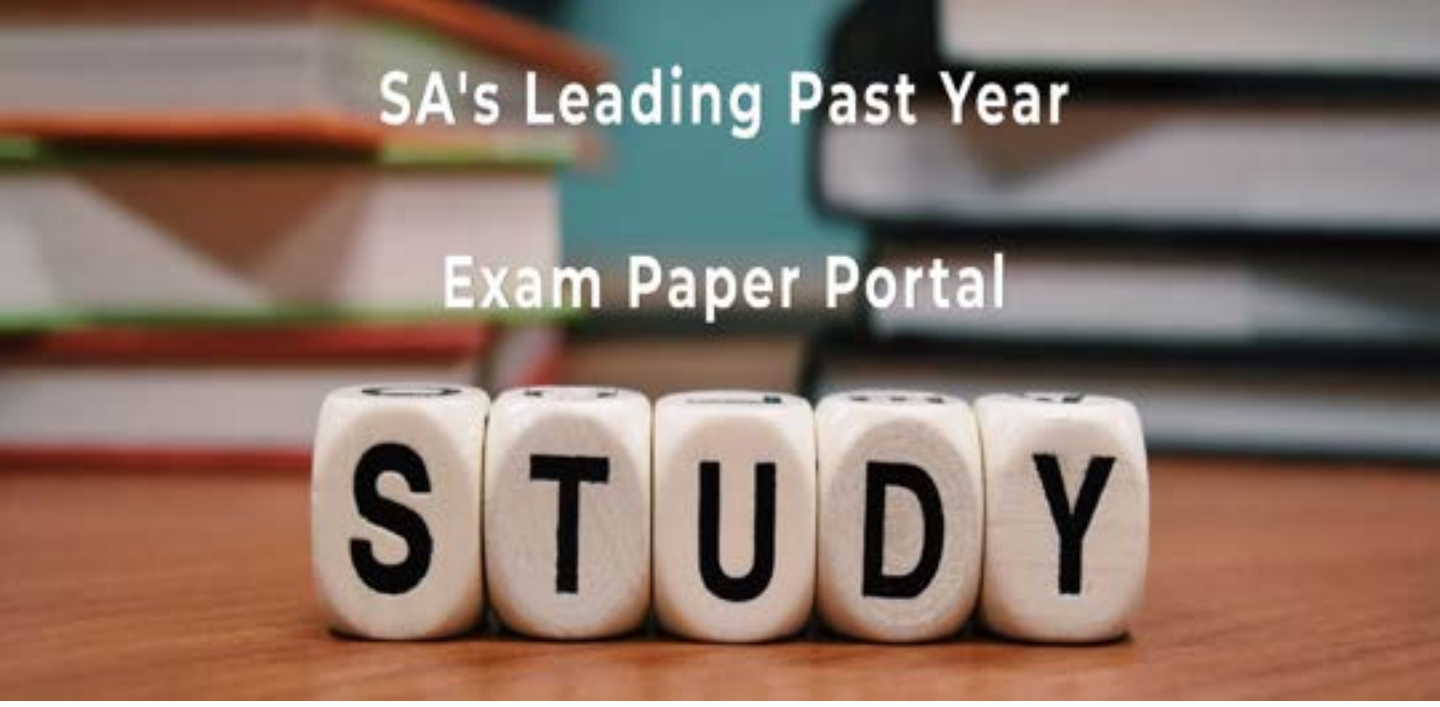


You have Downloaded, yet Another Great Resource to assist you with your Studies ☺

Thank You for Supporting SA Exam Papers

Your Leading Past Year Exam Paper Resource Portal

Visit us @ www.saexampapers.co.za

Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2022

TECHNICAL MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

This question paper consists of 15 pages, including 3 answer sheets and a 2-page information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTIONS 3.3.3, 4.2.4 and 7.5 on the ANSWER SHEETS provided. Write your name and school's name in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for *x*:

1.1.1 $(2x+3)(3x+1)=0$ (1)

1.1.2 $5x+1=-2x^2$ (Correct to ONE decimal place) (4)

1.1.3 $x^2-5x-6>0$ (3)

1.2 Solve for *x* and *y* if:

$y=8x-1$ and $y=3x^2+4$ (5)

1.3 The equivalent resistance R of two resistors, R_1 and R_2, connected in parallel is expressed as:

$$R=\frac{R_1R_2}{R_1+R_2} \quad \text{where } R_1=x^{\frac{3}{2}} \text{ and } R_2=\sqrt{x}$$

1.3.1 Express *R* in simplified terms of *x* (3)

1.3.2 Determine *R* if $x=25\,\Omega$ (2)

1.4 Given the decimal numbers: A = 75 and B = 5

Determine the value of $75\div5$ and then convert your answer to a binary number. (2)
[20]

QUESTION 2

2.1 Describe the nature of roots of $0=2x^2+5x+8$ without solving the equation. (3)

2.2 Determine ONE value of smallest positive integer *k* for which the equation $5x^2-kx-2=0$ will have rational roots. (4)
[7]

QUESTION 3

3.1 Simplify the following WITHOUT using a calculator:

3.1.1 $\dfrac{3^{2x}-3^{x}-6}{3^{x}+2}$ (2)

3.1.2 $81^{\frac{3}{4}}-\sqrt[3]{27^{2}}+\sqrt[4]{625}$ (4)

3.1.3 $\dfrac{\log_2 16-\log_2 8}{\log_2 4}$ (3)

3.2 Solve for x: $(\log x-4)\times\log(x-4)=0$ (4)

3.3 Given the complex number $z=-3+4i$.

Determine:

3.3.1 The modulus (2)

3.3.2 The size of θ (3)

3.3.3 Hence express z in polar form (where θ is in degrees) and then represent in an Argand diagram (3)

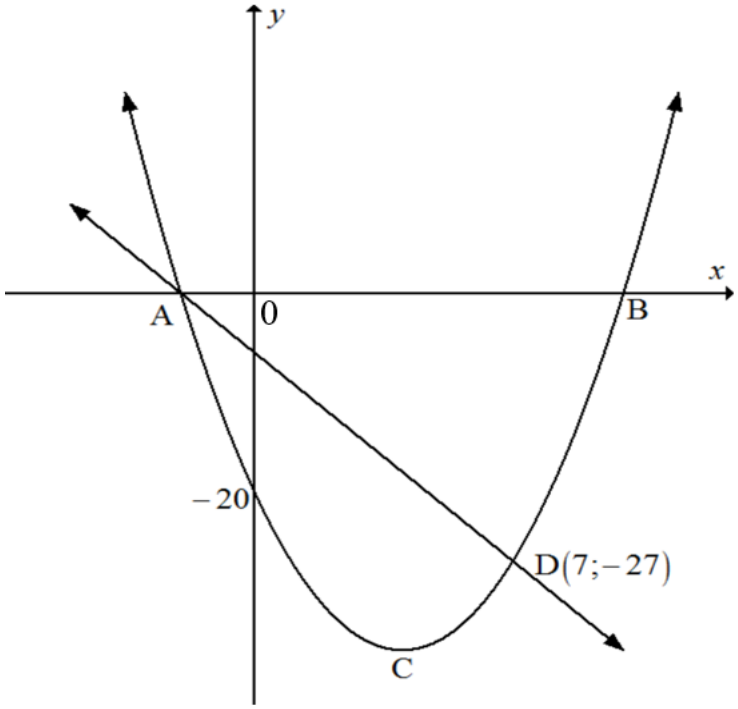
3.4 Solve for p and q if $2p-qi-8i=-2i(3i+7)$ (4)

[25]

QUESTION 4

4.1 The diagram below shows sketch graphs of functions defined by $f(x) = x^2 - 8x - 20$ and $y = mx + c$

A and B are the *x*-intercepts of *f*.
C is the turning point of *f*.
D (7; – 27) is the point on *f*.



4.1.1 Determine the coordinates of A and B. (3)

4.1.2 Write down the equation of the axis of symmetry. (2)

4.1.3 Hence, determine the coordinates of C. (2)

4.1.4 Determine the equation of the straight-line AD in the form y = … (3)

4.1.5 Determine the value(s) of *x* for which $f(x) > -20$. (2)

4.2 Given: $g(x)=\frac{1}{x-3}+2$ and $h(x)=3^x$

4.2.1 Write down the equation of the vertical asymptote of *g*. (1)

4.2.2 Determine the *x*-intercepts of *g*. (3)

4.2.3 Determine the *y*-intercepts of *g* and *h*. (3)

4.2.4 Sketch the graph of *g* and *h* on the same set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes and any asymptote(s). (6)

4.2.5 Use your graph to:

(a) Write down the domain of *g* (1)

(b) Determine the values of *x* for which $g(x) - h(x) = 0,67$ (1)

(c) Determine the equation of *k* if *k* is obtained by shifting the graph of *h*, 2 units downwards (1)

[28]

QUESTION 5

5.1 A pharmaceutical company is growing an organism to be used in a vaccine as shown in the picture below.

The organism grows at a rate of 5,5 % per hour. Determine how many units (rounded off to the nearest tenth) must they begin with, in order to have 1 200 units at the end of a 10-day period. (5)

5.2 Thabile won R500 000 in the Young Entrepreneurs Competition and decides to invest all the money into a savings account at a rate of 7,5% per annum, compounded monthly. At the end of the 2 years, she withdraws R365 000 to buy a light truck.

5.2.1 Calculate the remaining amount in the savings account after the withdrawal. (5)

5.2.2 Thabile reinvests the remaining amount at 6,75 % per annum, compounded quarterly for a period of 5 years. Calculate the final amount Thabile will have at the end of a 5-year investment period. (3)

[13]

QUESTION 6

6.1 Determine $f'(x)$ by using FIRST PRINCIPLES if $f(x) = -6x + 3$ (5)

6.2 Determine:

6.2.1 $D_x\left[\frac{x^2 - x - 12}{x - 4}\right]$ (3)

6.2.2 $\frac{dy}{dx}$ if $y = \frac{2}{3x^3} - 7x^2 + x$ (4)

6.3 Determine the average gradient of $g(x) = x^2 - 2x + 1$ between the points $(1;0)$ and $(3;4)$. (2)

[14]

QUESTION 7

Given: $f(x) = x^3 - 2x^2 - 5x + 6$

7.1 Write down the *y*-intercept of f. (1)

7.2 Show that $x - 1$ is a factor of f (2)

7.3 Determine the *x*-intercepts of f. (3)

7.4 Determine the coordinates of the turning point of f. (5)

7.5 Sketch the graph of f on the ANSWER SHEET provided. Clearly show all the coordinates of the turning points and intercepts with the axis. (4)

7.6 Determine the size of the acute angle θ between the tangent to the curve at the point where $x = 3$ and the *x*–axis. (4)

[19]

QUESTION 8

The picture below shows a loaded courier van that Ethan drives to deliver goods. He works for a courier company which pays him R145 per hour. Ethan has to deliver goods to a town which is 75 km away from the depot. If he travels at a constant speed of x km/h, the fuel costs R18/litre and the rate of fuel consumption per hour is calculated as $\left(2+\frac{x^2}{100}\right)$.

8.1 Determine Ethan's wages per hour in terms of x. (2)

8.2 Show that the fuel costs for the return trip can be expressed as: $\mathrm{P}(x)=\frac{5400}{x}+27x$ (3)

8.3 Hence, show that the total cost (in Rands) for the return trip can be expressed as:

$$\mathrm{C}(x)=\frac{27150}{x}+27x$$ (2)

8.4 Determine the speed at which Ethan needs to drive to minimise the cost of the trip. (5)

[12]

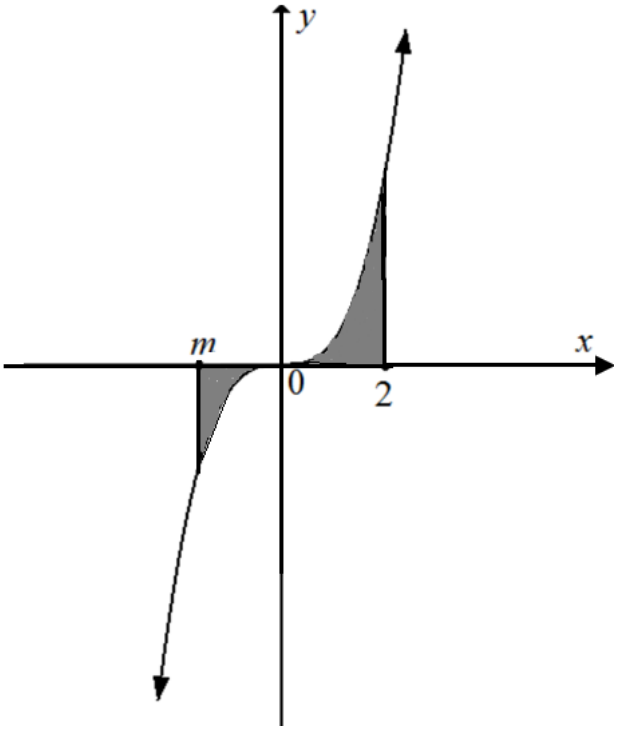
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int\left(\frac{1}{x}-1\right)dx$ (3)

9.1.2 $\int\left(x^5+6x\right)dx$ (2)

9.2 The sketch below represents the shaded bounded area of the curve of the function defined by $f(x)=x^3$ and the x-axis where $x=m$ and $x=2$.



Determine the value of m if the bounded area is 3,75 square units. (7)

[12]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1 \qquad \int k x^n \, dx = k. \frac{x^{n+1}}{n+1} + C, \; n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \; x > 0 \qquad \int \frac{k}{x} dx = k. \ln x + C, \; x > 0$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \; a > 0 \qquad \int k a^{nx} \, dx = k. \frac{a^{nx}}{n \ln a} + C, \; a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$\text{Area of } \Delta ABC = \frac{1}{2} ab. \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency
Angular velocity $= \omega = 360°n$ where n = rotation frequency

Circumferential velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

Arc length = $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2}$ where r = radius, s = arc length and θ = central angle in radians

Area of a sector $= \frac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

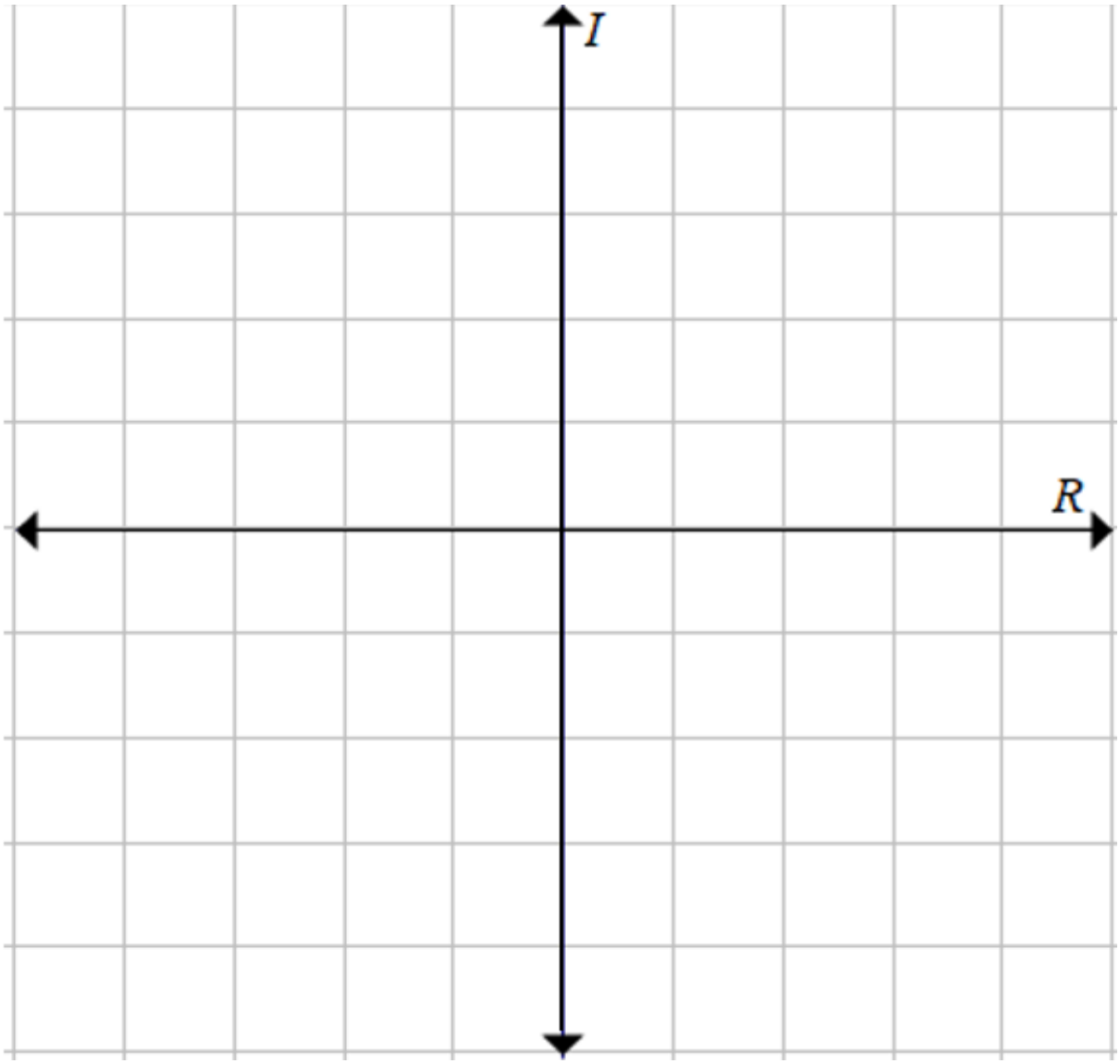
OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

NAME: _____ **SCHOOL:_____**

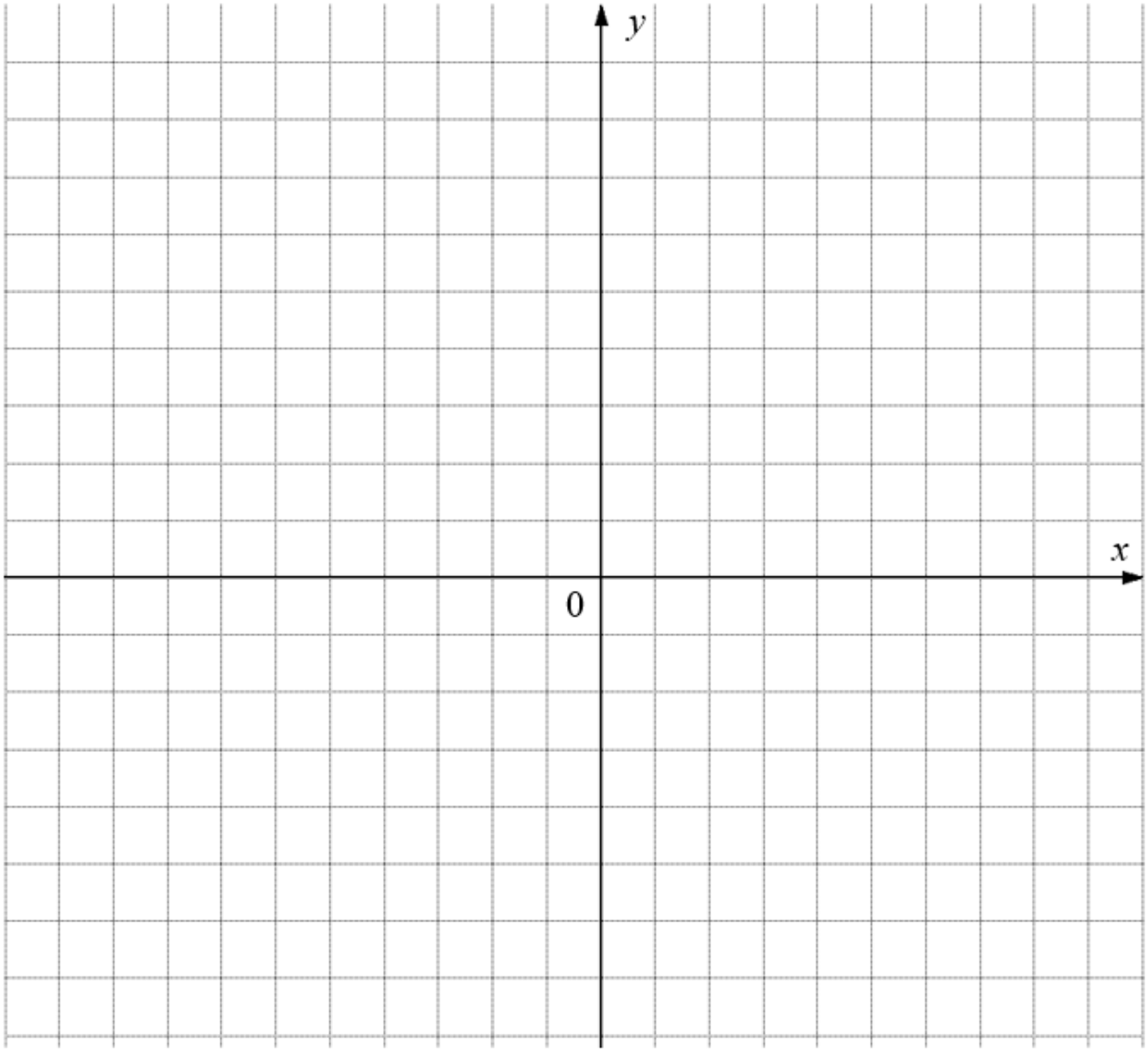
QUESTION 3.3.3



ANSWER SHEET

NAME: _____ SCHOOL: _____

QUESTION 4.2.4



ANSWER SHEET

NAME: _____ SCHOOL: _____

QUESTION 7.5