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**NATIONAL
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SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

JUNE/JUNIE 2022

**TECHNICAL MATHEMATICS P2/TEGNIESE WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 18 pages./
Hierdie nasienriglyn bestaan uit 18 bladsye.

NOTE:

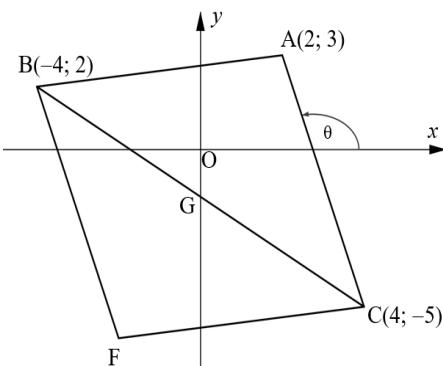
- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method/ <i>Metode</i>
A	Accuracy/ <i>Akkuraatheid</i>
AO	Answer only/ <i>Slegs antwoord</i>
CA	Consistent accuracy/ <i>Deurlopende akkuraatheid</i>
F	Formula/ <i>Formule</i>
I	Identity/ <i>Identiteit</i>
R	Rounding/ <i>Afronding</i>
S	Simplification/ <i>Vereenvoudiging</i>
ST	Statement/ <i>Bewering</i>
RE	Reason/ <i>Rede</i>
ST RE	Statement and correct reason/ <i>Bewering en korrekte rede</i>
SF	Substitution correctly in correct formula/ <i>Korrekte vervanging in die korrekte formule</i>
NPU	No penalty for omitting units/ <i>Geen penalisering vir eenhede uitgelaat</i>

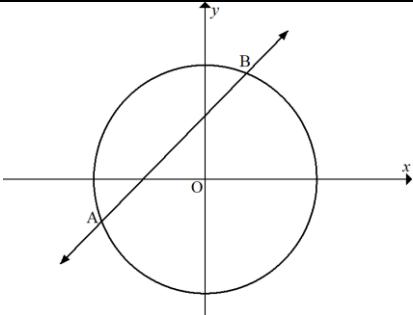
QUESTION/VRAAG 1



1.1	$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 4)^2 + (3 + 5)^2} = \sqrt{(4 - 2)^2 + (-5 - 3)^2} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$ <p style="text-align: right;">AO: Full marks / Volpunte</p>	✓ SF A ✓ S C
1.2	$\begin{aligned} m_{AC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 + 5}{2 - 4} = \frac{-5 - 3}{4 - 2} \\ &= -4 \end{aligned}$ <p>Equation of / Vergelyking van AC:</p> <p>Substitute / Vervang (2;3) or / of (4;-5) into / in $y - y_1 = m(x - x_1)$</p> $\begin{aligned} y - 3 &= -4(x - 2) & y + 5 &= -4(x - 4) \\ y &= -4x + 8 + 3 & y &= -4x + 16 - 5 \\ y &= -4x + 11 & y &= -4x + 11 \end{aligned}$ <p style="text-align: right;">OR / OF</p>	✓ SF A ✓ m_{AC} CA ✓ SF CA ✓ S CA
1.3	$\begin{aligned} \tan \theta &= m_{AC} = -4 \\ \therefore \text{Ref. Verw. } \angle &= 75,96^\circ \\ \therefore \theta &= 180^\circ - 75,96^\circ \approx 104,04^\circ \end{aligned}$	✓ M ✓ Ref / Verw \angle . ✓ Obtuse \angle / stomphoekige \angle NPU
1.4	$G\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) = \left(\frac{-4+4}{2}; \frac{2-5}{2}\right) = \left(0; -\frac{3}{2}\right)$ <p style="text-align: right;">AO: Full marks / Volpunte</p>	✓ x-coordinate / afsnit ✓ y-coordinate / afsnit
1.5	$\begin{aligned} \left(\frac{x_F + x_A}{2}; \frac{y_F + y_A}{2}\right) &= (x_G; y_G) \\ \therefore \frac{x_F + 2}{2} &= 0 \quad \text{and/en} \quad \frac{y_F + 3}{2} = -\frac{3}{2} \\ \therefore x_F &= -2 \quad \text{and/en} \quad y_F = -6 \end{aligned}$	✓ M ✓ x_F CA ✓ y_F CA

1.6	$m_{AG} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3 - (\frac{3}{2})}{2 - 0} = \frac{-\frac{3}{2} - 3}{0 - 2}$ $= \frac{9}{4}$ $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{2 - (-5)}{-4 - 4} = \frac{-5 - 2}{4 - (-4)}$ $= -\frac{7}{8}$ $m_{BC} \times m_{AG} = -\frac{7}{8} \times \frac{9}{4} \neq -1$ $\therefore BC \text{ not / nie } \perp AG$	✓ m_{AG} CA ✓ m_{BC} CA ✓ M ✓ conclusion/ gevolgtrekking (4)
		[18]

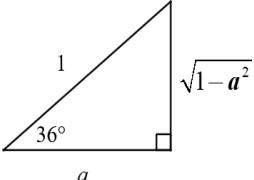
QUESTION/VRAAG 2

2.1		
2.1.1	$x^2 + (x+3)^2 = 29$ $x^2 + x^2 + 6x + 9 - 29 = 0$ $2x^2 + 6x - 20 = 0$ $x^2 + 3x - 10 = 0$ $(x+5)(x-2) = 0$ $x = -5 \text{ or } x = 2$ $y = -2 \text{ or } y = 5$	✓ substitution / vervanging ✓ expansion / uitbreiding ✓ standard form / standaardvorm ✓ Factorise / faktoriseer ✓ values of x / waardes van x ✓✓ values of y / waardes van y (7)
2.1.2 (a)	Substitute $(-5; 2)$ into circle equation <i>Vervang $(-5; 2)$ in sirkel vergelyking:</i> $r^2 = (-5)^2 + (2)^2 = 25 + 4 = 29$ \therefore the point lie on the circle / die punt lê op die sirkel	✓ M ✓ conclusion / gevolgtrekking (2)
2.1.2 (b)	$xx_1 + yy_1 = r^2$ $x(-5) + y(2) = 29$ $-5x + 2y = 29$ $y = \frac{5}{2}x + \frac{29}{2}$ <p style="text-align: center;">OR / OF</p> $m_{rad} = \frac{2}{-5}.$ $m_{tan} = \frac{5}{2}. \quad (\text{rad} \perp \tan)$ <p>Equation / Vergelyking:</p> $y - y_1 = m(x - x_1)$ $y - 2 = \frac{5}{2}(x + 5)$ $y = \frac{5}{2}x + \frac{29}{2}$	✓ F ✓ SF A ✓ S CA ✓ eq of line / vergl van lyn OR / OF ✓ m_{rad} A ✓ m_{tan} CA ✓ SF CA ✓ eq of line / vergl van lyn (4)

2.2 $\frac{x^2}{(2\sqrt{10})^2} + \frac{y^2}{8^2} = 1$	<ul style="list-style-type: none"> ✓ elliptical shape with vertical axis as major / Elliptiese vorm met groter-as die vertikale as ✓ x-intercepts/ x-afsnitte A ✓ y-intercepts/ y-afsnitte A <p style="text-align: right;">(3)</p>
	[16]

QUESTION/VRAAG 3

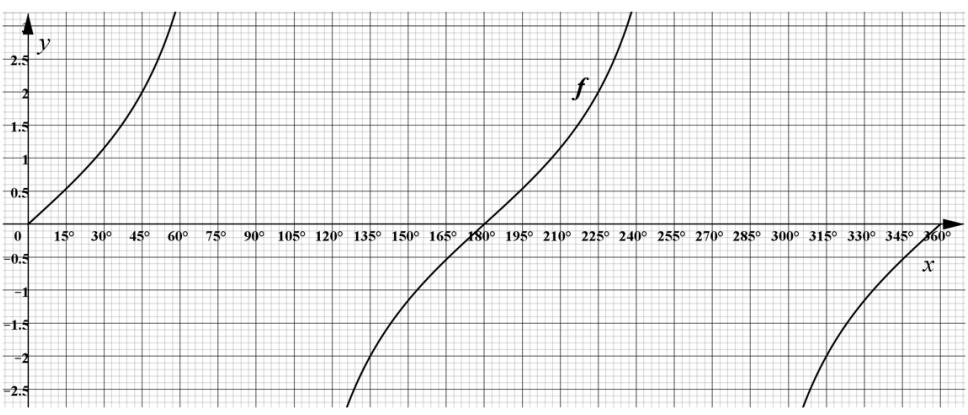
3.1		
3.1.1	$OP^2 = r^2 = (-8)^2 + (-15)^2 = 289$ $\therefore OP = 17$	✓ Subst/Vervang P A ✓ OP^2 A ✓ OP CA (3)
3.1.2	$\tan \theta = \frac{-15}{-8} = \frac{15}{8}$	✓ A (1)
3.1.3	$\cos ec^2 \theta - 1 = \cot^2 \theta$ $= \left[\frac{-8}{-15} \right]^2$ $= \frac{64}{225} =$ <p style="text-align: center;">OR / OF</p> $\cos ec^2 \theta - 1 = \left[\frac{17}{-15} \right]^2 - 1$ $= \frac{289}{225} - 1$ $= \frac{64}{225}$	✓ I A ✓ ratio / verh A ✓ S CA OR/OF ✓ ratio / verh A ✓ S CA ✓ S CA (3)
3.2.1	$\sin \left(\frac{180^\circ}{2} - 15,7^\circ \right) \approx 0,963$ <p style="text-align: center;">Penalty for units / Penaliseer vir eenhede AO: Full marks / Volpunte</p>	✓ $\pi = 180^\circ$ ✓ answer / antwoord A (2)
3.2.2	$\sec(135,5^\circ + 15,7^\circ) \approx -1,141$ <p style="text-align: center;">Penalty for rounding / Penaliseer vir afronding</p>	✓✓ answer / antwoord (2)
3.3.1	$\sin x + 1 = 0,587$ $\sin x = -0,413$ $\therefore \text{Ref. / Verw. } \angle = 24,4^\circ$ $\therefore x = 180^\circ + 24,9^\circ \text{ or / of } 360^\circ - 24,9^\circ$ $\therefore x = 204,4^\circ \text{ or / of } 335,6^\circ$	✓ S A ✓ Ref / Verw \angle CA ✓ $204,4^\circ$ CA ✓ $335,6^\circ$ CA NPU (4)

3.3.2	$\tan 2x = \frac{1}{2,114} = 0,473$ $\therefore \text{Ref. / Verw. } \angle = 25,3^\circ$ $2x = 180^\circ + 25,3^\circ$ $x = 102,7^\circ$	✓ S A ✓ Ref/Verw \angle CA ✓ Quadrant / Kwadrant A ✓ value of x / waarde van x CA (4)
3.4.1	 $\tan 36^\circ = \frac{\sqrt{1-a^2}}{a}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned}\tan 36^\circ &= \frac{\sin 36^\circ}{\cos 36^\circ} \\ &= \frac{\sqrt{1-\cos^2 36^\circ}}{\cos 36^\circ} \\ &= \frac{\sqrt{1-a^2}}{a}\end{aligned}$	✓✓ M complete Δ / voltooi Δ A ✓ ratio / verh CA OR/OF ✓ $\frac{\sin 36^\circ}{\cos 36^\circ}$ I A ✓ $\sqrt{1 - \cos^2 36^\circ}$ I A ✓ ratio i.t.o a / verh i.t.v a CA (3)
3.4.2	$\sec^2 144^\circ = \sec^2 36^\circ$ $= \frac{1}{a^2}$	✓ $\sec^2 36^\circ$ A ✓ ratio / verh. CA (2)
		[24]

QUESTION/VRAAG 4

4.1	1	✓ A (1)
4.2	$\frac{\sec x}{\cos(360^\circ - x)} + \frac{\tan^2(180^\circ - x)}{\sin(180^\circ + x) \operatorname{cosec}(180^\circ - x)}$ $= \frac{\sec x}{\cos x} + \frac{\tan^2 x}{(-\sin x)(\operatorname{cosec} x)}$ $= \frac{\sec x}{\frac{1}{\sec x}} - \frac{\tan^2 x}{(\sin x) \left(\frac{1}{\sin x}\right)} \quad \text{OR/OF} \quad \frac{1}{\cos x} - \frac{\tan^2 x}{(\sin x) \left(\frac{1}{\sin x}\right)} = \frac{1}{\cos^2 x} - \tan^2 x$ $= \sec^2 x - \tan^2 x$ $= 1$	✓ cos x A ✓ tan ² x A ✓ - sin x A ✓ cosec x A ✓ $\frac{1}{\sec x}$ I A ✓ $\frac{1}{\sin x}$ I A ✓ sec ² x - tan ² x CA ✓ 1 CA (8)
4.3	LHS/LK = sin(360° - x) cot(180° - x) = (- sin x) (- cot x) = sin x × $\frac{\cos x}{\sin x}$ = cos x = RHS/RK	✓ - sin x A ✓ - cot x A ✓ $\frac{\cos x}{\sin x}$ A ✓ S CA (4)
		[13]

QUESTION/VRAAG 5

		
5.1	$a = 2$	✓ A (1)
5.2	$x = 90^\circ$ or/of $x = 270^\circ$	✓ $x = 90^\circ$ A ✓ $x = 270^\circ$ A (2)
5.3	180°	✓ A (1)

5.4	<p>(60, 1)</p> <p>(240, -1)</p>	<ul style="list-style-type: none"> ✓ shape with end points / vorm met eindpunte A ✓ x-intercepts / x-afsnitte A ✓ turning points / draaipunte A (3)
5.5	1	✓ A (1)
5.6.1	for f / vir: $x \in \{0^\circ; 180^\circ; 360^\circ\}$ for /vir $g: x \in \{150^\circ; 330^\circ\}$	<ul style="list-style-type: none"> ✓ set of values for f / versameling van waardes vir f A ✓ set of values for g / versameling van waardes vir g CA (2)
5.6.2	$0^\circ \leq x < 90^\circ$ or/of $180^\circ \leq x < 270^\circ$ OR/OF $x \in (0^\circ; 90^\circ)$ or/of $x \in (180^\circ; 270^\circ)$	$0^\circ \leq x < 90^\circ$ <ul style="list-style-type: none"> ✓ end points / eindpunte A ✓ interval notation / notasie A $180^\circ \leq x < 270^\circ$ <ul style="list-style-type: none"> ✓ end points / eindpunte A ✓ interval notation / notasie A (4)
		[14]

QUESTION/VRAAG 6

6.1	$p^2 = q^2 + r^2 - 2qr \cos P$ OR/OF $q^2 = p^2 + r^2 - 2pr \cos Q$ OR/OF $r^2 = q^2 + p^2 - 2qp \cos R$	✓ A (1)
6.2		
6.2.1	<p>In $\triangle ABC$:</p> $b^2 = a^2 + c^2 - 2ac \cos B$ $10^2 = 8^2 + 9^2 - 2(8)(9)\cos B$ $\cos B = \frac{64 + 81 - 100}{144} = 0,3125$ $\hat{B} = 71,79^\circ$	✓ F A ✓ SF A ✓ value of / waarde van $\cos B$ CA ✓ answer / antwoord CA NPU (4)
6.2.2	$A\widehat{O}C = 143,58^\circ \dots$ (\angle a centre = $2 \times \angle$ at circum.) \angle midpts $\angle = 2 \times$ omtreks. \angle	✓ ST CA ✓ RE A (2)

6.2.3	$O\hat{A}C = O\hat{C}A$ $\angle s$ opp. = sides $\angle e$ teenoor. = sye $O\hat{A}C = \frac{180^\circ - 143,58^\circ}{2} \dots$ int. $\angle s$ of Δ binne $\angle e$ of Δ $= 18,2^\circ$ <p>In ΔAOC:</p> $\frac{a}{\sin A} = \frac{o}{\sin O}$ $\frac{a}{\sin 18,2^\circ} = \frac{o}{\sin 143,58^\circ}$ $a = \frac{10 \sin 18,2^\circ}{\sin 143,58^\circ}$ $a = 5,26$ $\therefore \text{diameter / middellyn} = 10,52 \text{ units / eenhede}$	✓ $O\hat{A}C = 18,2^\circ = O\hat{C}A$ ✓ sin rule/reël M ✓ SF CA ✓ value of / waarde van a CA ✓ value of diameter / waarde van middellyn CA ✓ NPU OR/OF ✓ cos rule/reël M ✓ SF CA ✓ simplification / vereenvoudiging ✓ value of / waarde van r CA ✓ value of diameter / waarde van middellyn CA (5)
6.2.4	$\text{Area } \Delta ABC = \frac{1}{2} ac \sin B$ $= \frac{1}{2} (8)(9)\sin 71,79^\circ$ $= 34,20 \text{ units}^2/\text{eenhede}^2$	✓ F ✓ SF CA ✓ answer / antwoord NPU (3)
		[15]

QUESTION/VRAAG 7

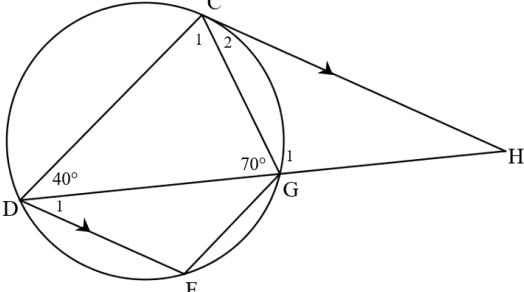
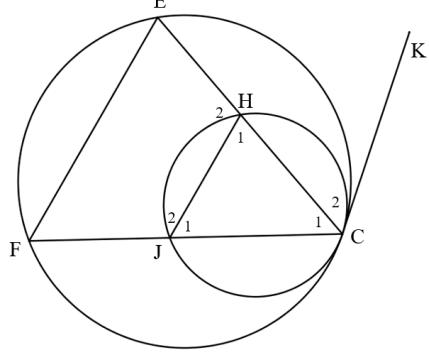
7.1		
7.1.1	$(x + 2)^2 = x^2 + 4^2 \dots \text{Pythagoras}$ $x^2 + 4x + 4 = x^2 + 16$ $4x = 12$ $x = 3$ $BC = 3 \text{ units/eenhede}$	✓ Pythagoras ✓ S CA ✓ answer /antwoord NPU (3)
7.1.2	AB (chord) subtends a 90° angle / koord onderspan 90° hoek OR/OF Converse \angle in semi-circle / omgekeerde \angle in semi-sirkel	✓ reason / rede (1)
7.2		
7.2.1	$\widehat{B}_3 = 35^\circ \dots \left(\begin{matrix} \angle \text{s opp = sides /} \\ \angle \text{e teenoor = sye} \end{matrix} \right)$ $\widehat{B}\widehat{O}\widehat{C} = 110^\circ \dots \left(\begin{matrix} \text{Int } \angle \text{s of } \Delta / \\ \text{Binne } \angle \text{e van } \Delta \end{matrix} \right)$ $\widehat{O}_1 = 110^\circ \dots \left(\begin{matrix} \text{Revolutions /} \\ \text{Omwenteling} \end{matrix} \right)$	✓ ST RE ✓ ST RE ✓ ST RE (3)
7.2.2	$\widehat{B}_2 = \frac{180^\circ - 140^\circ}{2} \dots \left(\begin{matrix} \text{Int } \angle \text{ of } \Delta \text{ and } \angle \text{s opp - sides /} \\ \text{Binne } \angle \text{e van } \Delta \text{ en } \angle \text{e teenoor = sye} \end{matrix} \right)$ $= 20^\circ$ $\widehat{B}_1 = 90^\circ - (35^\circ - 20^\circ) \quad \left(\begin{matrix} \angle \text{ in semi-circle /} \\ \angle \text{ in semi-sirkel} \end{matrix} \right)$ $= 35^\circ$	✓ ST RE ✓ ST ✓ RE (3)

7.2.3	$\widehat{C_1} = \widehat{B_1}$ ($\angle s$ in same segm. /) $\angle e$ in dies. segm. $= 35^\circ$ $= AC\widehat{B}$ $\therefore AC$ bisect/halveer $D\widehat{C}B$	✓ ST RE ✓ ST (2) [12]
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QUESTION/VRAAG 8

8.1	Interior opposite angle / teenoorstaande binnehoek	✓ reason / rede (1)
8.2		
8.2.1	$\widehat{C} = 85^\circ \dots \begin{pmatrix} \text{Int } \angle s \text{ of } \Delta \\ \text{Binne } \angle e \text{ van } \Delta \end{pmatrix}$ $\widehat{F}_2 = 85^\circ \dots \begin{pmatrix} \text{ext } \angle \text{ of cyclic quad} \\ \text{buite } \angle \text{ van kdvh} \end{pmatrix}$ $\widehat{G} = 51^\circ \dots \begin{pmatrix} \angle s \text{ opp = sides} \\ \angle e \text{ teenoor = sye} \end{pmatrix}$ $\widehat{D}_2 = 44^\circ \dots \begin{pmatrix} \text{Int } \angle s \text{ of } \Delta \\ \text{Binne } \angle e \text{ van } \Delta \end{pmatrix}$ OR/OF $\widehat{D}_1 = 180^\circ - 2 \times 51^\circ - 44^\circ \dots \begin{bmatrix} \text{opp. } \angle s \text{ cyclic quad} \\ \text{teenoorst. } \angle e \text{ van kdvk} \end{bmatrix}$ $= 34^\circ$ $\widehat{F}_1 = 95^\circ \dots \begin{pmatrix} \text{Int } \angle s \text{ of } \Delta \\ \text{Binne } \angle e \text{ van } \Delta \end{pmatrix}$ $\widehat{G} = 51^\circ \dots \begin{pmatrix} \angle s \text{ opp = sides} \\ \angle e \text{ teenoor = sye} \end{pmatrix}$ $\widehat{D}_2 = 95^\circ - 51^\circ \quad \begin{bmatrix} \text{ext. } \angle \text{ of } \Delta \\ \text{buite } \angle \text{ van } \Delta \end{bmatrix}$ $= 44^\circ$	✓ ST RE ✓ ST ✓ RE ✓ ST RE ✓ ST RE OR/OF ✓ ST ✓ RE ✓ ST RE ✓ ST RE ✓ ST RE ✓ ST RE ✓ ST RE (5)
8.2.2	In $\triangle GFD$ and/en $\triangle BCD$: (1) $GD = BD \dots \text{(given / gegee)}$ (2) $\widehat{D}_2 = \widehat{BDC} \dots \begin{pmatrix} \text{proved in 8.2.1 above} \\ \text{bewys in 8.2.1 hierbo} \end{pmatrix}$ (3) $FD = CD \dots \begin{pmatrix} \text{converse } \angle s \text{ in same segm.} \\ \text{omgekeerde } \angle e \text{ in dies. segm.} \end{pmatrix}$ $\therefore \triangle GFD \equiv \triangle BCD \dots \text{(SAS)}$	✓ ST RE ✓ ST RE ✓ ST ✓ RE ✓ RE (5)
		[11]

QUESTION/VRAAG 9

9.1	Radius / diameter <i>Radius / middellyn</i>	✓ A (1)
9.2		
9.2.1	$\widehat{C}_2 = 40^\circ \dots (\tan - \text{chord/koord})$	✓ ST ✓ RE (2)
9.2.2	$\widehat{C}_1 = 70^\circ \dots (\text{Int } \angle \text{s of } \Delta \text{ Binne } \angle \text{e van } \Delta)$ $\widehat{F} = 110^\circ \dots (\text{opp } \angle \text{s of cyclic quad teenoorst. } \angle \text{e van kdvk})$	✓ ST RE ✓ ST ✓ RE (3)
9.2.3	$\widehat{H} = 30^\circ \dots (\text{Int. } \angle \text{s of } \Delta \text{ Binne } \angle \text{e van } \Delta)$ $\widehat{D}_1 = 30^\circ \dots (\text{alt/verw. } \angle \text{s; } CH \parallel DF)$ OR/OF $\widehat{C} + \widehat{D} = 180^\circ \dots (\text{co-int/ko-binne } \angle \text{s; } CH \parallel DH)$ $\widehat{D}_1 = 30^\circ$	✓ ST RE ✓ ST RE OR/OF ✓ RE ✓ ST (2)
9.3		
	$\widehat{C}_2 = \widehat{F} \dots (\tan - \text{chord/koord})$ $\widehat{C}_2 = \widehat{J}_1 \dots (\tan - \text{chord/koord})$ $\therefore \widehat{F} = \widehat{J}_1$ $\therefore EF \parallel JH \dots (= \text{corrsp. } \angle \text{s} (= \text{ooreenk. } \angle \text{e}))$	✓ ST ✓ RE ✓ ST RE ✓ RE (4)
		[12]

QUESTION/VRAAG 10

10.1	Parallel to the third side / ewewydig aan die derde sy	✓ A (1)
10.2		
10.2.1	$\frac{FC}{DF} = \frac{EC}{AE} \dots \text{(prop th/ewerh.; } EF \parallel AD\text{)}$ $\frac{FC}{3} = \frac{10}{4}$ $FC = 7,5 \text{ units/eenhede}$	✓ ST ✓ RE ✓ ST (3)
10.2.2	$\frac{BD}{DC} = \frac{AE}{EC} \text{ (prop th/ewerh.; } AD \parallel ED\text{)}$ $\frac{BD}{10,5} = \frac{4}{10}$ $BD = 4,2 \text{ units/eenhede}$	✓ ST ✓ RE ✓ ST CA ✓ ST CA (4)
10.3		
10.3.1	In ΔABD and/en ΔCBA : <ol style="list-style-type: none"> (1) \widehat{B} is common/gemeen (2) $\widehat{BAD} = \widehat{C}$... (given / gegee) $\therefore \Delta ABD \sim \Delta CBA \dots (\text{AAA})$	✓ ST ✓ ST ✓ RE (3)
10.3.2	$\frac{AB}{CB} = \frac{BD}{BA} = \frac{AD}{CA} \dots (\sim \Delta s)$ $\therefore \frac{8}{4+DC} = \frac{4}{8}$ $\therefore 4(4+DC) = 64$ $\therefore 4+DC = 16$ $\therefore DC = 12 \text{ units/eenhede}$	✓ ST ✓ RE ✓ ST ✓ ST (4)
		[15]
	TOTAL/TOTAAL:	150