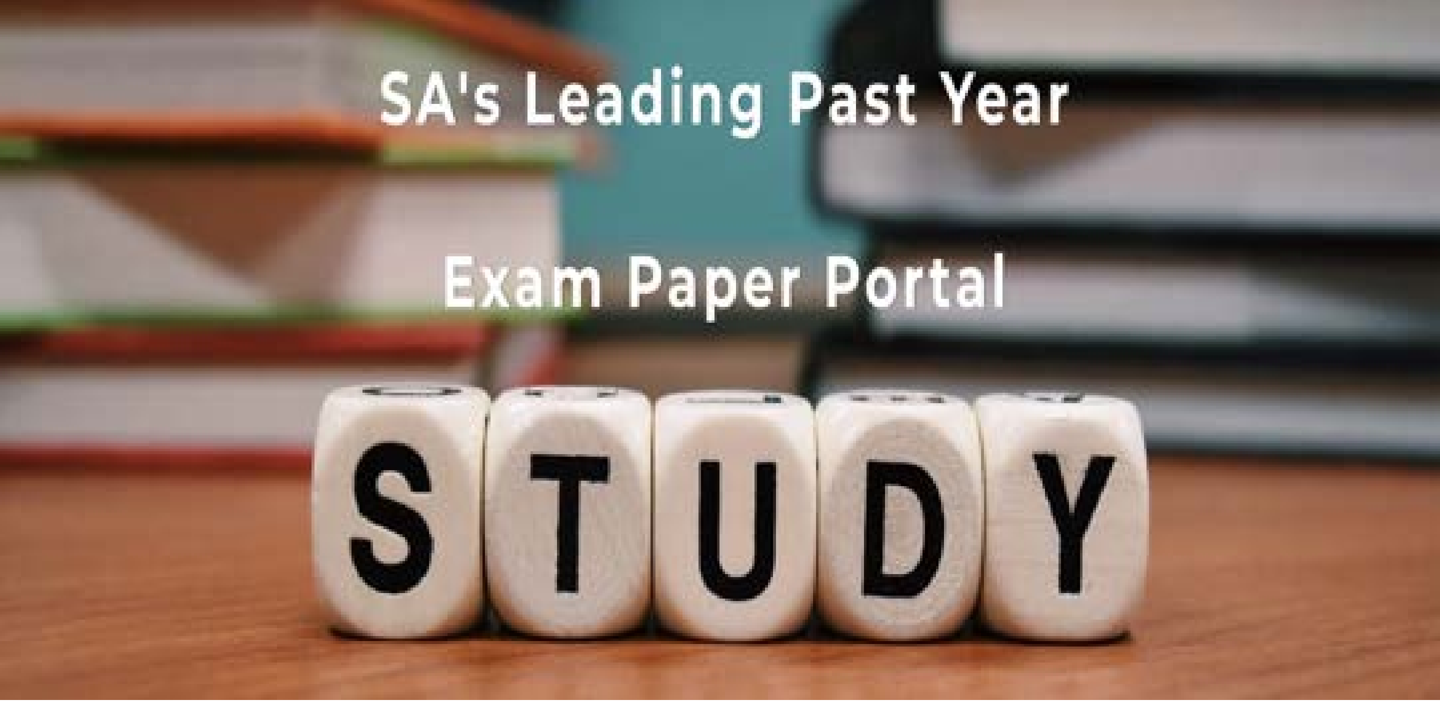
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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

EXEMPLAR 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 3 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. THREE diagram sheets for QUESTION 2.1, QUESTION 8.2, QUESTION 9, QUESTION 10.1, and QUESTION 10.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

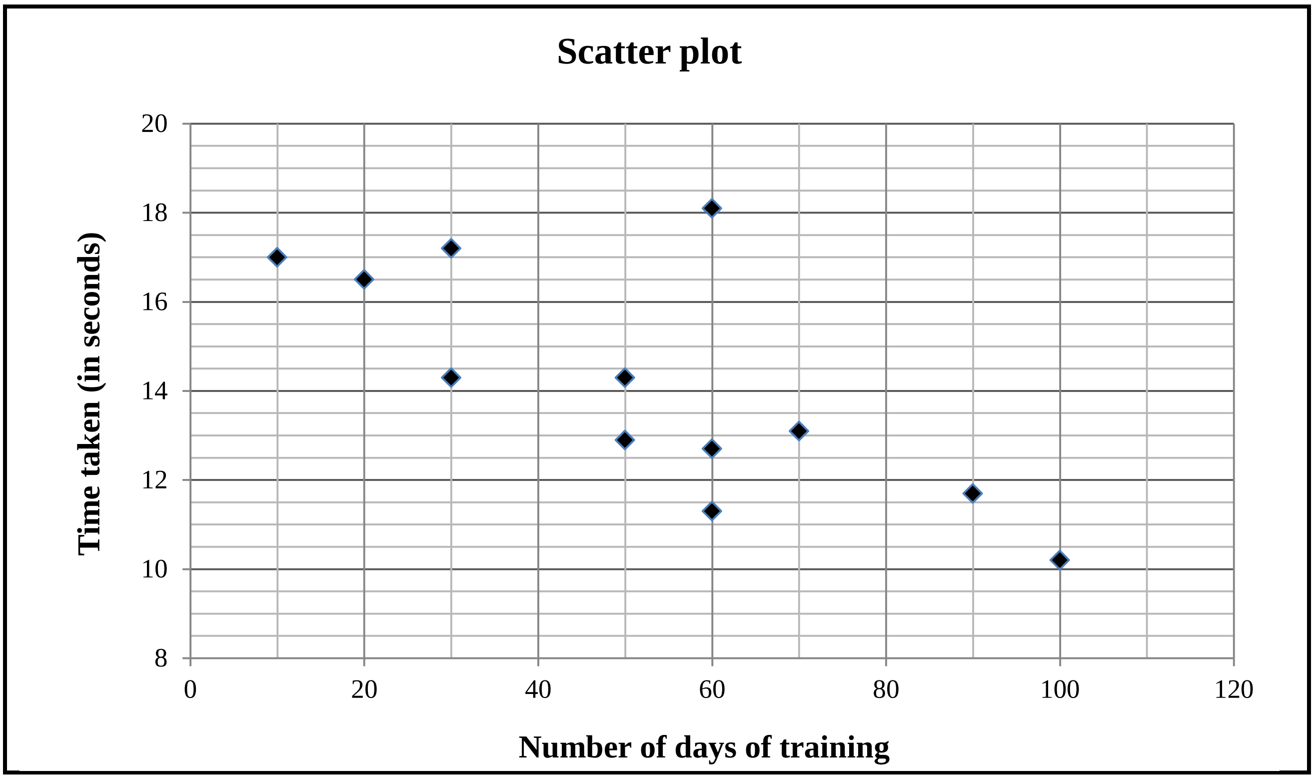
8. Number the answers correctly according to the numbering system used in this question paper.

9. Write neatly and legibly.

QUESTION 1

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



1.1 Discuss the trend of the data collected. (1)

1.2 Identify any outlier(s) in the data. (1)

1.3 Calculate the equation of the least squares regression line. (4)

1.4 Predict the time taken to run the 100 m sprint for an athlete training for 45 days. (2)

1.5 Calculate the correlation coefficient. (2)

1.6 Comment on the strength of the relationship between the variables. (1)
[11]

QUESTION 2

The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time (hours)	**Cumulative frequency**
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

2.1 Draw an ogive (cumulative frequency curve) on DIAGRAM SHEET 1 to represent the above data. (3)

2.2 Write down the modal class of the data. (1)

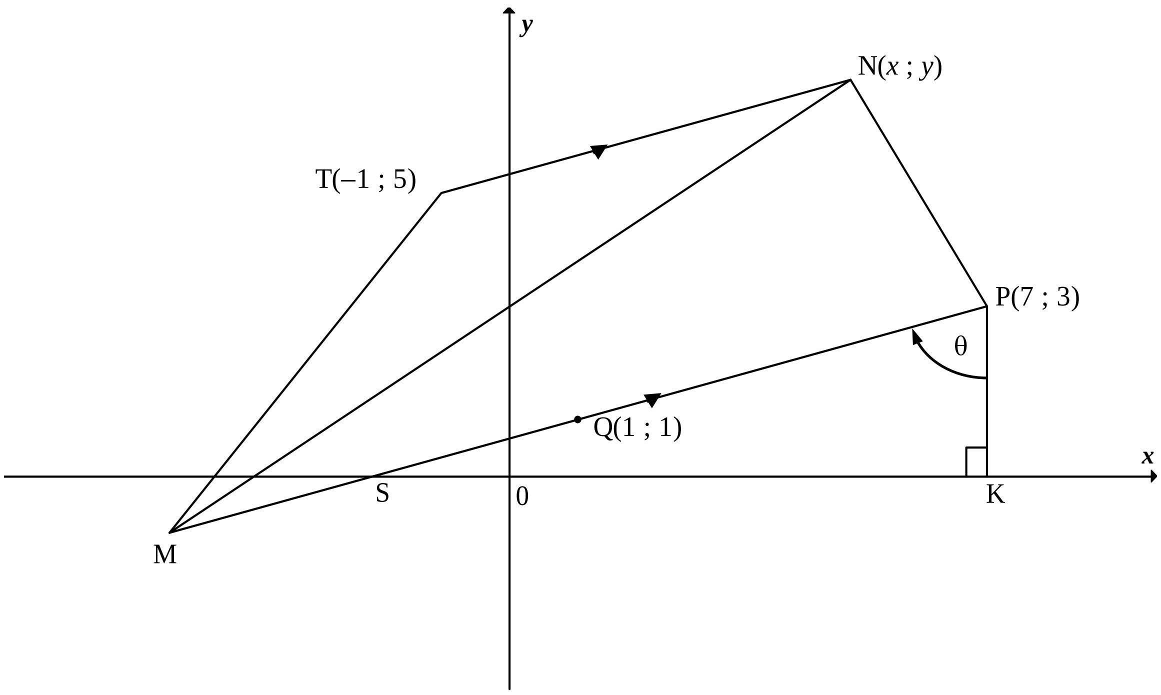
2.3 Use the ogive (cumulative frequency curve) to estimate the number of learners who watched television more than 80% of the time. (2)

2.4 Estimate the mean time (in hours) that learners spent watching television during 3 weeks of the holiday. (4)

[10]

QUESTION 3

In the diagram below, M, T(–1 ; 5), N(x ; y) and P(7 ; 3) are vertices of trapezium MTNP having TN | | MP. Q(1 ; 1) is the midpoint of MP. PK is a vertical line and $\hat{SPK} = \theta$. The equation of NP is $y = -2x + 17$.



3.1 Write down the coordinates of K. (1)

3.2 Determine the coordinates of M. (2)

3.3 Determine the gradient of PM. (2)

3.4 Calculate the size of θ. (3)

3.5 Hence, or otherwise, determine the length of PS. (3)

3.6 Determine the coordinates of N. (5)

3.7 If A(a ; 5) lies in the Cartesian plane:

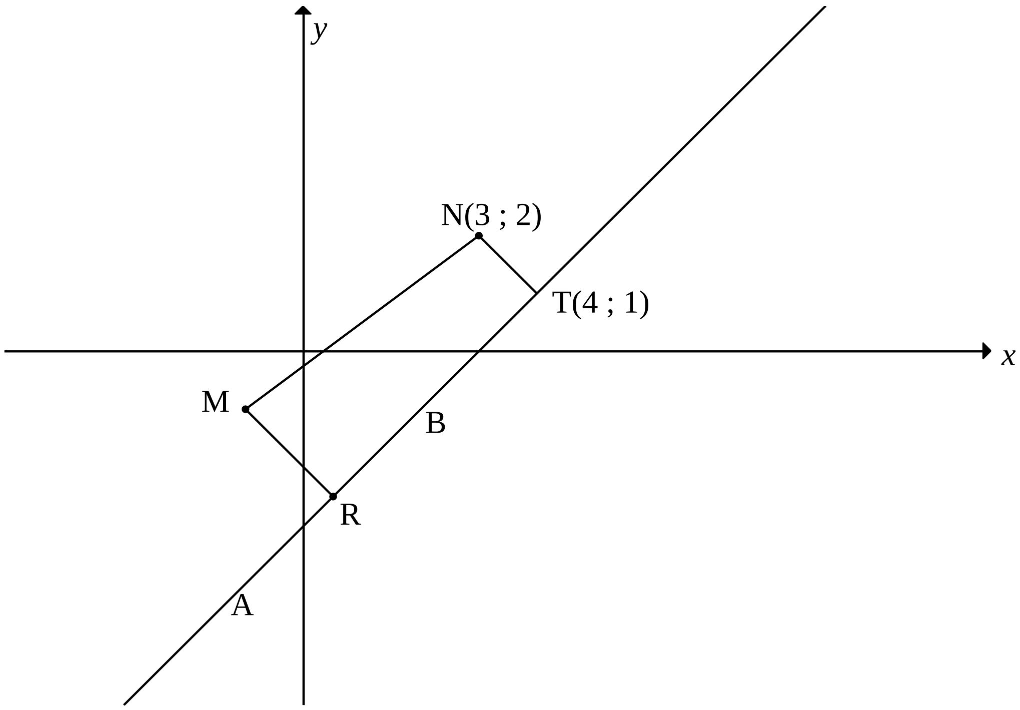
3.7.1 Write down the equation of the straight line representing the possible positions of A. (1)

3.7.2 Hence, or otherwise, calculate the value(s) of a for which $\hat{TAQ} = 45°$. (5)

[22]

QUESTION 4

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3 ; 2) at point T(4 ; 1).



4.1 Write down the coordinates of M. (1)

4.2 Determine the equation of AT in the form $y = mx + c$. (5)

4.3 If it is further given that $MR = \frac{\sqrt{10}}{2}$ units, calculate the length of AB. Leave your answer in simplest surd form. (4)

4.4 Calculate the length of MN. (2)

4.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3)

[15]

QUESTION 5

5.1 Given that $\sin \alpha = -\frac{4}{5}$ and $90° < \alpha < 270°$.

WITHOUT using a calculator, determine the value of each of the following in its simplest form:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos \alpha$ (2)

5.1.3 $\sin(\alpha - 45°)$ (3)

5.2 Consider the identity: $\dfrac{8\sin(180° - x)\cos(x - 360°)}{\sin^2 x - \sin^2(90° + x)} = -4\tan 2x$

5.2.1 Prove the identity. (6)

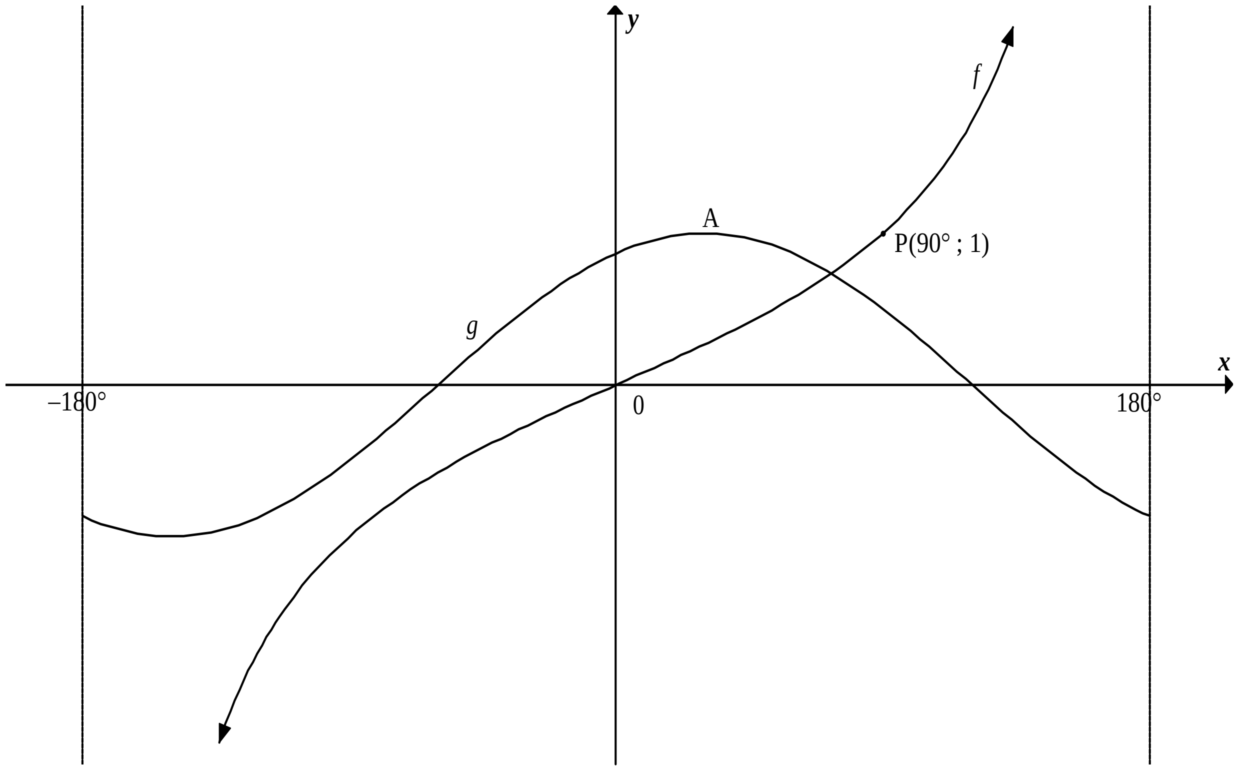
5.2.2 For which value(s) of x in the interval $0° < x < 180°$ will the identity be undefined? (2)

5.3 Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)

[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \leq x \leq 180°$. The point P(90° ; 1) lies on f. Use the diagram to answer the following questions.



6.1 Determine the value of b. (1)

6.2 Write down the coordinates of A, a turning point of g. (2)

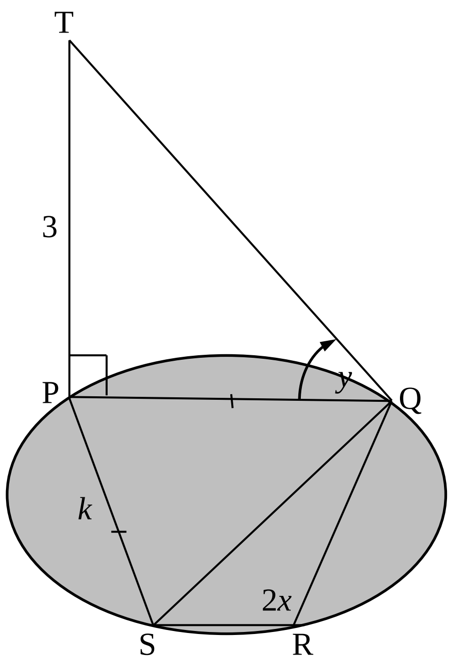
6.3 Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$. (1)

6.4 Determine the range of h if $h(x) = 2g(x) + 1$. (2)
[6]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$. (5)

7.2 The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is $y°$. PQ = PS = k units, TP = 3 units and $S\hat{R}Q = 2x°$.



7.2.1 Show, giving reasons, that $P\hat{S}Q = x$. (2)

7.2.2 Prove that $SQ = 2k \cos x$. (4)

7.2.3 Hence, prove that $SQ = \dfrac{6\cos x}{\tan y}$. (2)

[13]

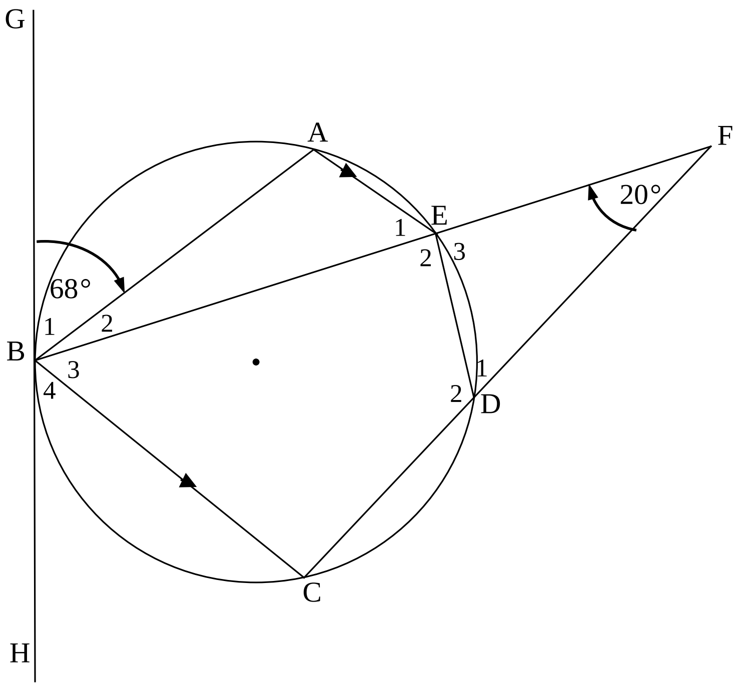
Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to ... (1)

8.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE | | BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68°$ and $\hat{F} = 20°$.



Determine the size of each of the following:

8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

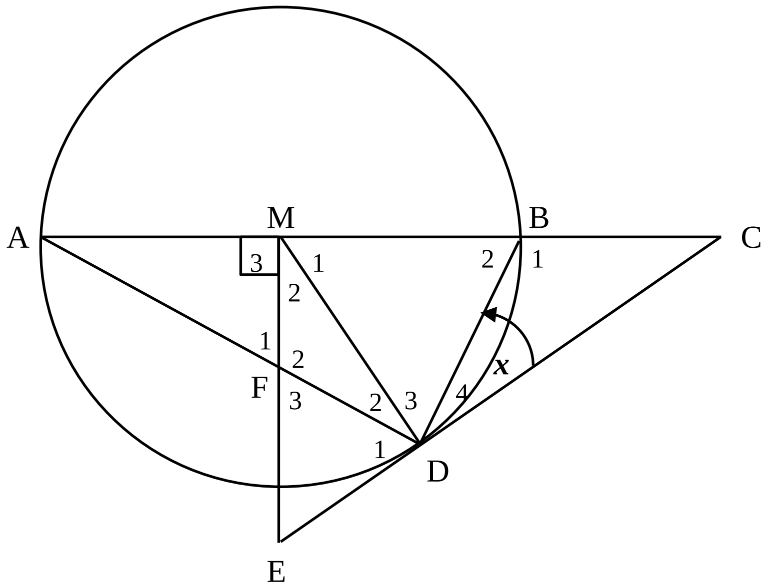
8.2.3 $\hat{D}_1$ (2)

8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2)
[9]

QUESTION 9

In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



9.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

9.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

9.3 Prove that FMBD is a cyclic quadrilateral. (3)

9.4 Prove that $DC^2 = 5BC^2$. (3)

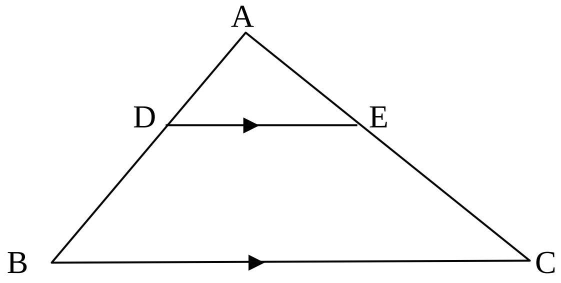
9.5 Prove that $\Delta DBC \mathbin{|||} \Delta DFM$. (4)

9.6 Hence, determine the value of $\dfrac{DM}{FM}$. (2)

[19]

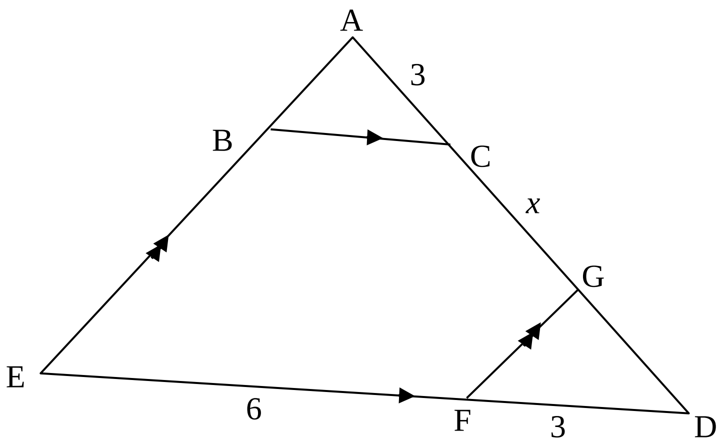
QUESTION 10

10.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE | | BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 In the diagram, ADE is a triangle having BC | | ED and AE | | GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



Calculate, giving reasons:

10.2.1 The length of CD (3)

10.2.2 The value of x (4)

10.2.3 The length of BC (5)

10.2.4 The value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5)

[23]

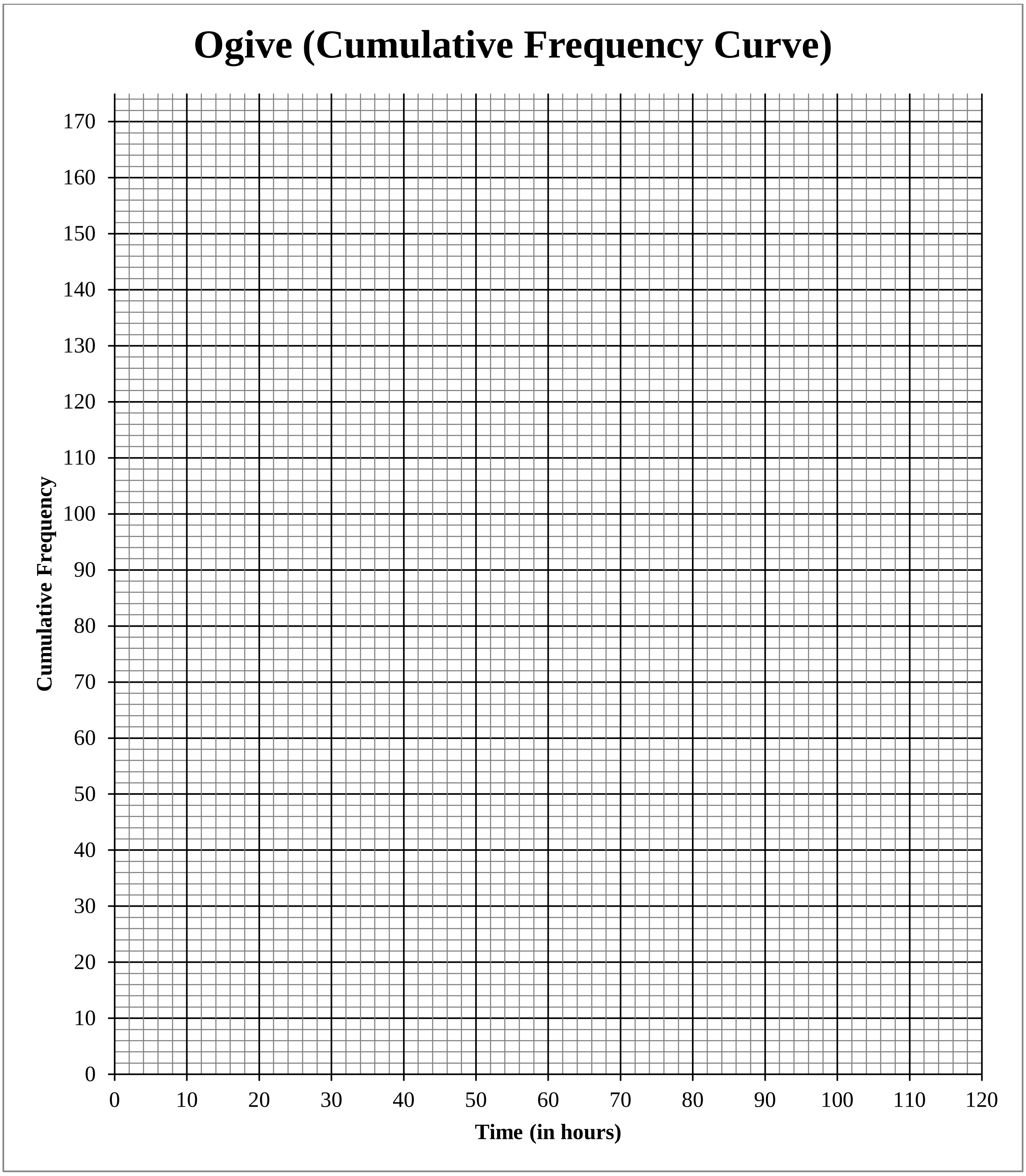
TOTAL: 150

NAME:

GRADE/CLASS:

DIAGRAM SHEET 1

QUESTION 2.1

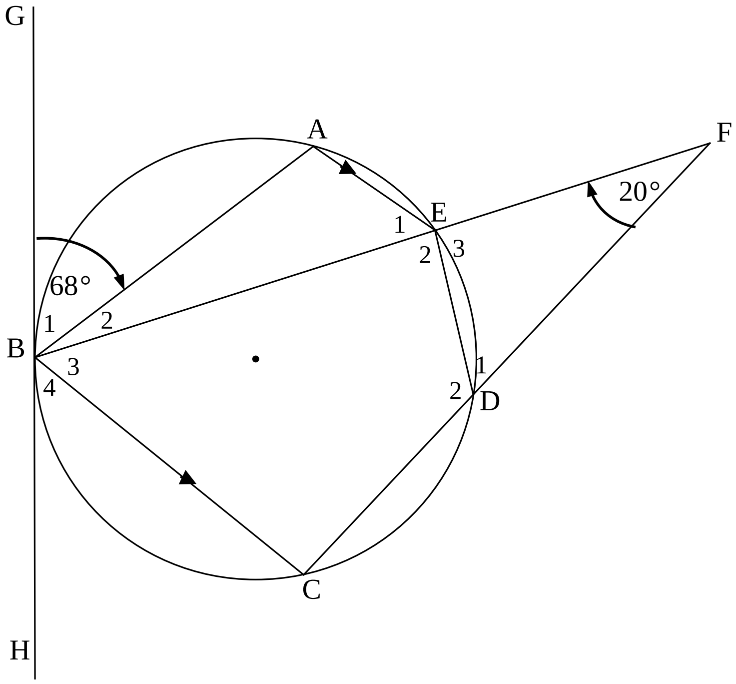


NAME:

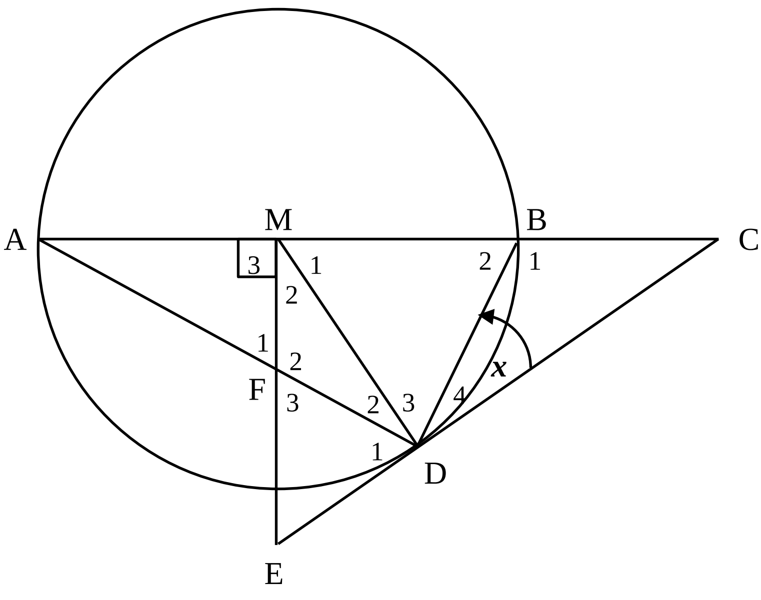
GRADE/CLASS:

DIAGRAM SHEET 2

QUESTION 8.2



QUESTION 9



NAME:

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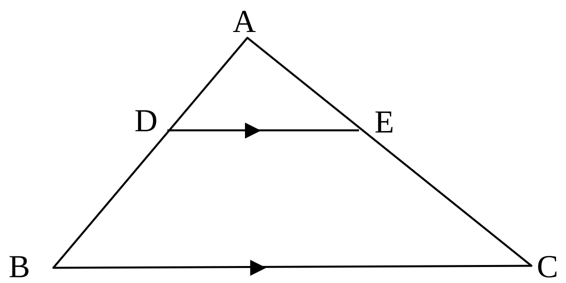
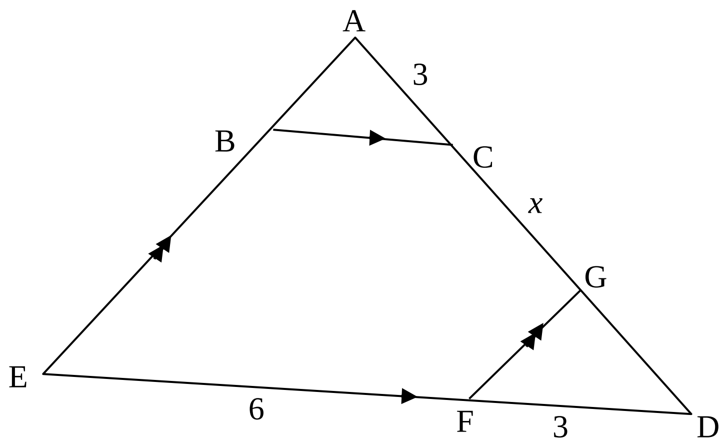
DIAGRAM SHEET 3

QUESTION 10.1



QUESTION 10.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$