basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2011

POSSIBLE ANSWERS

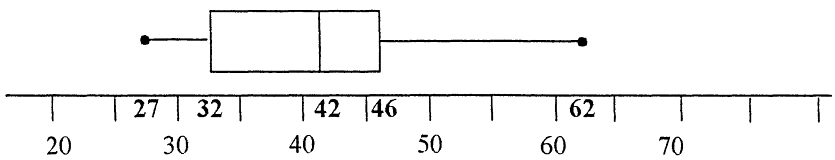
MARKS: 150

This memorandum consists of 22 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is not acceptable.

QUESTION 1

1.1	Median = 42	✓answer (1)
1.2	Lower quartile = 32 Upper quartile = 46 Inter quartile range = 46 – 32 = 14 Answer only: FULL MARKS	✓ lower quartile ✓ upper quartile ✓ answer (3)
1.3		✓ box-and-whisker with a median ✓ skewness ✓ indicating <u>5 number summary</u> 27; 32; 42; 46; 62 or correct scale (3)
1.4	There is a **greater spread** of scores to the right of the median (42). **OR** There is a **greater spread** of scores in the top 50%. **OR** The spread of the scores on the left hand side of the median is closer to each other. **OR** The greatest spread of scores lies between Q_3 and the maximum value. **Note:** • Description about the spread based on the box-and-whisker diagram must be accepted. • If it is indicated that it is skewed to the left because the mean is less than the median: full marks	✓greater spread ✓ right of median (42) (2) ✓greater spread ✓ top 50% (2) ✓spread closer ✓ left of median (2) ✓greater spread ✓ between Q_3 and max (2) **[9]**

QUESTION 2

2.1	Mean $= \dfrac{\sum_{i=1}^{n} x_i}{n} = \dfrac{580}{8} = 72,5$ Answer only: FULL MARKS **Note**: If rounded off to 73: 1 mark	✓ 580 ✓ answer (2)
2.2	Standard deviation (σ) = 2,78 (2,783882181...) **Note**: If rounded off to 2,8: 1 mark	✓✓ answer (2)
2.3	$\therefore$ 2 golfers' scores lie outside 1 standard deviation of the mean. The interval for 1 standard deviation of the mean is (72,5 – 2,78 ; 72,5 + 2,78) = (69,72 ; 75,28) Answer only: FULL MARKS	✓ interval ✓ number (2) **[6]**

QUESTION 3

3.1	30	✓ 30 (1)
3.2	Linear, the points seem to form a straight line.	✓ linear ✓ reason (2)
3.3	The greater the number of hours spent watching TV, the lower the test scores **OR** The less time a person spends watching TV, the higher the test score. **OR** Negative correlation between the variables **OR** Indirect relationship between the variables	✓ deduction (1)
3.4	60 marks. (Accept 50 -70 marks)	✓✓ deduction (2) **[6]**

QUESTION 4

4.1

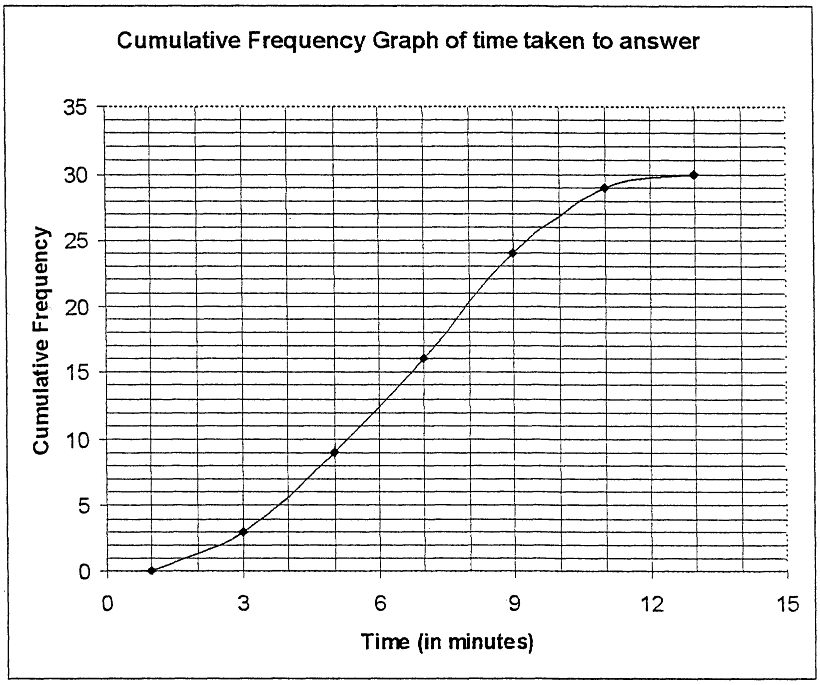
TIME	FREQUENCY	CUMULATIVE FREQUENCY
$1 \leq t < 3$	3	3
$3 \leq t < 5$	6	9
$5 \leq t < 7$	7	16
$7 \leq t < 9$	8	24
$9 \leq t < 11$	5	29
$11 \leq t < 13$	1	30

Note: Only cumulative frequency column – full marks

One mark for every two correct cumulative frequency values

(3)

4.2



✓upper limit
✓ cumulative frequency (at least 4 of 6 y-values correctly plotted)

✓ grounding at (1 ; 0)

✓ shape (not joined by a ruler; smooth curve)

(4)

4.3

Estimated number of learners that took less than 4 minutes:
approximately 5 learners (Accept 6)
Approximate percentage = 16,67% (Accept 20%)

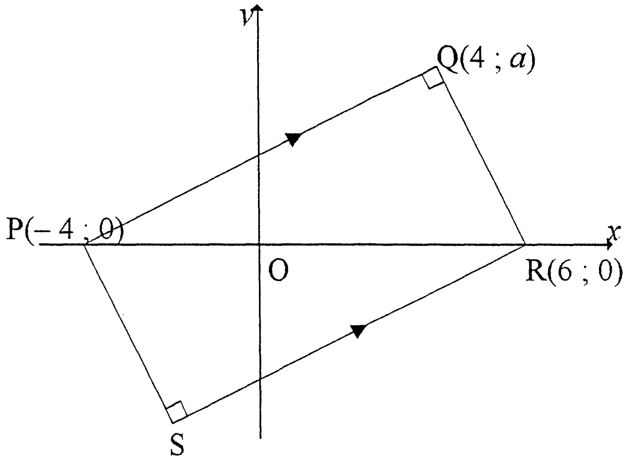
Note:
If using 9 learners and approximate percentage = 30%: 1 mark
If using 5,5 learners and approximate percentage = 18,33%: 1 mark

✓ 5 learners
✓ 16,67%

(2)

[9]

QUESTION 5



5.1	$m_{PQ} \times m_{QR} = -1$ $\left(\frac{a-0}{4+4}\right)\left(\frac{a-0}{4-6}\right) = -1$ $\left(\frac{a}{8}\right)\left(\frac{a}{-2}\right) = -1$ $\frac{a^2}{-16} = -1$ $a^2 = 16$ $a = \pm 4$ $a = 4$; since $a > 0$	✓ $\frac{a-0}{4+4}$ or $\frac{a}{8}$ ✓ $\frac{a-0}{4-6}$ or $\frac{a}{-2}$ ✓ using gradient of perpendicular lines ✓ $a^2 = 16$ (4)
	OR	
	$PQ^2 + QR^2 = PR^2$ $(8^2 + a^2) + (a^2 + 2^2) = 10^2$ $\therefore\ 2a^2 = 32$ $\therefore\ a^2 = 16$ $\therefore\ a = 4$	✓using Pythagoras ✓$(8^2 + a^2)$ $+ (a^2 + 2^2)$ ✓ 10^2 ✓ $a^2 = 16$ (4)
	OR	
	Let A be the midpoint of diagonal PR. Then $A(\frac{-4+6}{2};\frac{0+0}{2}) = A(1\,;0)$. AQ = AR (diagonals equal and bisect each other) $AQ^2 = AR^2$ $(1-4)^2 + (0-a)^2 = 5^2$ $9 + a^2 = 25$ $a^2 = 16$ $a = 4$	✓ (1 ; 0) is centre ✓ AQ = AR ✓ $3^2 + a^2 = 5^2$ ✓ $a^2 = 16$ (4)
	Note: If candidate uses $a = 4$ at the beginning, then zero marks.	

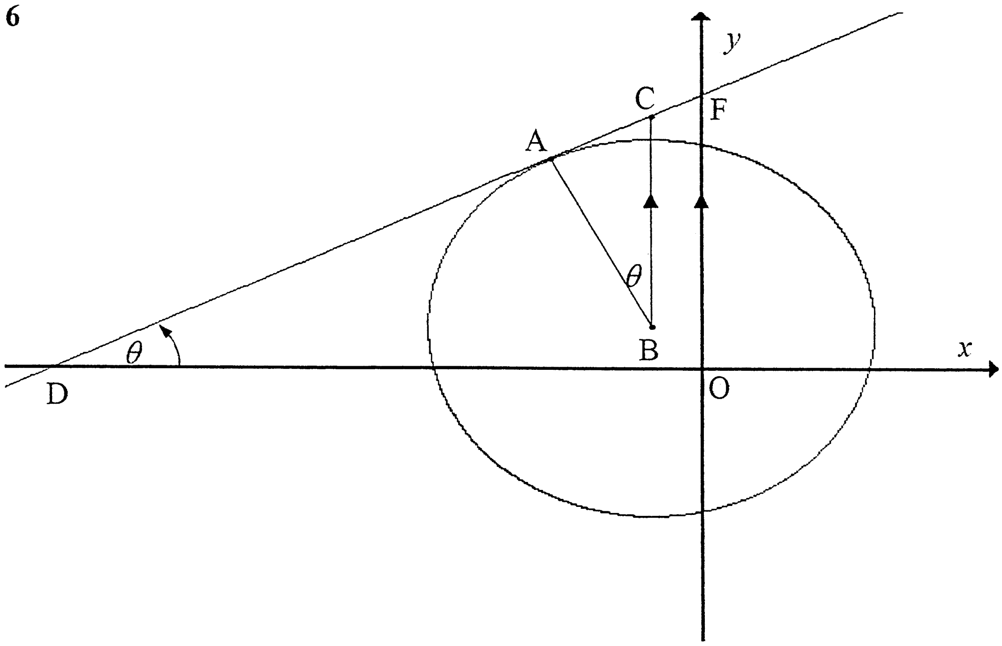
5.2	Equation of line SR: $m_{PQ} = \frac{4-0}{4-(-4)} = \frac{1}{2}$ $m_{SR} = m_{PQ} = \frac{1}{2}$ PQ | | SR $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{2}(x-6)$ $y = \frac{1}{2}x - 3$ **OR**	✓ $m_{PQ} = \frac{1}{2}$ ✓ $m_{SR} = \frac{1}{2}$ ✓ substitution of m and (6 ; 0) ✓ standard form (4)

	$m_{PQ} = \frac{1}{2}$ $m_{PQ} = m_{SR} = \frac{1}{2}$ PQ | | SR $y = \frac{1}{2}x + c$ $0 = \left(\frac{1}{2}\right)\left(\frac{6}{1}\right) + c$ $-3 = c$ $y = \frac{1}{2}x - 3$ **OR** S(– 2 ; – 4) (translation) $m_{RS} = \frac{0+4}{6+2} = \frac{1}{2}$ $\therefore y + 4 = \frac{1}{2}(x+2)$ $\therefore y = \frac{1}{2}x - 3$	✓ $m_{PQ} = \frac{1}{2}$ ✓ $m_{SR} = \frac{1}{2}$ ✓ substitution of m and (6 ; 0) ✓ standard form ✓ S(– 2 ; – 4) ✓ $m_{SR} = \frac{1}{2}$ ✓ substitution of m and (– 2 ; – 4) ✓ standard form (4)
5.3	Eq. of RS: $y = \frac{1}{2}x - 3$ Eq. of SP: $y - 0 = -2(x+4)$ $\therefore \frac{1}{2}x - 3 = -2(x+4)$ $\therefore x = -2$ $y = -4$ Answer only: FULL MARKS **OR**	✓ m = – 2 ✓ eq. of SP ✓ value of x ✓ value of y (4)

	Midpoint PR $= \text{M}\left(\frac{-4+6}{2};\frac{0+0}{2}\right) = (1\ ;\ 0)$ Let $\text{S}(x;y)$. Then since M(1 ; 0) is this, the midpoint of QS is: $\frac{x_1+x_2}{2}=1$ $\quad\frac{y_1+y_2}{2}=0$ $\therefore \frac{x+4}{2}=1$ and $\frac{y+4}{2}=0$ $x+4=2$ $\quad y+4=0$ $x=-2$ $\quad y=-4$ **OR** The translation that sends Q(4 ; 4) to R(6 ; 0) also sends P(– 4 ; 0) to S. (6 ; 0) = (4 + 2 ; 4 – 4) $\therefore$ S = (– 4 + 2 ; 0 – 4) = (– 2 ; – 4) **OR** The translation that sends Q(4 ; 4) to P(– 4 ; 0) also sends R(6 ; 0) to S. (– 4 ; 0) = (4 – 8 ; 4 – 4) $\therefore$ S = (6 – 8 ; 0 – 4) = (– 2 ; – 4) **OR** $m_{PQ}=m_{SR}$ $\frac{1}{2}=\frac{y}{x-6}$ $2y=x-6$ (1) $m_{PS}=m_{SR}$ $\frac{y}{x+4}=\frac{4}{-2}$ $-2y=4x+16$ (2) $(1)+(2):0=5x+10$ $x=-2$ $\textit{Substitute}:2y=-2-6=-8$ $y=-4$	✓ $\frac{x+4}{2}=1$ ✓ $\frac{y+4}{2}=0$ ✓ value of x ✓ value of y (4) ✓method ✓ 2 or $x+2$ ✓ -4 or $y-4$ ✓ answer (4) ✓method ✓ -8 or $x-8$ ✓ -4 or $y-4$ ✓ answer (4) ✓equations using the gradient ✓adding the equations ✓ value of x ✓ value of y (4)
5.4	$PR=6-(-4)$ $=10$ **OR** Answer only: FULL MARKS $PR^2=(6+4)^2+(0-0)^2$ $PR=10$	✓ $6-(-4)$ ✓ 10 (2) ✓ substitution in correct formula ✓ 10

5.5	midpoint PR= $\left(\frac{6+(-4)}{2}\ ;\ \frac{0+0}{2}\right) = (1;\ 0)$ radius of circle = $\frac{1}{2}$PR = 5 units $\therefore (x-1)^2+(y-0)^2=5^2$ $(x-1)^2+y^2=25$ Answer only: FULL MARKS	✓ midpoint ✓ radius ✓ eq. of circle in correct form (3)
5.6	$(x-1)^2+y^2=25$ substitute Q(4 ; 4): LHS $=(4-1)^2+4^2$ $=25$ = RHS $\therefore$ Q is a point on the circle **Note**: If substitute point into equation resulting in 25 = 25: 1 mark No conclusion: 1 mark	✓ substitute Q(4;4) ✓ LHS = RHS (2)
	OR	
	Distance from centre (1 ; 0) to Q(4 ; 4) $\therefore$ Q is a point on circle, r = 5	✓ = 5 ✓ conclusion (2)
	OR	
	PR is the diameter of circle PQR therefore Q lies on circle ($P\hat{Q}R=90°$)	✓ diameter ✓ $P\hat{Q}R=90°$) (2)
	OR	
	$(4-1)^2+y^2=25$ $y^2=16$ $\therefore y=4$ $\therefore$ Q is a point on the circle	✓ substitute $x=4$ ✓ conclusion (2)
	OR	
	$(x-1)^2+4^2=25$ $(x-1)^2=9$ $x-1=3$ $x=4$ $\therefore$ Q is a point on the circle	✓ substitute $y=4$ ✓ conclusion (2)
5.7	P needs to shift at least 4 units to the right and S needs to shift at least 4 units up for the image of PQRS in first quadrant. $\therefore$ minimum value of k is 4 and minimum value of l is 4 $\therefore$ minimum value of $k+l$ is 8 Answer only: FULL MARKS **Note**: No CA mark applies in 5.7 if k and l are not minimums.	✓ $k=4$ ✓ $l=4$ ✓ $k+l=8$ (3) **[22]**

QUESTION 6



6.1	$x_C = x_B = -1$ $y_C = y_B + 5 = 6$ $\therefore C(-1\,;6)$	✓ value of x ✓ value of y (2)
6.2	$BA \perp CA$ (tangent $\perp$ radius) $\therefore\ CA^2 = BC^2 - AB^2$ (Pythagoras) $= (5)^2 - (\sqrt{20})^2 = 5$ $\therefore\ CA = \sqrt{5}$ or 2,24 units	✓ $BA \perp CA$ or $B\hat{A}C = 90°$ ✓ substitution into Pythagoras ✓ answer (3)
6.3	$\tan\theta = \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$	✓ tan ratio (in any form) (1)
6.4	$m_{DC} \times m_{AB} = -1$ $m_{DC} = \tan\theta = \frac{1}{2}$ $m_{DC} = \frac{1}{2}$ $m_{AB} = -2$	✓ $m_{DC} \times m_{AB} = -1$ ✓ $m_{DC} = \tan\theta = \frac{1}{2}$ (2)

6.5	Eq. of DC: $y-6=\frac{1}{2}(x+1)$ $y=\frac{1}{2}x+\frac{13}{2}$ Eq. of AB: $y-1=-2(x+1)$ $y=-2x-1$ $-2x-1=\frac{1}{2}x+\frac{13}{2}$ $-\frac{5}{2}x=\frac{15}{2}$ $x=-3$ $y=-2(-3)-1$ $y=5$ $\therefore$ A $(-3\ ;\ 5)$	Answer only: $(-3\ ;\ 5)$: 1 mark	✓ DC: subst m and $(-1\ ;\ 6)$ ✓ eq. of DC ✓ eq. of AB ✓ equating equations ✓ value of x ✓ value of y (6)
	OR		
	Eq. of DC: $y-6=\frac{1}{2}(x+1)$ $y=\frac{1}{2}x+\frac{13}{2}$ Eq. of AB: $y-1=-2(x+1)$ $y=-2x-1$ At A: $x-2(-2x-1)+13=0$ $x+4x+2+13=0$ $5x=-15$ $x=-3$ and $y=-2(-3)-1=5$ $\therefore$ A$(-3\ ;\ 5)$		✓ DC: subst m and $(-1\ ;\ 6)$ ✓ eq. of DC ✓subt m and $(-1;1)$ ✓ eq. of AB ✓ value of x ✓ value of y (6)
	OR		
	Eq. of DC: $y-6=\frac{1}{2}(x+1)$ $y=\frac{1}{2}x+\frac{13}{2}$ Eq. of circle: $(x+1)^2+(y-1)^2=20$ At A: $(x+1)^2+(\frac{1}{2}x+\frac{13}{2}-1)^2=20$ $(x+1)^2+(\frac{1}{2}x+\frac{11}{2})^2=20$ $1\frac{1}{4}x^2+\frac{15}{2}x+11\frac{1}{4}=0$ $\therefore x^2+6x+9=0$ $(x+3)^2=0$ $\therefore\ x=-3$ and $y=\frac{1}{2}(-3)+\frac{13}{2}=5$ $\therefore$ A$(-3\ ;\ 5)$		✓ DC: subst m and $(-1\ ;\ 6)$ ✓ eq. of DC ✓ substitution ✓ $x^2+6x+9=0$ ✓ value of x ✓ value of y (6)

OR

Draw AE $\perp$ BC

$$\cos\theta = \frac{2\sqrt{5}}{5} = \frac{AE}{\sqrt{5}} = \frac{BE}{2\sqrt{5}}$$

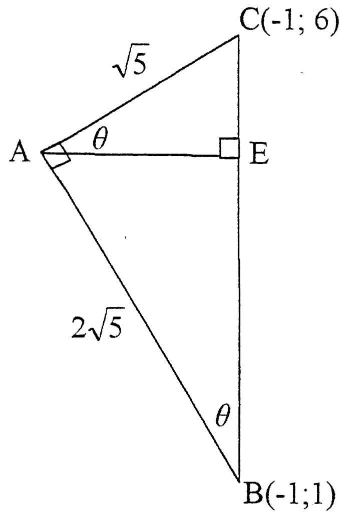
$$\therefore AE = \frac{2\times 5}{5} = 2$$

$$BE = \frac{4\times 5}{5} = 4$$

$$x_A = -1 - AE = -1 - 2 = -3$$

$$\therefore y_A = 1 + BE = 4 + 1 = 5$$

$\therefore$ A$(-3\ ;5)$



✓ $\frac{2\sqrt{5}}{5} = \frac{AE}{\sqrt{5}}$

✓AE = 2

✓ $\frac{2\sqrt{5}}{5} = \frac{BE}{2\sqrt{5}}$

✓ BE = 1

✓ -3

✓ 5

(6)

OR

$(x+1)^2 + (y-1)^2 = 20 \qquad (1)$

$y = -2x - 1 \qquad (2)$

$(x+1)^2 + (-2x-2)^2 = 20$

$x^2 + 2x + 1 + 4x^2 + 8x + 4 - 20 = 0$

$5x^2 + 10x - 15 = 0$

$x^2 + 10x - 15 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x \neq 1$

subst (1) in (2)

$\therefore y = 5$

✓ subst m and $(-1;1)$

✓eq of AB

✓eq of circle

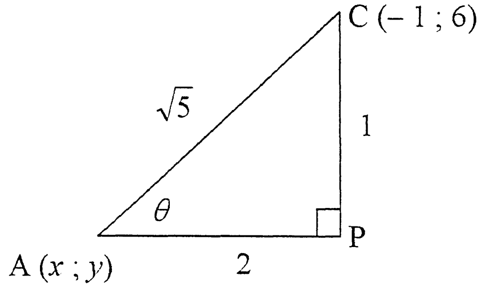
✓ substation

✓value of x

✓ value of y (6)

OR

Equation AC : $y = \frac{1}{2}x + 6\frac{1}{2}$



	$\tan\theta = \frac{1}{2}$	✓ $\theta = 26{,}57°$
	$\theta = 26{,}57°$	✓
	$\text{AP} = \sqrt{5}\cos 26{,}57°$	$\text{AP} = \sqrt{5}\cos 26{,}57°$
	$AP = 2$	✓ AP = 2
	$\text{CP} = \sqrt{5}\sin 26{,}57°$	
	$CP = 1$	✓ *CP = 1*
	$\therefore x = -1 - 2 = -3$	✓ value of x
	$y = 6 - 1 = 5$	✓ value of y
	$\therefore \text{A}(-3\,;5)$	(6)
6.6	Area $\Delta\text{ABC} = \frac{1}{2}(\sqrt{5})(\sqrt{20}) = 5$	✓ $\frac{1}{2}(\sqrt{5})(\sqrt{20})$
	Eqn. of DC is $y = \frac{1}{2}x + \frac{13}{2}$	
	Therefore $\text{OF} = \frac{13}{2}$ and OD = 13.	✓ $\text{OF} = \frac{13}{2}$ ✓OD = 13
	Area $\Delta\text{ODF} = \frac{1}{2}\left(\frac{13}{2}\right)(13) = \frac{169}{4}$	✓ $\frac{1}{2}\left(\frac{13}{2}\right)(13)$
	Area ΔABC: Area ΔODF = $5 : \frac{169}{4} = 20 : 169$	✓ answer (5)
	OR	
	$\text{DF}^2 = 13^2 + (\frac{13}{2})^2 = \frac{845}{4}$	✓ $= 13^2 + (\frac{13}{2})^2 = \frac{845}{4}$
	$\text{DF} = \frac{13.\sqrt{5}}{2}$	✓ $\text{DF} = \frac{13.\sqrt{5}}{2}$
	$\frac{\Delta ABC}{\Delta ODF} = \frac{\frac{1}{2}(5)(\sqrt{20})\sin\theta}{\frac{1}{2}(13)(\frac{13.\sqrt{5}}{2})\sin\theta}$	✓ $\frac{1}{2}(5)(\sqrt{20})\sin\theta$ ✓ $\frac{1}{2}(13)(\frac{13.\sqrt{5}}{2})\sin\theta$
	$= \frac{20}{169}$	✓ answer (5)

	OR ΔODF is an enlargement of ΔABC $\therefore$ area ΔABC : area ΔODF = $AB^2 : OD^2 = 20 : OD^2$ Equation of DC is $y = \frac{1}{2}x + \frac{13}{2}$ $x_D = -13$ $OD = 13$ $\therefore$ area ΔABC : area ΔODF = $AB^2 : OD^2 = 20 : 169$	✓ enlargement ✓✓ $AB^2:OD^2 = 20:OD^2$ ✓ – 13 ✓answer (5) **[19]**

QUESTION 7

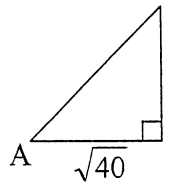
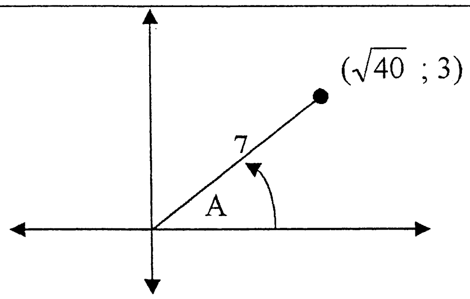
7.1	$(x;y) \rightarrow (x+4;y) \rightarrow (-x-4;-y)$ OR $(x;y) \rightarrow (-x-4;-y)$	✓$x + 4$ ✓ y ✓ $-x - 4$ ✓ $-y$ (4)
7.2	New centre = (–2 ; –5) $(x+2)^2 + (y+5)^2 = 16$ $x^2 + 4x + 4 + y^2 + 10y + 25 - 16 = 0$ $x^2 + y^2 + 4x + 10y + 13 = 0$	✓ (–2 ; –5) ✓ $(x + 2)^2 + (y + 5)^2$ ✓ 16 ✓ simplification (4) **[8]**

QUESTION 8

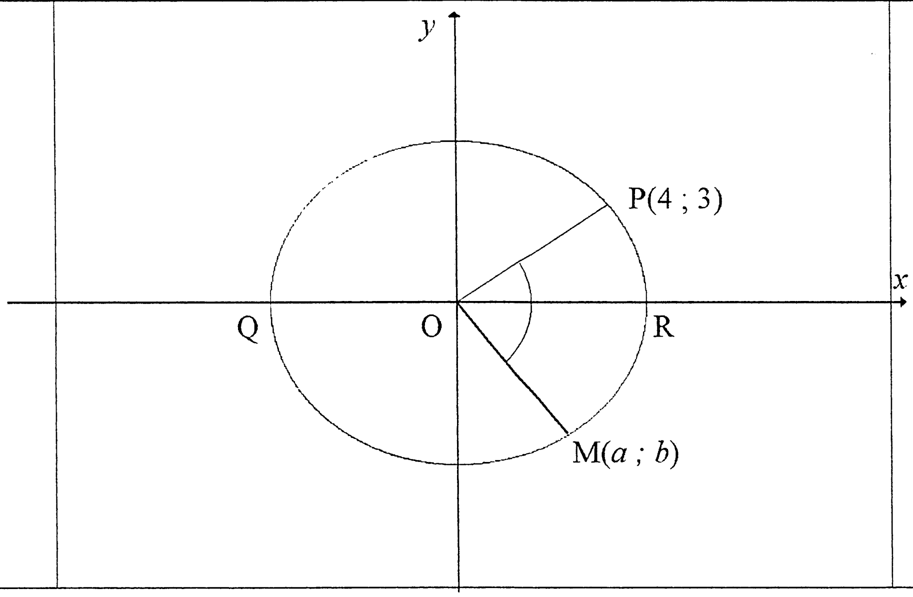
8.1	Rotation of 90° anticlockwise about the origin. **OR** Rotation of 270° clockwise about the origin. **Note:** if reflection of 90 anticlockwise: 0 marks	✓ rotation **90°** ✓ anticlockwise (2) ✓rotation **270°** ✓clockwise (2)
8.2	D(5 ; – 4) D$'$(4 ; 5)	✓ 4 ✓ 5 (2)
8.3	G (–7 ; –6)	✓ –7 ✓ –6 (2)
8.4	Area ABCD = 5 $\times$ 2 = 10 square units Area MNRP = $10 \times \left(\frac{3}{2}\right)^2 = \frac{45}{2}$ Area ABCD $\times$ Area MNRP $= 10 \times \frac{9}{4} \times 10$ $= 225\ (\text{units})^4$ **OR**	✓ area ABCD = 10 ✓ area MNRP $= \frac{45}{2}$ ✓ 225 (3)

	$\text{Product} = \left(\frac{3}{2}\right)^2 \times (\text{area ABCD})^2$ $= \frac{9}{4} \times (5 \times 2)^2$ $= 225\ (\text{units})^4$ Note: CA will apply if $\left(\frac{3}{2}\right)^2$ used in calculation.	✓ $\left(\frac{3}{2}\right)^2$ ✓ 10^2 ✓ 225 (3) **[9]**

QUESTION 9

9.1	9.1.1	 or  $r^2 = 40 + 9$ $r = 7$ $\cos A = \frac{\sqrt{40}}{7}$	✓ sketch ✓ $r = 7$ ✓ $\frac{\sqrt{40}}{7}$ (3)
	9.1.2	$\sin(180° + A)$ $= -\sin A$ $= -\frac{3}{7}$ **OR** $\sin(180° + A) = \sin 180°.\cos A + \cos 180°.\sin A$ $= 0.\cos A - 1.\sin A$ $= -\sin A$ $= -\frac{3}{7}$	✓ – sin A ✓ $-\frac{3}{7}$ (2) ✓ – sin A ✓ $-\frac{3}{7}$ (2)
9.2		$\frac{\cos 100° \times \tan^2 120°}{\sin(-10°)}$ $= \frac{(-\cos 80°)(-\tan 60°)^2}{(-\sin 10°)}$ $= \frac{(-\cos 80°) \times \left((-\sqrt{3})^2\right)}{(-\cos 80°)}$ $= 3$ **Note**: Answer only: 0 marks **Note**: If $\frac{+\cos 80°}{+\sin 10°}$ (assume two negatives cancelled), no penalty	✓ –cos 80° ✓ –tan 60° or $\tan^2 60°$ ✓ – sin10° ✓ $-\sqrt{3}$ ✓ sin10° = cos 80° ✓ 3 (6)

		OR $\dfrac{\cos 100° \times \tan^2 120°}{\sin(-10°)}$ $= \dfrac{(-\cos 80°)(-\tan 60°)^2}{(-\sin 10°)}$ $= \dfrac{(-\sin 10°)\times\left((-\sqrt{3})^2\right)}{(-\sin 10°)}$ $= 3$	✓ $-\cos 80°$ ✓ $-\sin 10°$ ✓ $-\tan 60°$ ✓ $-\sqrt{3}$ ✓ $\cos 80° = \sin 10°$ ✓ 3 (6)
		OR $\dfrac{\cos 100°}{\sin(-10°)} \times \tan^2 120°$ $= \dfrac{\cos(90° + 10°)}{-\sin(10°)} \times \tan^2 60°$ $= \dfrac{-\sin 10°}{-\sin 10°} \times \left(\sqrt{3}\right)^2$ $= 3$	✓ $\cos(90° + 10°)$ ✓ $-\sin 10°$ ✓ $-\sin 10°$ ✓ $\tan^2 60°$ ✓ $\sqrt{3}$ ✓ 3 (6)



9.3	9.3.1	$r = 5$ $\sin R\hat{O}P = \dfrac{3}{5} = 0{,}6$	✓ 5 ✓ ratio (2)
	9.3.2	$R\hat{O}P = 36{,}87°$ $Q\hat{O}P = 180° - 36{,}869....°$ $Q\hat{O}P = 143{,}13°$ Answer only: Full Marks	✓ $36{,}869....°$ ✓ $143{,}13°$ (2)

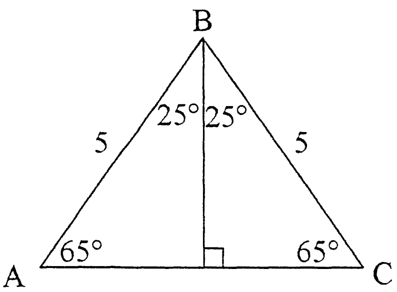
	9.3.3	$x_m = x\cos\theta + y\sin\theta$ $a = 4\cos 115° + 3\sin 115°$ $a = 1{,}03$ **Note**: Penalise 1 mark for rounding incorrectly **Note**: If incorrect angle is used in the x- formula: 1 mark **OR** Rotation of 115° clockwise = 245° anticlockwise $x_m = x\cos\theta - y\sin\theta$ $a = 4\cos 245° - 3\sin 245°$ $a = 1{,}03$ **OR** $\tan P\hat{O}R = \frac{3}{4}$ $P\hat{O}R = 36{,}86...°$ $M\hat{O}R = 78{,}13....°$ $\cos M\hat{O}R = \frac{a}{5}$ $a = 5\ \cos 78{,}13°$ $a = 1{,}03$	✓ formula ✓ substitution of values ✓ $a = 1{,}03$ (3) ✓ formula ✓ substitution of values ✓ $a = 1{,}03$ (3) ✓ 36,86° ✓ cos ratio ✓ $a = 1{,}03$ (3) **[18]**

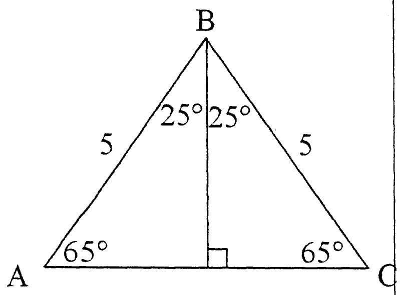
QUESTION 10

10.1	$f(225°) = 2$ $\therefore a\tan 225° = 2 \quad \therefore a = 2$ $g(0) = 4$ $\therefore b\cos 0° = 4 \quad \therefore b = 4$ Answer only: Full marks	✓ substitution ✓ $a = 2$ ✓ substitution ✓ $b = 4$ (4)
10.2	Minimum value of $g(x) + 2 = -4 + 2 = -2$ Answer only: Full marks	✓ -4 ✓ -2 (2)
10.3	Period $= \frac{180°}{\frac{1}{2}} = 360°$ Answer only: Full marks	✓ $\frac{180°}{\frac{1}{2}}$ ✓ 360° (2)

10.4	At P $f(\theta)=g(\theta)$ 2tan θ = 4cos θ for 180° – θ : 2tan (180° – θ) = –2tan θ and 4cos(180° – θ) = – 4cos θ $2\tan\theta = 4\cos\theta$ at P $\therefore -2\tan\theta = -4\cos\theta$ ∴ 2tan (180° – θ) = 4cos (180° – θ) at Q	✓2tan θ = 4cos θ ✓ 2tan (180° – θ) = –2tan θ ✓ 4cos(180° – θ) = – 4cos θ ✓ 2tan (180° – θ) = 4cos (180° – θ) (4)
	OR	
	$2\tan\theta = 4\cos\theta$ $\dfrac{\sin\theta}{\cos\theta} = 2\cos\theta$ $\sin\theta = 2\cos^2\theta$ $= 2(1-\sin^2\theta)$ $2\sin^2\theta + \sin\theta - 2 = 0$ $\sin\theta = \dfrac{-1\pm\sqrt{1-4(2)(-2)}}{4}$ $\sin\theta = 0{,}78077...$ $\theta = 51{,}33°$ or $128{,}67°$ $\therefore$ the x-coordinate of Q is $180° - x_p$	✓ equation ✓ $\sin\theta = 0{,}78077...$ ✓51,33° ✓ 128,67° (4) **[12]**

QUESTION 11

11.1	Area ΔABC = $\frac{1}{2}.AB.BC.\sin 50°$ $= \frac{1}{2}(5)(5)\sin 50°$ $= 9{,}58$ *units*^{2}	✓ substitution into correct formula ✓ answer (2)
	OR Area of ΔABC $= \frac{1}{2}(2)(5)\sin 25°)(5\cos 25°)$ $= 9{,}58\ units^2$ 	✓ base and height in terms of 5 and 25° ✓ answer (2)
	OR Area of ΔABC $= [\frac{1}{2}(5\cos 65°)(5\sin 65°)](2)$ $= 9{,}58\ units^2$	✓ base and height in terms of 5 and 65° ✓ answer (2)

11.2	$AC^2 = 5^2 + 5^2 - 2(5)(5)\cos 50°$ $AC^2 = 17,86061952$ $AC = 4,23\ units$ **OR** $\hat{A} = \hat{C} = 65°$ (angles opposite equal sides) $\frac{\sin 65°}{5} = \frac{\sin 50°}{AC}$ $AC = \frac{5\sin 50°}{\sin 65°}$ $= 4,23\ units$ **OR** $\sin 25° = \frac{\frac{1}{2}(AC)}{5}$ $AC = 2(5)\sin 25°$ $= 4,23\ units$  **OR** $\cos 65° = \frac{\frac{1}{2}(AC)}{5}$ $AC = 2(5)\cos 65°$ $AC = 4,23\ units$	✓ use of cosine rule ✓ substitution ✓ answer (3) ✓ use of sine rule ✓ substitution ✓ answer (3) ✓ sketch/diagram ✓ $\sin 25° = \frac{\frac{1}{2}AC}{5}$ ✓ answer (3) ✓ sketch/diagram ✓ $\cos 65° = \frac{\frac{1}{2}(AC)}{5}$ ✓ answer (3)
11.3	$\tan 25° = \frac{CF}{AC}$ $\therefore CF = 4,23 \times \tan 25°$ $\therefore CF = 1,97\ units$ **OR** $\frac{FC}{\sin 25°} = \frac{4,23}{\sin 65°}$ $FC = \frac{4,23 \sin 25°}{\sin 65°}$ $= 1,97\ units$	✓ ratio ✓ CF as subject ✓ answer (3) ✓ sine rule ✓ FC as subject ✓ answer (3)

[8]

QUESTION 12

12.1	$LHS = \dfrac{\sin(360°+90°+x-\alpha)}{\cos(\alpha - x)}$ $= \dfrac{\sin(90°+x-\alpha)}{\cos(\alpha - x)}$ $= \dfrac{\cos(x-\alpha)}{\cos(\alpha - x)}$ $= \dfrac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ **OR** $LHS = \dfrac{\sin[90° - (\alpha - x)]}{\cos(\alpha - x)}$ $= \dfrac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ = RHS	✓ subtracting 360° ✓ cos (x - α) ✓ cos(α - x) (3) ✓ subtracting 360° ✓ writing as 90° - (α - x) ✓ cos(α - x) (3)
12.2	$\cos 2x = 1 - 3\cos x$ $2\cos^2 x - 1 = 1 - 3\cos x$ $2\cos^2 x + 3\cos x - 2 = 0$ $(2\cos x - 1)(\cos x + 2) = 0$ $\cos x = \dfrac{1}{2}$ or $\cos x = -2$ n/a $x = 60° + k.360°\ ;\ k \in Z$ or $x = 300° + k.360°\ ;\ k \in Z$ **OR** $x = \pm 60° + k.360°\ ;\ k \in Z$	✓ $\cos 2x = 2\cos^2 x - 1$ ✓ factorisation ✓ $\cos x = \dfrac{1}{2}$ ✓ 60° ✓ 300° ✓ + k.360° ✓ k ∈ Z (7)
12.3.1	LHS: $\dfrac{\sin A\cos B - \cos A\sin B}{\sin B\cos B}$ $= \dfrac{\sin(A-B)}{\sin B\cos B}$ RHS $= \dfrac{2\sin(A-B)}{2\sin B\cos B}$ $= \dfrac{\sin(A-B)}{\sin B\cos B}$ = LHS	✓ writing as single fraction ✓ comp. angle expansion ✓ comp. angle expansion ✓simplification (4)

	OR LHS: $\dfrac{\sin A\cos B-\cos A\sin B}{\sin B\cos B}$ $=\dfrac{\sin(A-B)}{\sin B\cos B}$ $=\dfrac{2\sin(A-B)}{2\sin B\cos B}$ $=\dfrac{2\sin(A-B)}{\sin 2B}$ $=RHS$	✓ writing as single fraction ✓ comp. angle expansion ✓mult. by 2 ✓ comp. angle expansion (4)
	OR $RHS=\dfrac{2\sin(A-B)}{\sin 2B}$ $=\dfrac{2(\sin A\cos B-\cos A\sin B)}{2\sin B\cos B}$ $=\dfrac{\sin A\cos B-\cos A\sin B}{\sin B\cos B}$ $=\dfrac{\sin A\cos B}{\sin B\cos B}-\dfrac{\cos A\sin B}{\sin B\cos B}$ $=\dfrac{\sin A}{\sin B}-\dfrac{\cos A}{\cos B}$ $=LHS$	✓ expansion ✓ expansion ✓divide by 2 ✓ write as separate fractions (4)

12.3.2(a)	A = 5B $\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B} = \frac{2\sin(5B-B)}{\sin 2B}$ $= \frac{2\sin 4B}{\sin 2B}$ $= \frac{4\sin 2B\cos 2B}{\sin 2B}$ $= 4\cos 2B$	✓ recognising A = 5B ✓ substituting A = 5B ✓ sin 4B = 2sin 2B cos 2B (3)
	OR	
	$\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B}$ $= \frac{\sin 5B\cos B - \cos 5B\sin B}{\sin B\cos B}$ $= \frac{\sin(5B-B)}{\sin B\cos B}$ $= \frac{\sin 4B}{\frac{1}{2}(2)\sin B\cos B}$ $= \frac{2\sin 2B\cos 2B}{\frac{1}{2}\sin 2B}$ $= 4\cos 2B$	✓ writing as single fraction ✓ sin 4B = 2sin 2B cos 2B ✓compound angle in denominator (3)
12.3.2(b)	B = 18° $\frac{\sin 90°}{\sin 18°} - \frac{\cos 90°}{\cos 18°} = 4\cos 2(18)°$ $\therefore \frac{1}{\sin 18°} - 0 = 4\cos 36°$ $\therefore \frac{1}{\sin 18°} = 4\cos 36°$	✓ recognising B = 18° ✓ substituting B = 18° ✓ simplify (3)
12.3.2(c)	Let sin 18° = a $\frac{1}{\sin 18°} = 4\cos 36°$ $\frac{1}{\sin 18°} = 4(1 - 2\sin^2 18°)$ $\therefore \frac{1}{a} = 4(1-2a^2)$ $\therefore\ 1 = 4a - 8a^3$ $\therefore 8a^3 - 4a + 1 = 0$ Hence sin18° is a solution of $\therefore 8x^3 - 4x + 1 = 0$	✓ sin 18° = a ✓ cos 36° = 1 – 2 sin² 18° ✓ substitution of a ✓ simplification (4)
	OR	

	$\frac{1}{\sin 18°} = 4\cos 36°$ $\frac{1}{\sin 18°} = 4(1 - 2\sin^2 18°)$ $\frac{1}{\sin 18°} = 4 - 8\sin^2 18°$ $8(\sin 18°)^3 - 4(\sin 18) + 1 = 0$ Hence $\sin 18°$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$ **Note:** substituting $x = \sin 18°$ into $8x^3 - 4x + 1$ using a calculator showing equal to 0: 0 marks	✓ $\cos 36° = 1 - 2\sin^2 18°$ ✓ simplification ✓ equation i.t.o $\sin 18°$ ✓ replacing $\sin 18° = x$ (4) **[24]**

TOTAL: 150