36^\circ = \sqrt{1 - p^2}$ $\tan 36^\circ = \frac{\sqrt{1 - p^2}}{p}$ OR/OF $\cos^2 36^\circ = 1 - \sin^2 36^\circ$ $\cos 36^\circ = \sqrt{1 - (1 - p^2)}$ $= p$ $\tan 36^\circ = \frac{\sin 36^\circ}{\cos 36^\circ}$ $= \frac{\sqrt{1 - p^2}}{p}$	 ✓ method ✓ value of p ✓ answer ✓ method ✓ $\cos 36^\circ = p$ ✓ answer

5.3.2	$\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(2 \cos^2 36^\circ - 1)$ $= -2p^2 + 1$ OR/OF $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(1 - 2 \sin^2 36^\circ)$ $= -1 + 2(\sqrt{1 - p^2})^2$ $= -1 + 2(1 - p^2)$ $= -2p^2 + 1$ OR/OF $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(\cos^2 36^\circ - \sin^2 36^\circ)$ $= -\left(p^2 - (\sqrt{1 - p^2})^2\right)$ $= -(p^2 - (1 - p^2))$ $= -2p^2 + 1$ OR/OF $\cos 108^\circ$ $= \cos(2 \times 54^\circ)$ $= 2 \cos^2 54^\circ - 1$ $= 2(1 - p^2) - 1$ $= 1 - 2p^2$ OR/OF $\cos 108^\circ = \cos(72^\circ + 36^\circ)$ $= \cos 72^\circ \cos 36^\circ - \sin 72^\circ \sin 36^\circ$ $= (2 \cos^2 36^\circ - 1) \cos 36^\circ - (2 \sin 36^\circ \cos 36^\circ) \sin 36^\circ$ $= 2 \cos^3 36^\circ - \cos 36^\circ - 2 \cos 36^\circ \sin^2 36^\circ$ $= 2p^3 - p - 2p(\sqrt{1 - p^2})^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4) ✓ expansion ✓ both double angle identities ✓ value of $\sin 36^\circ$ ✓ answer i. t. o. p (4)
		[17]

QUESTION/VRAAG 7

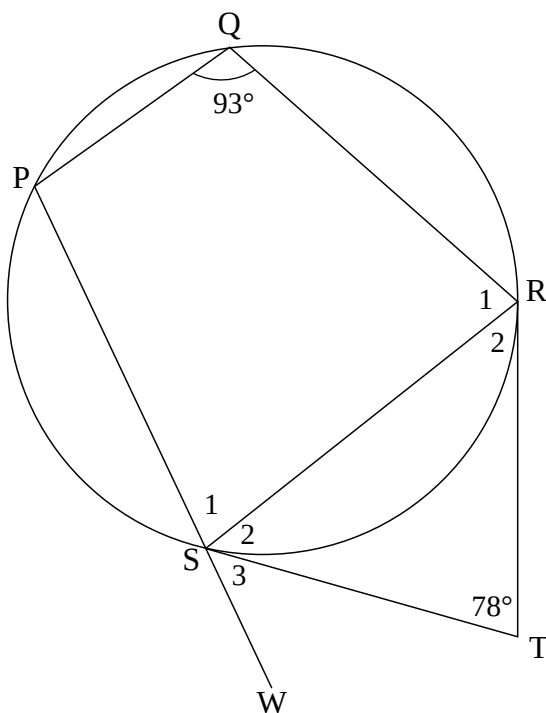
7.1		✓ both turning points ✓ both x intercepts (-30° & 150°) ✓ shape
7.2	Period = 120°	✓✓ answer
7.3	$x = -30^\circ$	✓ answer
7.4	Range of/waardeversameling van g: $y \in [-1; 1]$ Range of/Waardeversameling van $\frac{1}{2}g$: $y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$ Range of/Waardeversameling van $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2}; \frac{3}{2}\right]$ OR/OF Range of/Waardeversameling van $\frac{1}{2}g + 1$: $\frac{1}{2} \leq y \leq \frac{3}{2}$	✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
[8]		

QUESTION/VRAAG 8

8.1	In ΔSQR: $\frac{QS}{\sin x} = \frac{QR}{\sin(90^\circ + x)}$ $\frac{QS}{\sin x} = \frac{5}{\cos x}$ $QS = \frac{5 \sin x}{\cos x}$ $QS = 5 \tan x$	✓ correct use of sine rule ✓ $\sin(90^\circ + x) = \cos x$ ✓ $QS = \frac{5 \sin x}{\cos x}$ (3)
8.2	$\frac{QT}{\sin(180^\circ - 2x)} = \frac{TS}{\sin x}$ $\frac{QT}{\sin 2x} = \frac{5 \tan x}{\sin x}$ $QT = \frac{5 \tan x \sin 2x}{\sin x}$ $QT = \frac{5 \left(\frac{\sin x}{\cos x} \right) (2 \sin x \cos x)}{\sin x}$ $QT = \frac{5 \sin x (2 \sin x)}{\sin x}$ $QT = 10 \sin x$	✓ correct use of sine rule ✓ $TS = QS = 5 \tan x$ ✓ $QT = \frac{5 \tan x \sin 2x}{\sin x}$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\sin 2x = 2 \sin x \cos x$ (5)

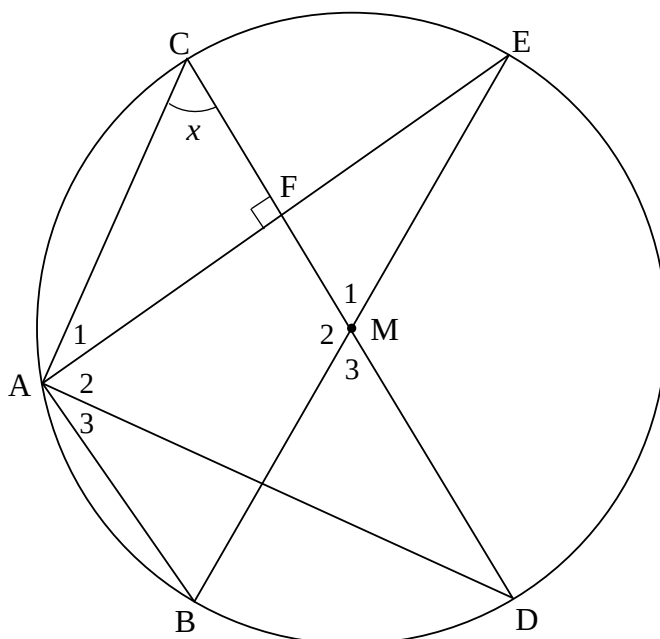
	OR/OF $QT^2 = QS^2 + TS^2 - 2QS.TS\cos\hat{QST}$ $QT^2 = (5 \tan x)^2 + (5 \tan x)^2 - 2(5 \tan x).(5 \tan x)\cos(180^\circ - 2x)$ $QT^2 = 50 \tan^2 x - 50 \tan^2 x(-\cos 2x)$ $QT^2 = 50 \tan^2 x(1 + \cos 2x)$ $QT^2 = 50 \tan^2 x(1 + 2 \cos^2 x - 1)$ $QT^2 = 50 \tan^2 x(2 \cos^2 x)$ $QT^2 = 100 \frac{\sin^2 x}{\cos^2 x} (\cos^2 x)$ $QT^2 = 100 \sin^2 x$ $QT = 10 \sin x$ OR/OF $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5 \tan x)^2 = (5 \tan x)^2 + TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ[TQ - 10 \tan x.\cos x]$ $TQ = 10 \tan x.\cos x \quad (TQ \neq 0)$ $= 10 \frac{\sin x}{\cos x} . \cos x$ $= 10 \sin x$	✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$ ✓ $\cos 2x = 2 \cos^2 x - 1$ & reduction ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $QT^2 = 100 \sin^2 x$ (5) ✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$ ✓ quadratic equation into TQ ✓ $TQ = 10 \tan x . \cos x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ (5)
8.3	Area of $\Delta TQR = \frac{1}{2} . TQ . QR \sin \hat{TQR}$ $= \frac{1}{2} (10 \sin 25^\circ)(5)(\sin 70^\circ)$ $= 9,93 \text{ unit}^2$	✓ correct substitution into the area rule ✓ answer (2)
[10]		

QUESTION/VRAAG 9



9.1	tangents from same(common) point/raaklyne vanaf dieselfde punt	✓ R	(1)
9.2.1	$\hat{S}_2 = \hat{SRT}$ $\therefore \hat{S}_2 = 51^\circ$	$[\angle s \text{ opp equal sides} / \angle e \text{ teenoor gelyke sye}]$ $[\text{sum of } \angle s \text{ in } \Delta / \text{som van } \angle e \text{ in } \Delta]$	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93^\circ$ $\hat{S}_3 = 42^\circ$ OR/OF $\hat{S}_1 = 87^\circ$ $\hat{S}_3 = 180^\circ - (87^\circ + 51^\circ)$ $\hat{S}_3 = 42^\circ$	$[\text{ext } \angle \text{ of cyclic quad} / \text{buite } \angle \text{ van koordevh}]$ $[\text{opp } \angle s \text{ of cyclic quad} / \text{teenoorst } \angle e \text{ v kdvh}]$ $[\angle s \text{ on a str line} / \angle e \text{ op reguitlyn}]$	✓ R ✓ answer (2) ✓ R ✓ answer (2)
[5]			

QUESTION/VRAAG 10

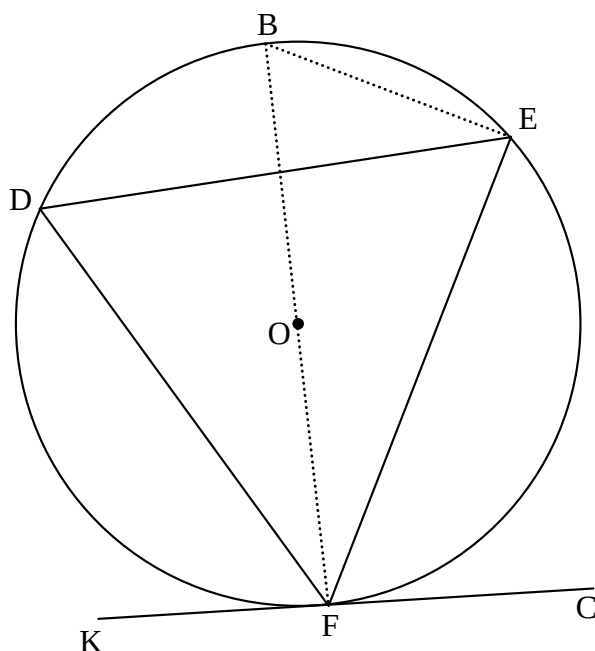


10.1	line from centre $\perp$ to chord/lyn vanaf middelpunt $\perp$ op koord	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90^\circ - x$ [sum of $\angle$ s in Δ /som van $\angle$ e in Δ] $\therefore \hat{M}_1 = 180^\circ - 2x$ [$\angle$ at centre = $2 \times$ at circumf/midpts $\angle$ = $2 \times$ omtreks $\angle$]	✓ S ✓ S ✓ R (3)
10.3	$\hat{C}\hat{A}\hat{D} = 90^\circ$ [$\angle$ in semi circle/ $\angle$ in halfsirkel] $\hat{A}_2 = 90^\circ - (90^\circ - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/omgek rkl-kd st.] OR/OF $\hat{E}\hat{M}\hat{D} = 2x$ [adj suppl $\angle$ s/aanligg suppl $\angle$ e] $\therefore \hat{A}_2 = x$ [$\angle$ at centre = $2 \times \angle$ at circumf/midpts $\angle$ = $2 \times$ omtreks $\angle$] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/omgek rkl-kd st.] OR/OF $\hat{M}_3 = 180^\circ - 2x$ [vert. opp/ regoorstaande $\angle$ e] $\therefore \hat{A}_3 = 90^\circ - x$ [$\angle$ at centre = $2 \times \angle$ at circumf/midpts $\angle$ = $2 \times$ omtreks $\angle$] $\hat{B}\hat{A}\hat{E} = 90^\circ$ [$\angle$ in semi-circle/ $\angle$ in halfsirkel] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/omgek rkl-kd st.] OR/OF	✓ S ✓ R ✓ S ✓ R ✓ S ✓ S ✓ R ✓ R ✓ S ✓ R ✓ S ✓ R (4) (4) (4)

	$CD \parallel AB$ [midpt. Thm/ <i>middelpuntst.</i>] $\hat{BAE} = 90^\circ$ [$\angle$ in semi-circle/$\angle$ in <i>halfsirkel</i>] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.$\angle$s; $CD \parallel AB$/verwiss $\angle$e] $\therefore \hat{A}_2 = x = C$ $\therefore AD$ is a tangent [converse tan-chord theorem/<i>omgek rkl-kd st.</i>] OR/OF $\hat{CAD} = 90^\circ$ [$\angle$ in semi circle/$\angle$ in <i>halfsirkel</i>] $AC = \text{diameter}$ [converse $\angle$ in semi circle/<i>omgek $\angle$ in halfsirkel</i>] $\therefore AD$ is a tangent [converse radius $\perp$ tangent/<i>omgek radius $\perp$ rkl</i>]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)
10.4	$AF = FE$ and $BM = ME$ [given & radii] $\therefore FM = \frac{1}{2} AB = 12$ units [Midpt Theorem/<i>middelpuntstelling</i>] $EM = MB = CM = 18$ units [radii] $\therefore EB = 36$ units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] $AE = 12\sqrt{5}$ or 26,83 units OR/OF $AF = FE$ and $BM = ME$ [given & radii] $\therefore FM = \frac{1}{2} AB = 12$ units [Midpt Theorem/<i>middelpuntstelling</i>] $EM = MB = CM = 18$ units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] $FE = 6\sqrt{5}$ $AE = 12\sqrt{5}$ or 26,83 units	$\checkmark$ FM = 12 $\checkmark$ R $\checkmark$ EB = 36 $\checkmark$ using Pyth correctly $\checkmark$ answer (5) $\checkmark$ FM = 12 $\checkmark$ R $\checkmark$ EM = 18 $\checkmark$ using Pyth correctly $\checkmark$ answer (5)
		[13]

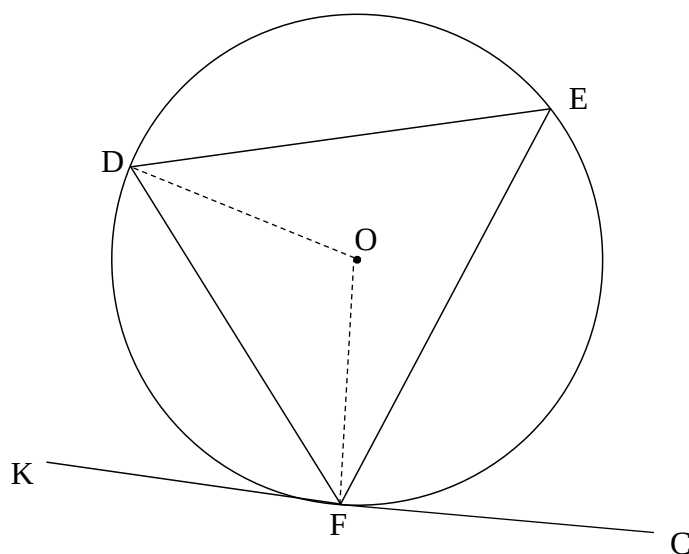
QUESTION/VRAAG 11

11.1



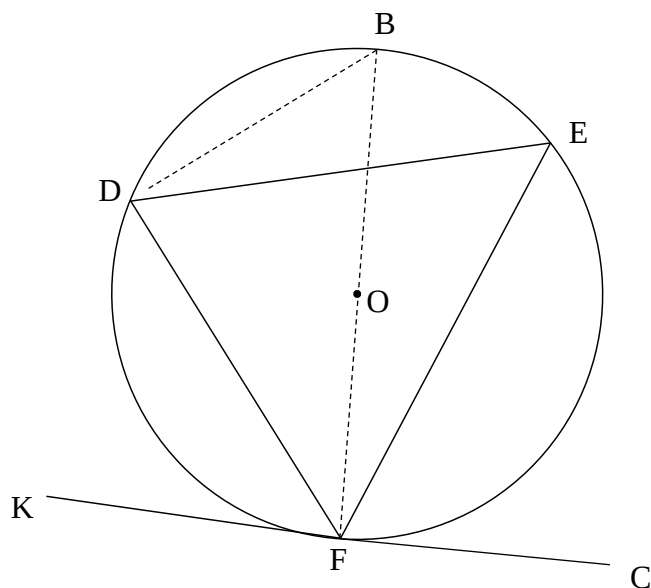
	Construction: Draw diameter BF and draw BE <i>Konstruksie: Trek middellyn BF en verbind BE</i> $\hat{B}FK = 90^\circ$ or $\hat{D}FK = 90^\circ - \hat{B}FD$ [radius $\perp$ tangent/raaklyn] $\hat{B}EF = 90^\circ$ [$\angle$ in semi-circle/semi-sirkel] $\therefore \hat{D}EF = 90^\circ - \hat{B}ED$ $= 90^\circ - \hat{B}FD$ [$\angle$s same segment/$\angle$e dieselfde segment] $\therefore \hat{D}FK = \hat{D}EF$	✓ Constr ✓ S ✓ R ✓ S ✓ S/R (5)
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OR/OF



	Construction: Draw radii DO and OF <i>Konstruksie: Trek radii DO en OF</i> $\hat{O}F\hat{K} = 90^\circ$ or $\hat{D}\hat{F}\hat{K} = 90^\circ - \hat{O}\hat{F}\hat{D}$ radius $\perp$ tangent/raaklyn] $\hat{O}\hat{D}\hat{F} = \hat{O}\hat{F}\hat{D}$ [$\angle$s opp = sides/$\angle$e teenoor = sye] $\therefore \hat{D}\hat{O}\hat{F} = 180^\circ - 2\hat{O}\hat{F}\hat{D}$ [$\angle$s of Δ/$\angle$e van Δ] $\hat{D}\hat{E}\hat{F} = 90^\circ - \hat{O}\hat{F}\hat{D}$ [$\angle$ at centre = $2 \times \angle$ circumf/ midpts $\angle = 2 \times$ omtreks $\angle$] $\therefore \hat{D}\hat{F}\hat{K} = \hat{D}\hat{E}\hat{F}$	✓ construction ✓ S ✓ R ✓ S ✓ S/R (5)
--	--	---

OR/OF



	Construction: Draw diameter BF and join BD. <i>Konstruksie: Trek middellyn BF en verbind BD.</i> $\hat{B}\hat{F}K = 90^\circ$ or $\hat{D}\hat{F}K = 90^\circ - \hat{B}\hat{F}D$ [radius $\perp$ tangent/raaklyn] $\hat{F}\hat{D}B = 90^\circ$ [$\angle$ in half circle/semi-sirkel] $\hat{B} = 90^\circ - \hat{B}\hat{F}D$ $\therefore \hat{D}\hat{F}K = \hat{B}$ but $\hat{B} = \hat{E}$ [$\angle$s same segment/$\angle$e dieselfde segment] $\therefore \hat{D}\hat{F}K = \hat{E}$	✓ construction ✓ S ✓/R ✓ S ✓ S/R (5)
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	$NKL = 180^\circ - K_4$ [adj. suppl.] $\therefore NKL = 180^\circ - NML$ [proved] $\therefore KLMN$ is a cyclic quad [opp. $\angle$ s supplementary]	$\checkmark$ R (1)
11.2.2	In $\triangle LKN \parallel \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$ s in the same seg / $\angle$ e in dieselfde sirkel segm] $\hat{L}_1 = \hat{M}_2$ [$\angle$ s in the same seg / $\angle$ e in dieselfde sirkel segm] $= \hat{K}_2$ [alt $\angle$ s; / verw $\angle$ e; $MN \parallel KS$] $N\hat{K}L = M\hat{S}K$ [$\angle$ s of $\triangle$ / $\angle$ e van $\triangle$] $\triangle LKN \parallel \triangle KSM$ OR/OF In $\triangle LKN \parallel \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$ s in the same seg / $\angle$ e in dieselfde sirkel segm] $N\hat{K}L = \hat{M}_1$ [ext $\angle$ of cyclic quad/buite $\angle$ van koordevh] $= \hat{S}_2$ [corresp $\angle$ s/ooreenk $\angle$ e; $KS \parallel NM$] $\triangle LKN \parallel \triangle KSM$ [$\angle$, $\angle$, $\angle$] OR/OF In $\triangle LKN \parallel \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$ s in the same seg / $\angle$ e in dieselfde sirkel segm] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [$\angle$ s on straight line / $\angle$ e op reguitlyn] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\triangle LKN \parallel \triangle KSM$ [$\angle$, $\angle$, $\angle$]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S/R $\checkmark$ S (5) $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R (5) $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\triangle LKN \parallel \triangle KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	$\checkmark$ S $\checkmark$ R $\checkmark$ substitution $\checkmark$ answer (4)
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line $\parallel$ one side of $\triangle$ / lyn $\parallel$ een sy v $\triangle$] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10,95$	$\checkmark$ $SM = 6$ $\checkmark$ S $\checkmark$ R $\checkmark$ answer (4)
		[23]

TOTAL/TOTAAL: 150