basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS
NATIONAL SENIOR CERTIFICATE EXAMI ATIONS

MATHEMATICS - MAY TEST 2022
P1

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.

   Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 2x - 15 = 0$ (3)

1.1.2 $5x^2 - x - 9 = 0$ (Leave your an er correct to TWO decimal places.) (3)

1.1.3 $x^2 \leq 3x$ (4)

1.2 Given: $a + \frac{64}{a} = 16$

1.2.1 Solve for a. (3)

1.2.2 Hence, solve for x: $2^x + 2^{6-x} = 16$ (3)

1.3 **Without using a calculator**, calculate the value of $\sqrt{\frac{2^{1002} + 2^{1006}}{17(2)^{998}}}$ (4)

1.4 Solve for x and y simultaneously:

$2x - y = 2$ and $\frac{1}{x} - 3y = 1$ (6)

[26]

QUESTION 2

2.1 The first term of an arithmetic sequence is -1 and the 7^{th} term is 35.

Determine:

2.1.1 the common difference of the sequence (2)

2.1.2 the number of terms in the sequence if the last term of the sequence is 473 (3)

2.1.3 The sum of the first 40 terms in this sequence (2)

2.2 75 ; 53 ; 35 ; 21 ; ... is a quadratic number pattern.

2.2.1 Write down the FIFTH term of the number pattern. (1)

2.2.2 Determine the n^{th} term of the number pattern. (4)

2.2.3 Determine the maximum value of the following number pattern:

$-15 ; -\frac{53}{5} ; -7 ; -\frac{21}{5} ; \ldots$ (4)

[16]

QUESTION 3

3.1 Consider the following geometric sequence: 1 024 ; 256 ; 64 ; ...

Calculate:

3.1.1 The 10^{th} term of the sequence (2)

3.1.2 $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ (4)

3.2 The first two terms of a geometric sequence are:

$-t^2 - 6t - 9$ and $\dfrac{t^3 + 9t^2 + 27t + 27}{2}$

Determine the values of t for which the sequence will converge. (5)

[11]

QUESTION 4

The graph of $g(x) = a\left(\dfrac{1}{3}\right)^x + 7$ passes through point E(–2 ; 10).

4.1 Calculate the value of a. (3)

4.2 Calculate the coordinates of the y-intercept of g. (2)

4.3 Consider: $h(x) = \left(\dfrac{1}{3}\right)^x$

4.3.1 Describe the translation from g to h. (2)

4.3.2 Determine the equation of the inverse of h, in the form $y = \ldots$ (2)

[9]

QUESTION 5

Consider: $g(x)=\dfrac{a}{x+p}+q$

The following information of g is given:

- Domain: $x \in R;\ x \neq -2$
- x-intercept at $K(1\ ;\ 0)$
- y-intercept at $N\left(0\ ;\ -\dfrac{1}{2}\right)$

5.1 Show that the equation of g is given by: $g(x)=\dfrac{-3}{x+2}+1$ (6)

5.2 Write down the range of g. (1)

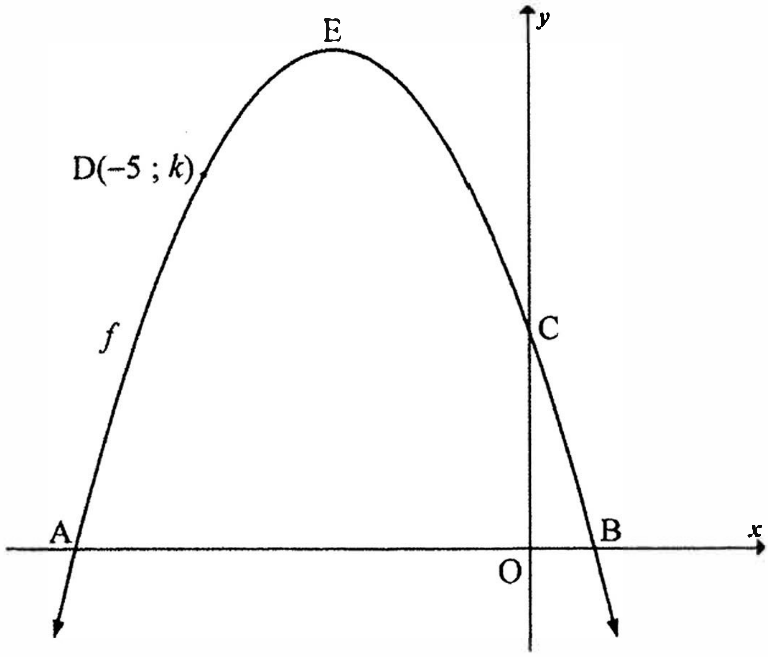
5.3 Determine the equation of h, the axis of symmetry of g, in the form $y=mx+c$, where $m>0$. (3)

5.4 Write down the coordinates of $K^{/}$, the image of K reflected over h. (2)

[12]

QUESTION 6

The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D(-5 ; k) is a point on f.



6.1	Calculate the coordinates of E, the turning point of f.	(3)
6.2	Write down the value of k.	(1)
6.3	Determine the equation of the straight line passing through C and D.	(4)
6.4	A tangent, parallel to CD, touches f at P. Determine the coordinates of P.	(4)
6.5	For which values of x will $f(x) - 12 > 0$?	(2)

[14]

QUESTION 7

7.1 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (4)

7.2 A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

7.2.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method. (4)

7.2.2 The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years? (2)

7.2.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.

Calculate the monthly payment into the sinking fund. (5)

[15]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$, if it is given that $f(x) = 4x^3 - 5x^2$ (2)

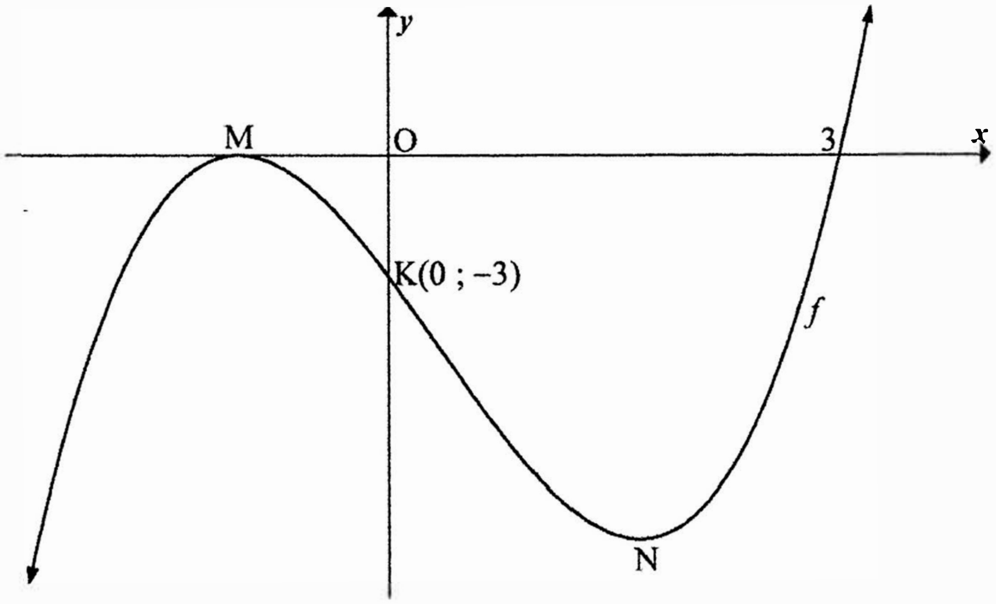
8.2.2 $D_x\left[\dfrac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4)

[11]

QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x-intercepts of f are at $(3\,;0)$ and M, where M lies on the negative x-axis.

$K(0\,;-3)$ is the y-intercept of f.
M and N are the turning points of f.



9.1	Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$.	(5)
9.2	Calculate the coordinates of N.	(5)
9.3	For which values of x will:	
	9.3.1 $f(x) < 0$	(2)
	9.3.2 f be increasing	(2)
	9.3.3 f be concave up	(3)
9.4	Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$.	(6)
		[23]

QUESTION 10

10.1 Flags from four African countries and three European countries were displayed in a row during the 2021 Olympics.

Determine:

10.1.1 The total number of possible ways in which all 7 flags from these countries could be displayed (2)

10.1.2 The probability that the flags from the African countries were displayed next to each other (3)

10.2 A and B are two independent events.

P(A) = 0,4 and P(A or B) = 0,88

Calculate P(B). (3)

10.3 There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$. Calculate the probability that the first passenger will choose a cheese sandwich. (5)

[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$