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GRADE 12

MATHEMATICS P1

NOVEMBER 2012

MEMORANDUM

MARKS: 150

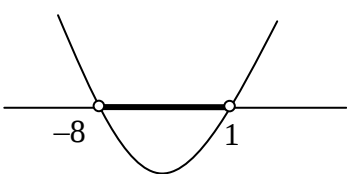
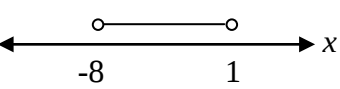
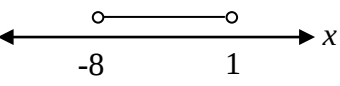
This memorandum consists of 30 pages.

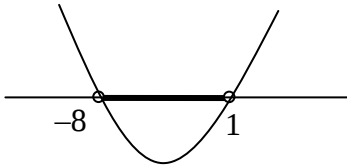
NOTE:

- If a candidate answered a question TWICE, mark the FIRST attempt ONLY.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed out question.
- Consistent accuracy applies in ALL aspects of the memorandum.

QUESTION 1

1.1.1	$(2x-1)(x+4)=0$ $x = \frac{1}{2} \quad \text{or} \quad -4$	✓ answer ✓ answer (2)
1.1.2	$3x^2 - x = 5$ $3x^2 - x - 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-5)}}{2(3)}$ $= \frac{1 \pm \sqrt{61}}{6}$ $= 1,47 \quad \text{or} \quad -1,14$ Note: if a candidate uses incorrect formula award max 1 mark (for standard form) Note: if a candidate has not rounded off correctly, penalise 1 mark OR $3x^2 - x = 5$ $x^2 - \frac{1}{3}x = \frac{5}{3}$ $\left(x - \frac{1}{6}\right)^2 = \frac{5}{3} + \frac{1}{36}$ $\left(x - \frac{1}{6}\right) = \pm \sqrt{\frac{61}{36}}$ $x = \frac{1}{6} \pm \sqrt{\frac{61}{36}}$ $= 1,47 \quad \text{or} \quad -1,14$ OR	✓ standard form ✓ subs into correct formula ✓✓ answer (4) ✓ division by 3 ✓ $\left(x - \frac{1}{6}\right) = \pm \sqrt{\frac{61}{36}}$ ✓✓ answer (4)

	$3x^2 - x = 5$ $3x^2 - x - 5 = 0$ $x^2 - \frac{x}{3} - \frac{5}{3} = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-\frac{1}{3}) \pm \sqrt{(-\frac{1}{3})^2 - 4(1)(-\frac{5}{3})}}{2(1)}$ $= \frac{\frac{1}{3} \pm \sqrt{\frac{61}{9}}}{2}$ $= 1,47 \quad \text{or} \quad -1,14$	✓ standard form ✓ subs into correct formula ✓✓ answer (4)																																						
1.1.3	$x^2 + 7x - 8 < 0$ $(x+8)(x-1) < 0$ <table style="border-collapse: collapse; margin-right: 10px;"> <tr> <td style="padding: 0 10px;">+</td> <td style="padding: 0 10px;">0</td> <td style="padding: 0 10px;">-</td> <td style="padding: 0 10px;">0</td> <td style="padding: 0 10px;">+</td> </tr> <tr> <td style="padding: 0 10px;"></td> <td style="padding: 0 10px;">-8</td> <td style="padding: 0 10px;"></td> <td style="padding: 0 10px;">1</td> <td style="padding: 0 10px;"></td> </tr> </table> OR  OR <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px 10px;">x</td> <td style="border-left: 1px solid black; padding: 5px 10px;"></td> <td style="padding: 5px 10px;"></td> <td style="border-left: 1px solid black; padding: 5px 10px;"></td> <td style="padding: 5px 10px;"></td> <td style="border-left: 1px solid black; padding: 5px 10px;"></td> <td style="padding: 5px 10px;"></td> </tr> <tr> <td style="padding: 5px 10px;">$x+8$</td> <td style="border-left: 1px solid black; padding: 5px 10px;">-</td> <td style="padding: 5px 10px;">0</td> <td style="border-left: 1px solid black; padding: 5px 10px;">+</td> <td style="padding: 5px 10px;">+</td> <td style="border-left: 1px solid black; padding: 5px 10px;">+</td> <td style="padding: 5px 10px;">+</td> </tr> <tr> <td style="padding: 5px 10px;">$x-1$</td> <td style="border-left: 1px solid black; padding: 5px 10px;">-</td> <td style="padding: 5px 10px;">-</td> <td style="border-left: 1px solid black; padding: 5px 10px;">-</td> <td style="padding: 5px 10px;">0</td> <td style="border-left: 1px solid black; padding: 5px 10px;">+</td> <td style="padding: 5px 10px;">+</td> </tr> <tr> <td style="padding: 5px 10px;">$(x+8)(x-1)$</td> <td style="border-left: 1px solid black; padding: 5px 10px;">+</td> <td style="padding: 5px 10px;">0</td> <td style="border-left: 1px solid black; padding: 5px 10px;">-</td> <td style="padding: 5px 10px;">0</td> <td style="border-left: 1px solid black; padding: 5px 10px;">+</td> <td style="padding: 5px 10px;">+</td> </tr> </table> Therefore the solution is: $-8 < x < 1$ OR $x \in (-8; 1)$ OR  OR $x^2 + 7x - 8 < 0$ $(x+8)(x-1) < 0$ $\therefore x+8 < 0$ and $x-1 > 0$ or $x+8 > 0$ and $x-1 < 0$ $x < -8$ and $x > 1$ $x > -8$ and $x < 1$ No solution Therefore the solution is: $-8 < x < 1$ OR $x \in (-8; 1)$ OR 	+	0	-	0	+		-8		1		x							$x+8$	-	0	+	+	+	+	$x-1$	-	-	-	0	+	+	$(x+8)(x-1)$	+	0	-	0	+	+	✓ factors ✓ $-8, 1$ ✓✓ answer (4) ✓ factors ✓ $-8, 1$ ✓✓ answer (4)
+	0	-	0	+																																				
	-8		1																																					
x																																								
$x+8$	-	0	+	+	+	+																																		
$x-1$	-	-	-	0	+	+																																		
$(x+8)(x-1)$	+	0	-	0	+	+																																		

	NOTE: In this alternative, award max 3/4 marks since there is no conclusion $x^2 + 7x - 8 < 0$ $(x + 8)(x - 1) < 0$ 	✓ factors ✓ - 8, 1 ✓ graph with bolded line
1.2.1	$4y - x = 4$ and $xy = 8$ $x = 4y - 4$ $(4y - 4)y = 8$ $(y - 1)y = 2$ $y^2 - y - 2 = 0$ $(y + 1)(y - 2) = 0$ $y = -1 \text{ or } y = 2$ $x = -8 \text{ or } x = 4$ $(x ; y) = (-8 ; -1) \text{ or } (4 ; 2)$ OR $4y - x = 4$ and $xy = 8$ $x = 4y - 4$ $(4y - 4)y = 8$ $(y - 1)y = 2$ By inspection $y = -1$ or $y = 2$ $x = -8 \text{ or } x = 4$ $(x ; y) = (-8 ; -1) \text{ or } (4 ; 2)$ OR $4y - x = 4$ and $xy = 8$ $x = 4y - 4$ $(4y - 4)y = 8$ $(y - 1)y = 2$ $y^2 - y - 2 = 0$ $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ $y = -1 \text{ or } y = 2$ $x = -8 \text{ or } x = 4$ $(x ; y) = (-8 ; -1) \text{ or } (4 ; 2)$	✓ $x = 4y - 4$ ✓ substitution ✓ factors ✓ y-values ✓✓ x-values (6) ✓ $x = 4y - 4$ ✓ substitution ✓✓ y-values ✓✓ x-values (6) ✓ $x = 4y - 4$ ✓ substitution ✓ subs into correct formula ✓ y-values ✓✓ x-values (6)

OR

$$4y - x = 4 \quad \text{and} \quad xy = 8$$

$$y = \frac{x}{4} + 1$$

$$x\left(\frac{x}{4} + 1\right) = 8$$

$$\frac{x^2}{4} + x - 8 = 0$$

$$x^2 + 4x - 32 = 0$$

$$(x + 8)(x - 4) = 0$$

$$x = -8 \quad \text{or} \quad x = 4$$

$$y = -1 \quad \text{or} \quad y = 2$$

$$(x ; y) = (-8 ; -1) \quad \text{or} \quad (4 ; 2)$$

OR

$$4y - x = 4 \quad \text{and} \quad xy = 8$$

$$y = \frac{x}{4} + 1$$

$$x\left(\frac{x}{4} + 1\right) = 8$$

$$\frac{x^2}{4} + x - 8 = 0$$

$$x^2 + 4x - 32 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-32)}}{2(1)}$$

$$x = -8 \quad \text{or} \quad x = 4$$

$$y = -1 \quad \text{or} \quad y = 2$$

$$(x ; y) = (-8 ; -1) \quad \text{or} \quad (4 ; 2)$$

OR

$$xy = 8 \quad \text{and} \quad 4y - x = 4$$

$$x = \frac{8}{y}$$

$$4y - \frac{8}{y} = 4$$

$$4y^2 - 4y - 8 = 0$$

$$y^2 - y - 2 = 0$$

$$(y - 2)(y + 1) = 0$$

$$y = -1 \quad \text{or} \quad y = 2$$

$$x = -8 \quad \text{or} \quad x = 4$$

$$(x ; y) = (-8 ; -1) \quad \text{or} \quad (4 ; 2)$$

$$\checkmark \quad y = \frac{x}{4} + 1$$

✓ substitution

✓ factors

✓ x-values

✓✓ y-values

(6)

$$\checkmark \quad y = \frac{x}{4} + 1$$

✓ substitution

✓ subs into correct formula

✓ x-values

✓✓ y-values

(6)

$$\checkmark \quad x = \frac{8}{y}$$

✓ substitution

✓ factors

✓ y-values

✓✓ x-values

(6)

OR

$$xy = 8 \quad \text{and} \quad 4y - x = 4$$

$$x = \frac{8}{y}$$

$$4y - \frac{8}{y} = 4$$

$$4y^2 - 4y - 8 = 0$$

$$y^2 - y - 2 = 0$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$$

$$y = -1 \quad \text{or} \quad y = 2$$

$$x = -8 \quad \text{or} \quad x = 4$$

$$(x; y) = (-8; -1) \quad \text{or} \quad (4; 2)$$

$$\checkmark x = \frac{8}{y}$$

✓ substitution

✓ subs into correct formula

✓ y-values

✓✓ x-values

(6)

OR

$$xy = 8 \quad \text{and} \quad 4y - x = 4$$

$$y = \frac{8}{x}$$

$$4\left(\frac{8}{x}\right) - x = 4$$

$$0 = x^2 + 4x - 32$$

$$0 = (x+8)(x-4)$$

$$x = -8 \quad \text{or} \quad x = 4$$

$$y = -1 \quad \text{or} \quad y = 2$$

$$(x; y) = (-8; -1) \quad \text{or} \quad (4; 2)$$

$$\checkmark y = \frac{8}{x}$$

✓ substitution

✓ factors

✓ x-values

✓✓ y-values

(6)

OR

$$xy = 8 \quad \text{and} \quad 4y - x = 4$$

$$y = \frac{8}{x}$$

$$4\left(\frac{8}{x}\right) - x = 4$$

$$0 = x^2 + 4x - 32$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-32)}}{2(1)}$$

$$x = -8 \quad \text{or} \quad x = 4$$

$$y = -1 \quad \text{or} \quad y = 2$$

$$(x; y) = (-8; -1) \quad \text{or} \quad (4; 2)$$

$$\checkmark y = \frac{8}{x}$$

✓ substitution

✓ subs into correct formula

✓ x-values

✓✓ y-values

(6)

1.2.2	$4x - y = 4$ OR $y = 4x - 4$ OR $x = \frac{y+4}{4}$ OR $4x - y - 4 = 0$ OR $x = \frac{1}{4}y + 1$	✓✓ interchanges x and y (2)
1.3.1	$\sqrt{2p+5} = 0$ $2p+5 = 0$ $2p = -5$ $p = -\frac{5}{2}$	✓ $2p+5 = 0$ or $\sqrt{2p+5} = 0$ or $\frac{-2 \pm \sqrt{0}}{7}$ ✓ answer (2)
1.3.2	$2p+5 < 0$ $p < -\frac{5}{2}$	✓ answer (1) [21]

QUESTION 2

2.1	$T_2 - T_1 = T_3 - T_2$ $2x - (3x + 1) = (3x - 7) - 2x$ $2x - 3x - 1 = 3x - 7 - 2x$ $-x - 1 = x - 7$ $-2x = -6$ $x = 3$ OR $T_2 = \frac{T_1 + T_3}{2}$ $2x = \frac{(3x + 1) + (3x - 7)}{2}$ $4x = 6x - 6$ $6 = 2x$ $x = 3$ OR $T_3 - T_1 = 2(T_2 - T_1)$ $(3x - 7) - (3x + 1) = 2(2x - (3x + 1))$ $-8 = -2x - 2$ $2x = 6$ $x = 3$	$\checkmark T_2 - T_1 = T_3 - T_2$ or $2x - (3x + 1) = (3x - 7) - 2x$ $\checkmark$ answer (2) $\checkmark T_2 = \frac{T_1 + T_3}{2}$ or $2x = \frac{(3x + 1) + (3x - 7)}{2}$ $\checkmark$ answer (2) $\checkmark T_3 - T_1 = 2(T_2 - T_1) \text{ or } (3x - 7) - (3x + 1) = 2(2x - (3x + 1))$ $\checkmark$ answer (2)
2.2.1	$T_n = a + (n - 1)d$ $T_{11} = 10 + (11 - 1)(-4)$ $= -30$ OR 10; 6; 2; -2; -6; -10; -14; -18; -22; -26; -30 ... $\therefore T_{11} = -30$	$\checkmark d = -4$ $\checkmark$ answer (2) $\checkmark$ expands sequence $\checkmark$ answer (2)

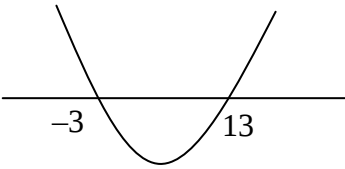
2.2.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $-560 = \frac{n}{2}[2(10) + (n-1)(-4)]$ $-1120 = -4n^2 + 24n$ $4n^2 - 24n - 1120 = 0$ $n^2 - 6n - 280 = 0$ $(n-20)(n+14) = 0$ $n = 20 \quad \text{or} \quad -14$ $\therefore n = 20 \quad \text{only}$ Note: if candidate substitutes into incorrect formula, award 0/6 OR $S_n = \frac{n}{2}[2a + (n-1)d]$ $-560 = \frac{n}{2}[2(10) + (n-1)(-4)]$ $-1120 = -4n^2 + 24n$ $4n^2 - 24n - 1120 = 0$ $n^2 - 6n - 280 = 0$ $n = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-280)}}{2(1)}$ $n = 20 \quad \text{or} \quad -14$ $\therefore n = 20 \quad \text{only}$ OR $S_n = \frac{n}{2}[2a + (n-1)d]$ $-560 = \frac{n}{2}[2(10) + (n-1)(-4)]$ $-560 = \frac{20n}{2} - \frac{4n^2}{2} + \frac{4n}{2}$ $2n^2 - 12n - 560 = 0$ $n^2 - 6n - 280 = 0$ $(n-20)(n+14) = 0$ $n = 20 \quad \text{or} \quad -14$ $\therefore n = 20 \quad \text{only}$	✓ correct formula ✓ substitution of a and d ✓ subs $S_n = -560$ ✓ $-4n^2 + 24n + 1120 = 0$ or $4n^2 - 24n - 1120 = 0$ or $n^2 - 6n - 280 = 0$ ✓ factors ✓ selects $n = 20$ only (6) ✓ correct formula ✓ substitution of a and d ✓ subs $S_n = -560$ ✓ $4n^2 - 24n - 1120 = 0$ or $-4n^2 + 24n + 1120 = 0$ or $n^2 - 6n - 280 = 0$ ✓ subs into correct formula ✓ selects $n = 20$ only (6) ✓ correct formula ✓ substitution of a and d ✓ subs $S_n = -560$ ✓ $2n^2 - 12n - 560 = 0$ or $-2n^2 + 12n + 560 = 0$ or $n^2 - 6n - 280 = 0$ ✓ factors ✓ selects $n = 20$ only (6)
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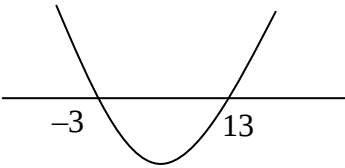
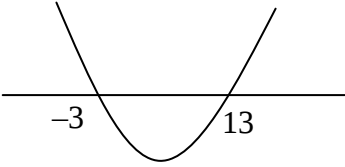
	OR $S_{11} = -110$ <table><tr><td>n</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>T_n</td><td>-34</td><td>-38</td><td>-42</td><td>-46</td><td>-50</td><td>-54</td><td>-58</td><td>-62</td><td>-66</td></tr><tr><td>S_n</td><td>-144</td><td>-182</td><td>-224</td><td>-270</td><td>-320</td><td>-374</td><td>-432</td><td>-494</td><td>-560</td></tr></table> $\therefore n = 20$	n	12	13	14	15	16	17	18	19	20	T_n	-34	-38	-42	-46	-50	-54	-58	-62	-66	S_n	-144	-182	-224	-270	-320	-374	-432	-494	-560	$\checkmark S_{11} = -110$ $\checkmark$ sequence expanded $\checkmark\checkmark$ series calculated $\checkmark\checkmark$ answer (6) [10]
n	12	13	14	15	16	17	18	19	20																							
T_n	-34	-38	-42	-46	-50	-54	-58	-62	-66																							
S_n	-144	-182	-224	-270	-320	-374	-432	-494	-560																							

QUESTION 3

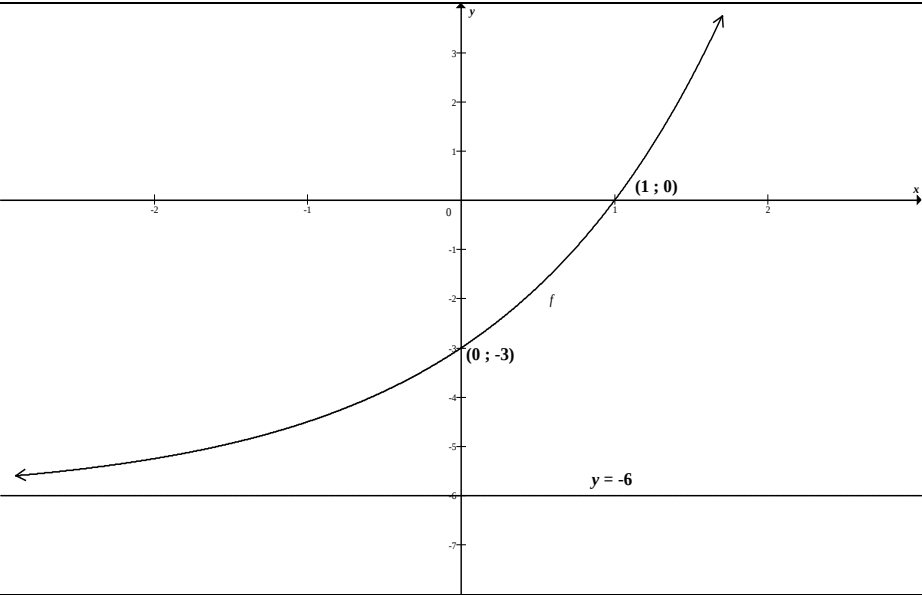
3.1.1	$T_n = ar^{n-1}$ $= 27\left(\frac{1}{3}\right)^{n-1}$ Note: The final answer can also be written as 3^{4-n} or $\left(\frac{1}{3}\right)^{n-4}$	✓ $a = 27$ and $r = \frac{1}{3}$ ✓ substitute into correct formula (2)
3.1.2	$-1 < r < 1$ or $r < 1$ OR The common ratio (r) is $\frac{1}{3}$ which is between -1 and 1. OR $-1 < \frac{1}{3} < 1$ Note: If candidate concludes series is not convergent, award 0 marks.	✓ answer (1) ✓ answer (1) ✓ answer (1)
3.1.3	$S_\infty = \frac{a}{1-r}$ $= \frac{27}{1-\frac{1}{3}}$ $= \frac{81}{2}$ or 40,5 or 41 Note: If $r > 1$ or $r < -1$ is substituted then 0/2 marks.	✓ substitution ✓ answer (2)

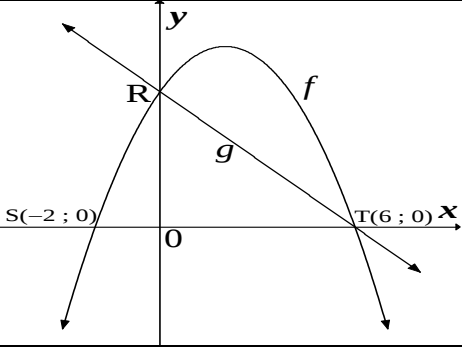
3.2	Let V be the volume of the first tank. $\frac{V}{2}; \frac{V}{4}; \frac{V}{8} \dots\dots$ Note: If candidate lets the volume of the first tank be a specific value (instead of a variable) and his/her argument follows correctly, award 4/4 marks Note: If candidate answers 'Yes' only with no justification: 1/4 marks $S_{19} = \frac{\frac{V}{2} \left[1 - \left(\frac{1}{2} \right)^{19} \right]}{1 - \frac{1}{2}}$ $= \frac{524287}{524288} V$ $= 0,9999980927 V$ $< V$ Yes, the water will fill the first tank without spilling over. OR Let V be the volume of the first tank. $\frac{V}{2}; \frac{V}{4}; \frac{V}{8} \dots\dots$ $S_{19} = \frac{\frac{V}{2} \left[1 - \left(\frac{1}{2} \right)^{19} \right]}{1 - \frac{1}{2}}$ $= V \left[1 - \left(\frac{1}{2} \right)^{19} \right]$ $< V \cdot 1$ $= V$ Yes, the water will fill the first tank without spilling over. OR Let V be the volume of the first tank. $\frac{V}{2}; \frac{V}{4}; \frac{V}{8} \dots\dots$ $S_{\infty} = \frac{\frac{V}{2}}{1 - \frac{1}{2}}$ $= V$ Since the first tank will hold the water from infinitely many tanks without spilling over, certainly: Yes, the first tank will hold the water from the other 19 tanks without spilling over.	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ answer ✓ conclusion (4) ✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ observes that $\left[1 - \left(\frac{1}{2} \right)^{19} \right] < 1$ ✓ conclusion (4) ✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓✓ correct argument (4)
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	OR If the tanks are emptied one by one, starting from the second, each tank will fill only half the remaining space , so the first tank can hold all the water from the other 19 tanks.	✓ Yes (explicit or understood from the argument.) ✓✓✓ argument (4)
3.3.1	$T_n = -2(n-5)^2 + 18$ Term 1 = -14 Term 2 = 0 Term 3 = 10	✓ -14 ✓ 0 ✓ 10 (3)
3.3.2	Term 5 OR $n = 5$ OR T_5	✓ answer (1)
3.3.3	Second difference = $2a$ Second difference = $2(-2)$ Second difference = -4 OR $\begin{array}{ccccccc} & -14 & & & 0 & & 10 \\ & \swarrow & & \swarrow & \swarrow & & \swarrow \\ & & 14 & & & 10 & \\ & \searrow & & \searrow & \searrow & & \searrow \\ & & & -4 & & & \end{array}$ Second difference = -4	✓ subs -2 into $2a$ ✓ answer (2) ✓ first differences ✓ second difference (2)
3.3.4	$-2(n-5)^2 + 18 < -110$ $-2(n-5)^2 + 128 < 0$ $-2n^2 + 20n - 50 + 128 < 0$ $-2n^2 + 20n + 78 < 0$ $n^2 - 10n - 39 > 0$ $(n-13)(n+3) > 0$ $\begin{array}{ccccccc} + & 0 & - & 0 & + \\ \hline & -3 & & 13 & \end{array}$ $n < -3 \quad \text{or} \quad n > 13$  $n \geq 14 ; n \in \mathbb{N}$ OR $n > 13 ; n \in \mathbb{N}$ OR	Note: Answer only award 2/6 marks ✓ $T_n < -110$ ✓ standard form ✓ factors ✓ critical values ✓ inequalities ✓ $n > 13$ (accept: $n \geq 14$) (6)

	$-2(n-5)^2 + 18 < -110$ $-2(n-5)^2 + 128 < 0$ $(n-5)^2 - 64 > 0$ $[(n-5)-8][(n-5)+8] > 0$ $(n-13)(n+3) > 0$ $\begin{array}{ccccccc} + & & 0 & & - & & 0 & & + \\ & & -3 & & & & 13 & & \end{array}$  $n < -3 \quad \text{or} \quad n > 13$ $n \geq 14 ; n \in \mathbb{N} \quad \text{OR} \quad n > 13 ; n \in \mathbb{N}$ OR $-2(n-5)^2 + 18 < -110$ $-2(n-5)^2 < -128$ $(n-5)^2 > 64$ $n-5 < -8 \quad \text{or} \quad n-5 > 8$ $n < -3 \quad \text{or} \quad n > 13$ $n \geq 14 ; n \in \mathbb{N} \quad \text{OR} \quad n > 13 ; n \in \mathbb{N}$ OR $T_n = -2(n-5)^2 + 18$ $T_n = -2n^2 + 20n - 32$ $-2n^2 + 20n - 32 < -110$ $-2n^2 + 20n - 78 < 0$ $n^2 - 10n - 39 > 0$ $(n-13)(n+3) > 0$ $\begin{array}{ccccccc} + & & 0 & & - & & 0 & & + \\ & & -3 & & & & 13 & & \end{array}$  $n < -3 \quad \text{or} \quad n > 13$ $n \geq 14 ; n \in \mathbb{N} \quad \text{OR} \quad n > 13 ; n \in \mathbb{N}$ OR $-14 ; 0 ; 10 ; 16 ; 18 ; 16 ; 10 ; 0 ; -14 ; -32 ; -54 ; -80 ; -110$ $n \geq 14 ; n \in \mathbb{N}$	$\checkmark T_n < -110$ $\checkmark (n-5)^2 - 64 > 0$ $\checkmark$ factors $\checkmark$ critical values $\checkmark$ inequalities $\checkmark n > 13$ (accept: $n \geq 14$) (6) $\checkmark T_n < -110$ $\checkmark 2(n-5)^2 > 128$ $\checkmark 8 \text{ and } -8$ $\checkmark n-5 > 8$ $\checkmark n-5 < -8$ $\checkmark n > 13$ (accept: $n \geq 14$) (6) $\checkmark T_n < -110$ $\checkmark$ standard form $\checkmark$ factors $\checkmark$ critical values $\checkmark$ inequalities $\checkmark n > 13$ (accept: $n \geq 14$) (6) $\checkmark \checkmark \checkmark \checkmark$ expansion $\checkmark \checkmark$ conclusion of $n \geq 14$ (accept $n > 13$) (6) [21]
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QUESTION 4

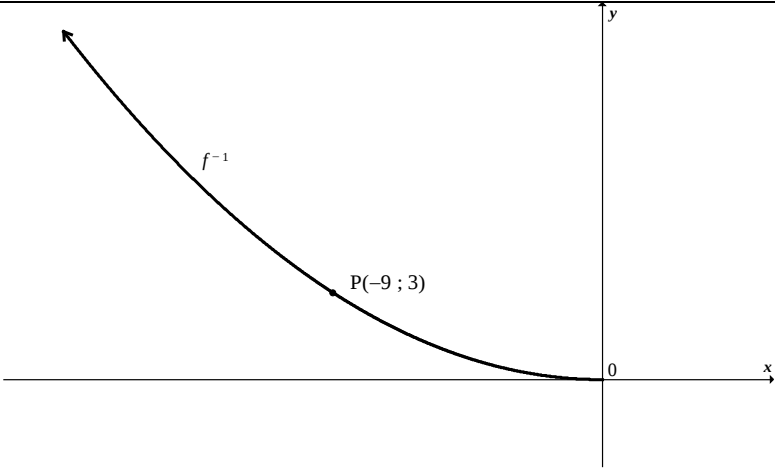
4.1.1	$y = 3 \cdot 2^0 - 6$ $y = 3 - 6$ $y = -3 \quad (0; -3)$	✓ answer (1)
4.1.2	$0 = 3 \cdot 2^x - 6$ $3 \cdot 2^x = 6$ $2^x = 2^1$ $x = 1 \quad (1; 0)$ Note: If a candidate interchanges question 4.1.1 and 4.1.2: 0/3 marks Note: If a candidate says that $3 \cdot 2^x = 6^x$ (i.e. wrong mathematics) s/he will arrive at correct answer BUT award max 1/2	✓ $y = 0$ ✓ x-value (2)
4.1.3		✓ intercepts ✓ asymptote ✓ shape (3)
4.1.4	$y > -6 \quad \text{OR} \quad (-6; \infty)$	✓ answer (1)

4.2		
4.2.1	$y = -2x + d$ $0 = (-2)(6) + d$ $d = 12$ OR $y - y_1 = m(x - x_1)$ $y - 0 = -2(x - 6)$ $y = -2x + 12$ $\therefore d = 12$ OR Since $m = -2$ and $m = \frac{-d}{6}$ $-2 = \frac{-d}{6}$ $d = 12$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
4.2.2	$y = a(x - 6)(x + 2)$ $12 = a(0 - 6)(0 + 2)$ $a = -1$ $y = -(x^2 - 4x - 12)$ $= -x^2 + 4x + 12$ OR $y = ax^2 + bx + 12$ $0 = a(-2)^2 + b(-2) + 12 \quad \text{i.e.} \quad 0 = 4a - 2b + 12$ $0 = a(6)^2 + b(6) + 12 \quad \text{i.e.} \quad 0 = 36a + 6b + 12$ $0 = 4a - 2(4) + 12$ $a = -1$ $y = -x^2 + 4x + 12$ Note: No marks for answer only.	✓ $y = a(x - 6)(x + 2)$ ✓ subs R(0 ; 12) ✓ a-value ✓ $y = -x^2 + 4x + 12$ (4) ✓ $y = ax^2 + bx + 12$ ✓ subs S(-2; 0) and T(6; 0) ✓ b-value ✓ $y = -x^2 + 4x + 12$ (4)

	OR $y = a(x - 2)^2 + q$ $0 = a(-2 - 2)^2 + q \quad \text{or} \quad 0 = a(6 - 2)^2 + q \quad \text{i.e.} \quad 0 = 16a + q$ $12 = a(0 - 2)^2 + q \quad \text{i.e.} \quad \frac{12 = 4a + q}{12 = -12a}$ $a = -1$ $q = 16$ $y = -(x - 2)^2 + 16$ $= -(x^2 - 4x + 4) + 16$ $= -x^2 + 4x + 12$ OR $y = a(x - 6)(x + 2)$ $= a(x^2 - 4x - 12)$ $= -(x^2 - 4x - 12)$ $= -x^2 + 4x + 12$	✓ $y = a(x - 2)^2 + q$ ✓ subs R(0 ; 12) and S(-2 ; 0) (or T(6 ; 0)) ✓ a-value ✓ $y = -x^2 + 4x + 12$ (4) ✓ $y = a(x - 6)(x + 2)$ ✓ expand ✓ a-value ✓ $y = -x^2 + 4x + 12$ (4)
4.2.3	$\frac{dy}{dx} = 0$ $-2x + 4 = 0$ $x = 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is (2 ; 16) OR $x = -\frac{b}{2a}$ $= -\frac{4}{2(-1)}$ $= 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is (2 ; 16) OR $f(x) = -(x - 2)^2 + 16$ TP of f is (2 ; 16)	✓ x-value ✓ y-value (2) ✓ x-value ✓ y-value (2) ✓ x-value ✓ y-value (2)

	OR $x = \frac{-2+6}{2}$ $= 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is (2 ; 16)	✓ x-value ✓ y-value (2)
4.2.4	$k < 16$ OR $(-\infty; 16)$	✓✓ answer (2)
4.2.5	Maximum value of $h(x) = 3^{f(x)-12}$ occurs at max value of $f(x)$ Maximum value = 3^{16-12} $= 81$ OR Maximum value of $h(x) = 3^{f(x)-12}$ occurs at max value of $f(x)$ $h(2) = 3^{f(2)-12}$ $= 3^{16-12}$ $= 3^4$ or 81 OR $f(x) - 12 = -x^2 + 4x$ $= x(4 - x)$ which has a maximum value of $f(2) = 4$ $\therefore$ Maximum value of $h(x)$ is 3^4 or 81	✓✓ subs 16 for $f(x)$ ✓ 3^4 or 81 (3) ✓✓ subs 16 for $f(x)$ ✓ 3^4 or 81 (3) ✓✓ subs $f(2) = 4$ ✓ 3^4 or 81 (3) [20]

QUESTION 5

5.1	$0 \leq x \leq 3$ OR $[0;3]$	Note: if the candidate gives $0 < x < 3$, award 1/2 marks	✓ $0 \leq x$ ✓ $x \leq 3$ (2)
5.2	$f^{-1}: \quad x = -\sqrt{27y}$ $x^2 = 27y$ $y = \frac{x^2}{27} \quad x \leq 0 \text{ OR } (-\infty; 0]$		✓ interchange x - and y - values ✓ $y = \frac{x^2}{27}$ ✓ $x \leq 0$ or $(-\infty; 0)$ (3)
5.3			✓ shape ✓ end at origin ✓ any other point on the graph (3)
5.4	Reflection about the x -axis OR $(x; y) \rightarrow (x; -y); x \geq 0$		✓ answer (1) ✓ answer (1) [9]

QUESTION 6

$f(x) = \frac{a}{x-5} + 1$ $0 = \frac{a}{(2)-5} + 1$ $-1 = \frac{a}{-3}$ $a = 3$ $f(x) = \frac{3}{x-5} + 1$ OR $(x-5)(y-1) = k$ $(2-5)(0-1) = k$ $k = 3$ $(x-5)(y-1) = 3$ $y = \frac{3}{x-5} + 1$	NOTE: $f(x) = \frac{x-2}{x-5}$ as an alternative simplified form.	✓ $x - 5$ ✓ $+ 1$ ✓ substitution of $(2 ; 0)$ ✓ $a = 3$ (4) ✓ $(x - 5)$ ✓ $(y - 1)$ ✓ substitution of $(2 ; 0)$ ✓ $k = 3$ (4) [4]
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QUESTION 7

7.1.1	$A = P(1-i)^n$ $= 120\,000(1-0,09)^5$ $= R74\,883,86$	NOTE: Incorrect formula (in 7.1.1 or 7.1.2) award max 1/3 marks	✓ i, n and P identified ✓ subs into correct formula ✓ answer (3)
7.1.2	$A = P(1+i)^n$ $= 120\,000(1+0,07)^5$ $= R168\,306,21$		✓ i, n and P identified ✓ subs into correct formula ✓ answer (3)
7.1.3	Sinking fund needed: $F_v = R\,90\,000$ $F_v = \frac{x[(1+i)^n - 1]}{i}$ $90\,000 = \frac{x \left[\left(1 + \frac{0,085}{12} \right)^{61} - 1 \right]}{\frac{0,085}{12}}$ $x = R\,1\,184,68$	NOTE: Incorrect formula award max 2/5 marks	✓ $F_v = R\,90\,000$ ✓ $i = \frac{0,085}{12} = \frac{17}{2400}$ in annuity formula ✓ $n = 61$ ✓ subs into correct formula ✓ answer (5)

OR

Consider the scenario as money deposited at the beginning of every month, but in the last month an additional payment was made at the end of the month:

$$F_v = \frac{x(1+i)\left[(1+i)^n - 1\right]}{i} + x$$

$$= x \left(\frac{(1+i)\left[(1+i)^n - 1\right]}{i} + 1 \right)$$

$$90\,000 = x \left(\frac{\left(1 + \frac{0,085}{12}\right) \left[\left(1 + \frac{0,085}{12}\right)^{60} - 1\right]}{\frac{0,085}{12}} + 1 \right)$$

$$x = \frac{90\,000 \left(\frac{0,085}{12}\right)}{\left(1 + \frac{0,085}{12}\right) \left[\left(1 + \frac{0,085}{12}\right)^{60} - 1\right] + \left(\frac{0,085}{12}\right)}$$

$$= R1184,68$$

OR

Present value of sinking fund needed:

$$90\,000 = P_v \left(1 + \frac{0,085}{12}\right)^{61}$$

$$P_v = R58513,03$$

Using the present value formula:

$$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$58513,03 = \frac{x \left[1 - \left(1 + \frac{0,085}{12}\right)^{-61}\right]}{\frac{0,085}{12}}$$

$$x = R1\,184,68$$

$$\checkmark i = \frac{0,085}{12} = \frac{17}{2400}$$

in annuity formula

$$\checkmark n = 60 \text{ in annuity formula}$$

$$\checkmark F_v = R\,90\,000$$

✓ subs into correct formula

✓ answer

(5)

$$\checkmark i = \frac{0,085}{12} = \frac{17}{2400}$$

in annuity formula

$$\checkmark n = 61$$

$$\checkmark P_v = R58513,03$$

✓ subs into correct formula

✓ answer

(5)

7.2	$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $900\,000 = \frac{18\,000 \left[1 - \left(1 + \frac{0,105}{12} \right)^{-n} \right]}{\frac{0,105}{12}}$ $1 - \frac{900\,000 \left(\frac{0,105}{12} \right)}{18\,000} = \left(1 + \frac{0,105}{12} \right)^{-n}$ $-n = \log_{\left(1 + \frac{0,105}{12} \right)} \frac{9}{16}$ $n = 66,04 \text{ months}$ She will be able to maintain her current lifestyle for a little more than 66 months using her pension money. OR $P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $900\,000 = \frac{18\,000 \left[1 - \left(1 + \frac{0,105}{12} \right)^{-n} \right]}{\frac{0,105}{12}}$ $1 - \frac{900\,000 \left(\frac{0,105}{12} \right)}{18\,000} = \left(1 + \frac{0,105}{12} \right)^{-n}$ $-n \log \left(1 + \frac{0,105}{12} \right) = \log \frac{9}{16}$ $n = 66,04 \text{ months}$ She will be able to maintain her current lifestyle for a little more than 66 months using her pension money.	Note: If F_v formula used, possibly award one each for x, i, use of logs: max 3/6 marks If any other incorrect formula is used, award 0/6 marks ✓ $x = 18\,000$ ✓ $i = \frac{0,105}{12}$ in annuity formula ✓ subs into correct formula ✓ simplification ✓ use of logs ✓ answer in months (6) Note: If candidate rounds off early in Question 7.2 (and obtain 58 months), penalise 1 mark ✓ $x = 18\,000$ ✓ $i = \frac{0,105}{12}$ in annuity formula ✓ subs into correct formula ✓ simplification ✓ use of logs ✓ answer in months (6)
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	OR $A = F_v$ $P(1+i)^n = \frac{x[(1+i)^n - 1]}{i}$ $900\,000\left(1 + \frac{0,105}{12}\right)^n = \frac{18\,000\left[\left(1 + \frac{0,105}{12}\right)^n - 1\right]}{\frac{0,105}{12}}$ $\frac{0,105}{12} \times 900\,000\left(1 + \frac{0,105}{12}\right)^n = 18\,000\left(1 + \frac{0,105}{12}\right)^n - 18\,000$ $18\,000 = 18\,000\left(1 + \frac{0,105}{12}\right)^n - \frac{0,105}{12} \times 900\,000\left(1 + \frac{0,105}{12}\right)^n$ $18\,000 = \left(1 + \frac{0,105}{12}\right)^n \left(18\,000 - \frac{0,105}{12} \times 900\,000\right)$ $18\,000 \div \left(18\,000 - \frac{0,105}{12} \times 900\,000\right) = \left(1 + \frac{0,105}{12}\right)^n$ $\frac{16}{9} = \left(1 + \frac{0,105}{12}\right)^n$ $-n = \log_{\left(1 + \frac{0,105}{12}\right)} \frac{9}{16}$ $n = 66,04 \text{ months}$ She will be able to maintain her current lifestyle for a little more than 66 months using her pension money.	✓ $x = 18\,000$ ✓ $i = \frac{0,105}{12}$ in annuity formula ✓ subs into correct formula ✓ simplification ✓ use of logs ✓ answer in months (6) [17]
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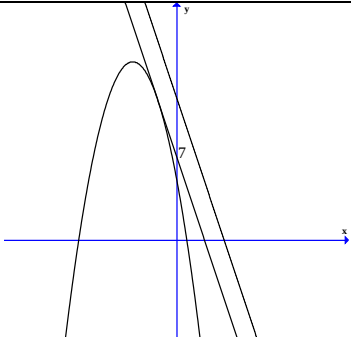
QUESTION 8

8.1	$f(x) = 2x^2 - 5$ $f(x+h) = 2(x+h)^2 - 5$ $= 2x^2 + 4xh + 2h^2 - 5$ $f(x+h) - f(x) = 4xh + 2h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h}$ $= \lim_{h \rightarrow 0} (4x + 2h)$ $= 4x$ OR	Note: If candidate makes a notation error Penalise 1 mark Note: If candidate uses differentiation rules Award 0/5 marks ✓ substitution of $x + h$ ✓ simplification to $4xh + 2h^2$ ✓ formula ✓ $\lim_{h \rightarrow 0} (4x + 2h)$ ✓ answer (5)
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	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{[2(x+h)^2 - 5] - (2x^2 - 5)}{h}$ $= \lim_{h \rightarrow 0} \frac{[2(x^2 + 2xh + h^2) - 5] - 2x^2 + 5}{h}$ $= \lim_{h \rightarrow 0} \frac{[2x^2 + 4xh + 2h^2 - 5] - 2x^2 + 5}{h}$ $= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h}$ $= \lim_{h \rightarrow 0} (4x + 2h)$ $= 4x$	✓ formula ✓ substitution of $x + h$ ✓ simplification to $\frac{4xh + 2h^2}{h}$ ✓ $\lim_{h \rightarrow 0} (4x + 2h)$ ✓ answer (5)
8.2	$\frac{dy}{dx} = -4x^{-5} + 6x^2 - \frac{1}{5}$ $= -\frac{4}{x^5} + 6x^2 - \frac{1}{5}$ Note: notation error penalise 1 mark Note: candidates do NOT need to give their answer with positive exponents	✓ $-4x^{-5}$ ✓ $6x^2$ ✓ $-\frac{1}{5}$ (3)
8.3.1	$g(x) = \frac{x^2 + x - 2}{x - 1}$ $= \frac{(x+2)(x-1)}{x-1}$ $= x + 2 \quad (x \neq 1)$ $g'(x) = 1 \quad (x \neq 1)$	✓ simplification ✓ answer (2)
8.3.2	The function is undefined at $x = 1$. OR Division by zero is undefined. OR The denominator cannot be zero. OR In the definition of the derivative, $g'(1) = \lim_{h \rightarrow 0} \frac{g(1+h) - g(1)}{h}$, but $g(1)$ does not exist.	✓ answer (1) [11]

QUESTION 9

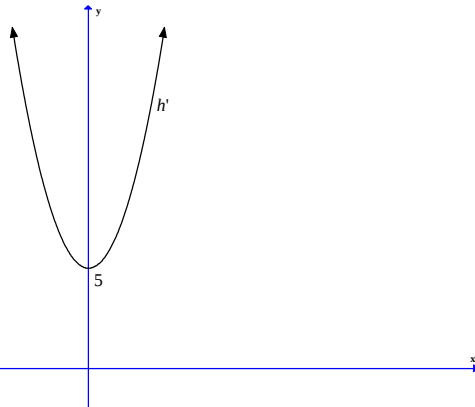
9.1.1	$f(x) = -x^3 - x^2 + 16x + 16$ $f'(x) = -3x^2 - 2x + 16$ $0 = -3x^2 - 2x + 16$ $3x^2 + 2x - 16 = 0$ $(3x + 8)(x - 2) = 0$ $x = -\frac{8}{3} \text{ or } x = 2$ OR $f(x) = -x^3 - x^2 + 16x + 16$ $f'(x) = -3x^2 - 2x + 16$ $0 = -3x^2 - 2x + 16$ $0 = 3x^2 + 2x - 16$ $x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-16)}}{2(3)}$ $x = -\frac{8}{3} \text{ or } x = 2$	Note: if neither $f'(x) = 0$ nor $0 = -3x^2 - 2x + 16$ explicitly stated, award maximum 3/4 marks $\checkmark f'(x) = -3x^2 - 2x + 16$ $\checkmark f'(x) = 0 \text{ or } 0 = -3x^2 - 2x + 16$ $\checkmark \text{ factors}$ $\checkmark x \text{ values}$ (4) $\checkmark f'(x) = -3x^2 - 2x + 16$ $\checkmark f'(x) = 0 \text{ or } 0 = -3x^2 - 2x + 16$ $\checkmark \text{ subs into formula}$ $\checkmark x \text{ values}$ (4)
9.1.2	$f''(x) = 0$ $-6x - 2 = 0$ $x = -\frac{1}{3}$ OR $x = \frac{-\frac{8}{3} + 2}{2}$ $x = -\frac{1}{3}$ OR $f'(x) = -3x^2 - 2x + 16$ $x = \frac{-(-2)}{2(-3)}$ $= -\frac{1}{3}$ OR	$\checkmark f''(x) = -6x - 2$ $\checkmark -6x - 2 = 0$ $\checkmark \text{ answer}$ (3) $\checkmark x = \frac{-\frac{8}{3} + 2}{2}$ $\checkmark \checkmark \text{ answer}$ (3) $\checkmark \checkmark x = \frac{-(-2)}{2(-3)}$ $\checkmark \text{ answer}$ (3)

	$f(x) = -x^3 - x^2 + 16x + 16$ $x = \frac{-(-1)}{3(-1)}$ $= -\frac{1}{3}$	$\checkmark\checkmark \quad x = \frac{-(-1)}{3(-1)}$ $\checkmark \text{ answer}$ (3)
9.2.1	$g(x) = -2x^2 - 9x + 5$ $g(-1) = -2(-1)^2 - 9(-1) + 5$ $= 12$ $g'(x) = -4x - 9$ $m_{\text{tan}} = -4(-1) - 9$ $= -5$ $y = -5x + c$ $12 = -5(-1) + c$ $c = 7$ $y = -5x + 7$ OR $g(x) = -2x^2 - 9x + 5$ $g(-1) = -2(-1)^2 - 9(-1) + 5$ $= 12$ $g'(x) = -4x - 9$ $m_{\text{tan}} = -4(-1) - 9$ $= -5$ $y - 12 = -5(x + 1)$ $y = -5x + 7$	$\checkmark \quad g(-1) = 12$ $\checkmark \quad g'(x) = -4x - 9$ $\checkmark \quad m_{\text{tan}} = -5$ $\checkmark \text{ answer}$ (4) $\checkmark \quad g(-1) = 12$ $\checkmark \quad g'(x) = -4x - 9$ $\checkmark \quad m_{\text{tan}} = -5$ $\checkmark \text{ answer}$ (4)
9.2.2	 $q > 7$ OR $y = -5x + q \text{ and } y = -2x^2 - 9x + 5$ $-5x + q = -2x^2 - 9x + 5$ $q = -2(x+1)^2 + 7$ $\therefore q > 7$	$\checkmark \text{ sketch}$ $\checkmark 7$ $\checkmark \text{ correct inequality}$ (3) $\checkmark \text{ method}$ $\checkmark 7$ $\checkmark \text{ correct inequality}$ (3)

	OR $y = -5x + q \text{ and } y = -2x^2 - 9x + 5$ $-5x + q = -2x^2 - 9x + 5$ $2x^2 + 4x + q - 5 = 0$ $x = \frac{-4 \pm \sqrt{16 - 4(2)(q - 5)}}{2(2)}$ $x = \frac{-4 \pm \sqrt{56 - 8q}}{4}$ $56 - 8q < 0$ $q > 7$ OR Since $g(-1) = 12$ and at $x = -1$, tangent equation is $y = -5x + 7$, $y = -5x + q$ not intersecting $g \Rightarrow$ $12 < -5(-1) + q$ $12 - 5 < q$ $7 < q$	✓ method ✓ 7 ✓ correct inequality (3) ✓ method ✓ 7 ✓ correct inequality (3)
9.3	$h'(x) = 12x^2 + 5$ For all values of x: $x^2 \geq 0$ $12x^2 \geq 0$ $12x^2 + 5 \geq 5$ $12x^2 + 5 > 0$ For all values of x: $h'(x) > 0$ All tangents drawn to h will have a positive gradient. It will never be possible to draw a tangent with a negative gradient to the graph of h. OR $h'(x) = 12x^2 + 5$ Suppose $h'(x) < 0$ and try to solve for x: $12x^2 + 5 < 0$ $x^2 < -\frac{5}{12}$ but x^2 is always positive $\therefore$ no solution for x $\therefore h'(x) \geq 0$ for all $x \in R$ i.e. there are no tangents with negative slopes	✓ $h'(x) = 12x^2 + 5$ ✓ clearly argues that $h'(x) > 0$ ✓ conclusion (3) ✓ $h'(x) = 12x^2 + 5$ ✓ clearly argues that $h'(x) < 0$ is impossible ✓ conclusion (3)

OR

$$h'(x) = 12x^2 + 5$$



Since clearly $h'(x) > 0$ for all $x \in \mathbb{R}$,
it will never be possible to draw a tangent with a negative gradient to
the graph of h .

✓ $h'(x) = 12x^2 + 5$

✓ argues $h'(x) > 0$ by
drawing a sketch

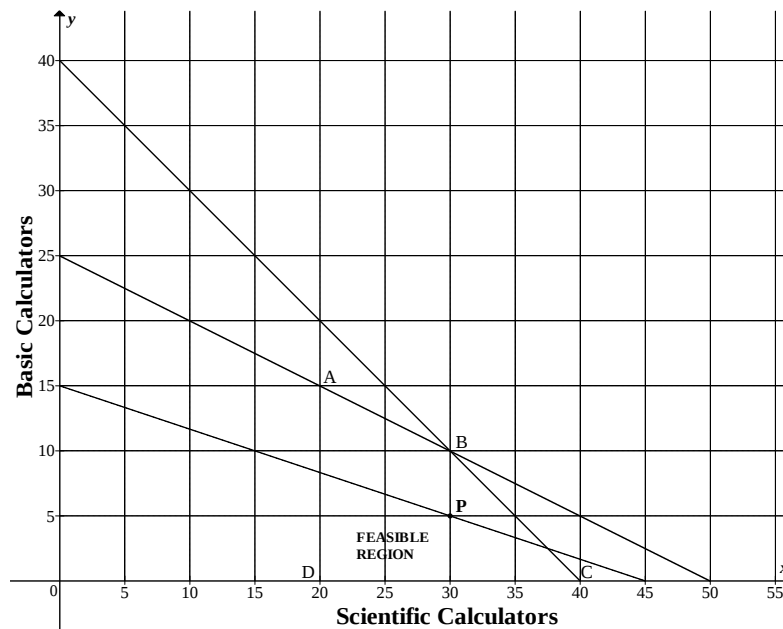
✓ conclusion

(3)

[17]

QUESTION 10

10.1	$s(t) = 2t^2 - 18t + 45$ $s'(t) = 4t - 18$ $s'(0) = 4(0) - 18$ $= -18 \text{ m/s}$	Note: answer only award 0/3 marks	✓ $s'(t)$ ✓ subs $t = 0$ into $s'(t)$ formula ✓ answer (3)
10.2	$s''(t) = 4 \text{ m/s}^2$		✓ answer (1)
10.3	$4t - 18 = 0$ $4t = 18$ $t = \frac{9}{2} \text{ seconds or } 4,5 \text{ seconds}$ OR $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ $t = \frac{9}{2} \text{ seconds or } 4,5 \text{ seconds}$ OR $s(t) = 2t^2 - 18t + 45$ $t = -\frac{-18}{2(2)}$ $t = \frac{9}{2} \text{ seconds or } 4,5 \text{ seconds}$		✓ $s'(t) = 0$ ✓ answer (2) ✓ $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ ✓ answer (2) ✓ $t = -\frac{-18}{2(2)}$ ✓ answer (2) [6]

QUESTION 11

11.1	No, because (15 ; 5) does not lie within the feasible region. OR No, because according to the constraints, the x-value (number of scientific calculators) must be at least 20.	✓ answer (with motivation) (1)
11.2	$ \begin{array}{l} x \geq 20 \\ x + 2y \leq 50 \\ x + y \leq 40 \\ y \geq 0 \end{array} \text{ OR } \begin{array}{l} x \geq 20 \\ y \leq -\frac{1}{2}x + 25 \\ y \leq -x + 40 \\ y \geq 0 \end{array} \text{ OR } \begin{array}{l} x \geq 20 \\ \frac{y}{25} + \frac{x}{50} \leq 1 \\ \frac{y}{40} + \frac{x}{40} \leq 1 \\ y \geq 0 \end{array} \text{ OR } \begin{array}{l} x \geq 20 \\ 40x + 40y \leq 1600 \\ 25x + 50y \leq 1250 \\ y \geq 0 \end{array} $	$ \begin{array}{l} \checkmark x \geq 20 \\ \checkmark\checkmark x + 2y \leq 50 \\ \checkmark\checkmark x + y \leq 40 \\ \checkmark y \geq 0 \end{array} $ (6)
11.3.1	A	✓ answer (1)
11.3.2	All points on the search line yield the same profit. Hence no such point exists. OR If such an $(x ; y)$ exists, $Q = x + 3y$ and $y = -\frac{1}{3}x + 15$ so $45 = 3y + x = Q$ $Q = 4\ 500$ Hence, there is no such point.	$ \begin{array}{l} \checkmark\checkmark \text{ No point exists} \\ \checkmark\checkmark \text{ No point exists} \end{array} $ (2) (2)

11.3.3	$Q = ax + by$ $y = -\frac{a}{b}x + \frac{Q}{b}$ $-1 \leq -\frac{a}{b} \leq -\frac{1}{2}$ $\frac{1}{2} \leq \frac{a}{b} \leq 1$ The maximum value of $\frac{a}{b}$ is 1.	$\checkmark y = -\frac{a}{b}x + \frac{Q}{b}$ $\checkmark -1 \leq -\frac{a}{b}$ $\checkmark \frac{a}{b} \leq 1$ $\checkmark$ answer (4) [14]
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TOTAL: 150