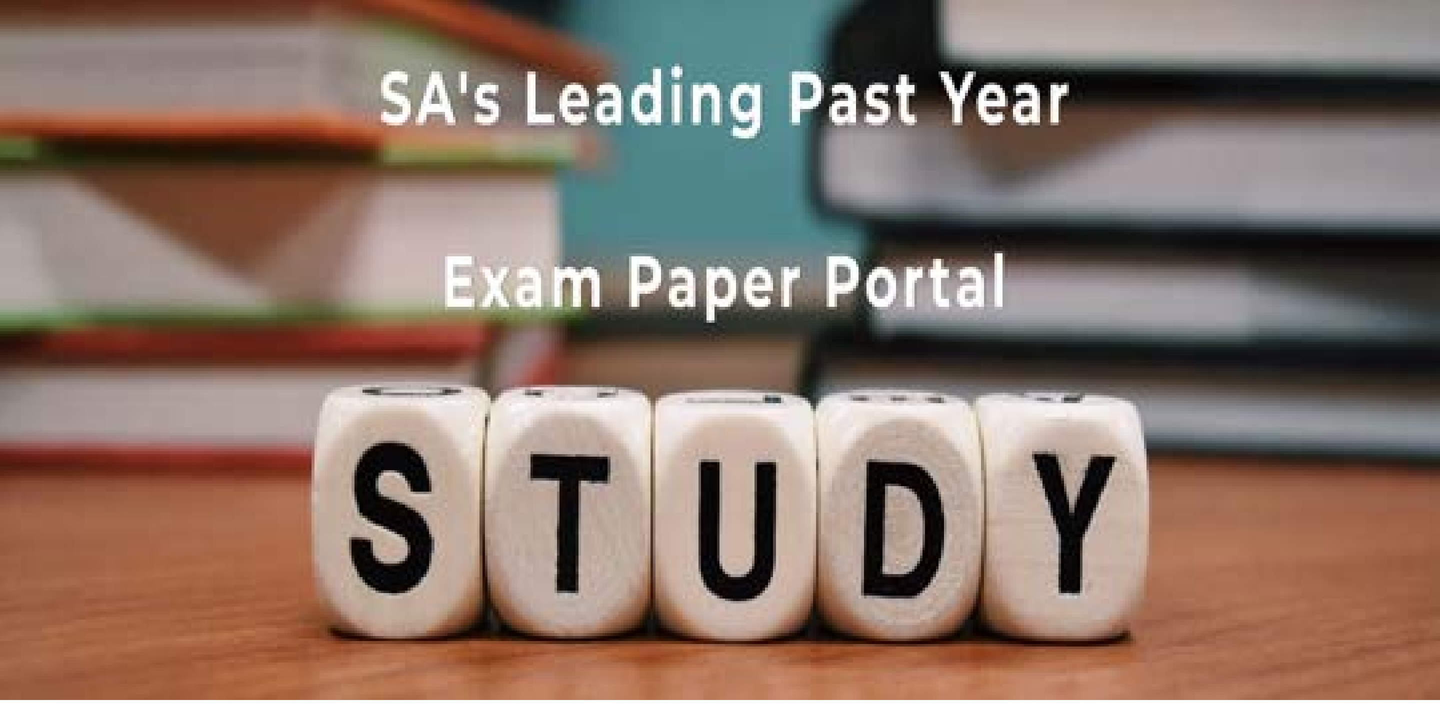
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SENIOR CERTIFICATE EXAMINATIONS/
NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P2

2022

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages and 2 information sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you used to determine your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

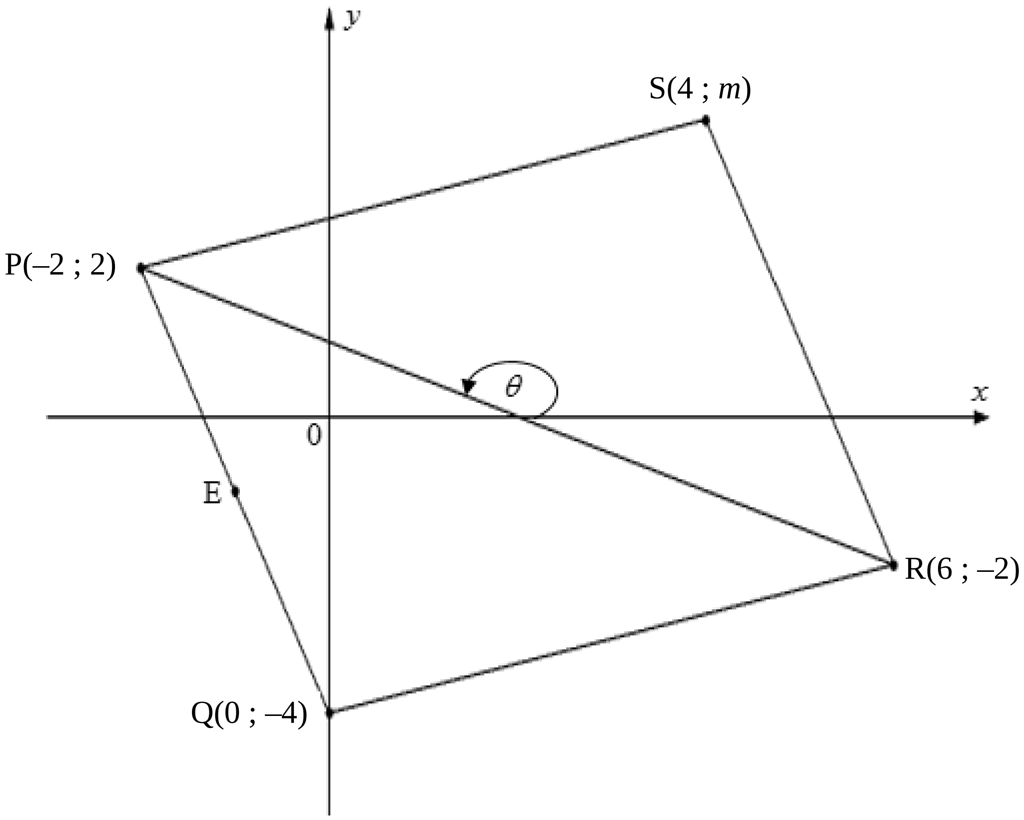
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

In the diagram below, PQRS is a quadrilateral with vertices P(–2 ; 2), Q(0 ; –4), R(6 ; –2) and S(4 ; *m*).
E is the midpoint of PQ.
The angle formed by PR and the positive *x*-axis is θ.



1.1 Determine:

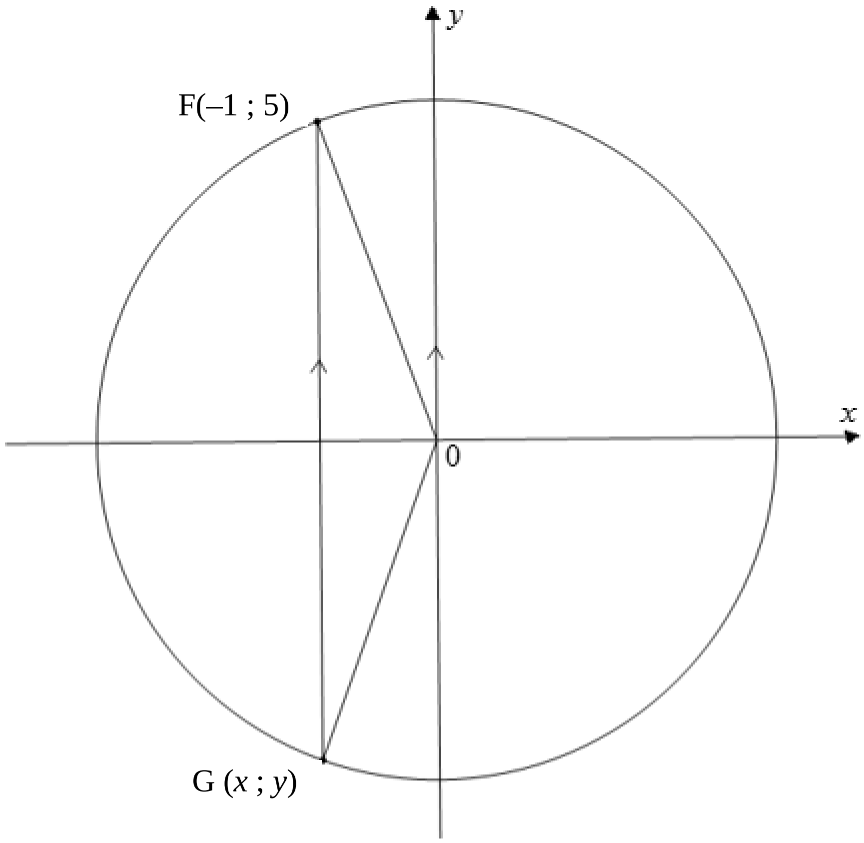
1.1.1	The gradient of PR	(2)
1.1.2	θ, the angle of inclination of PR	(3)
1.1.3	The length of QR (leave your answer in surd form)	(2)
1.1.4	The coordinates of E	(2)
1.1.5	The equation of SR, if $SR \parallel PQ$	(4)
1.1.6	The value of m	(2)

1.2 Show that ΔPQR is a right-angled triangle. (2)

[17]

QUESTION 2

2.1 In the diagram below, $F(-1\,;5)$ and $G\,(x\,;\,y)$ are points on the circle with the centre at the origin. FG is parallel to the y-axis.



2.1.1 Write down the coordinates of G. (2)

2.1.2 Determine:

(a) The gradient of OF (1)

(b) The equation of the tangent to the circle at F in the form $y = \ldots$ (3)

2.2 Draw, on the grid provided in the ANSWER BOOK, the graph defined by:

$$\frac{x^2}{7}+\frac{y^2}{64}=1$$

Clearly show ALL the intercepts with the axes. (3)

[9]

QUESTION 3

3.1 Given $\mathrm{Q} = 42°$ and $\mathrm{P} = 71°$

Determine:

3.1.1 $\cot(\mathrm{P} - \mathrm{Q})$ (3)

3.1.2 $\dfrac{\cos \mathrm{Q}}{\sec \mathrm{P}}$ (3)

3.2 Given $3\sec\beta - 5 = 0$ and $\beta \in [90° ; 360°]$

Determine $\sin^2\beta - \cos^2\beta$ with the aid of a diagram. (7)

3.3 Solve for x: $\cos 2x - \tan 29° = 0$ and $2x \in [0°;360°]$ (5)

[18]

QUESTION 4

4.1 Simplify: $\cot^2 \mathrm{A} \cdot \sin^2 \mathrm{A} + \cos^2 \mathrm{A} \cdot \tan^2 \mathrm{A}$ (4)

4.2 Prove that: $\dfrac{\sin^2(\pi + \theta) + \cos(180° - \theta)\cdot\sec(360° - \theta)}{\tan(2\pi - \theta)\cdot\cot(180° + \theta)} = \cos^2\theta$ (9)

[13]

QUESTION 5

Given the functions defined by $f(x) = \sin x$ and $g(x) = \cos 2x$, where $x \in [0°;180°]$

5.1 Write down the period of g. (1)

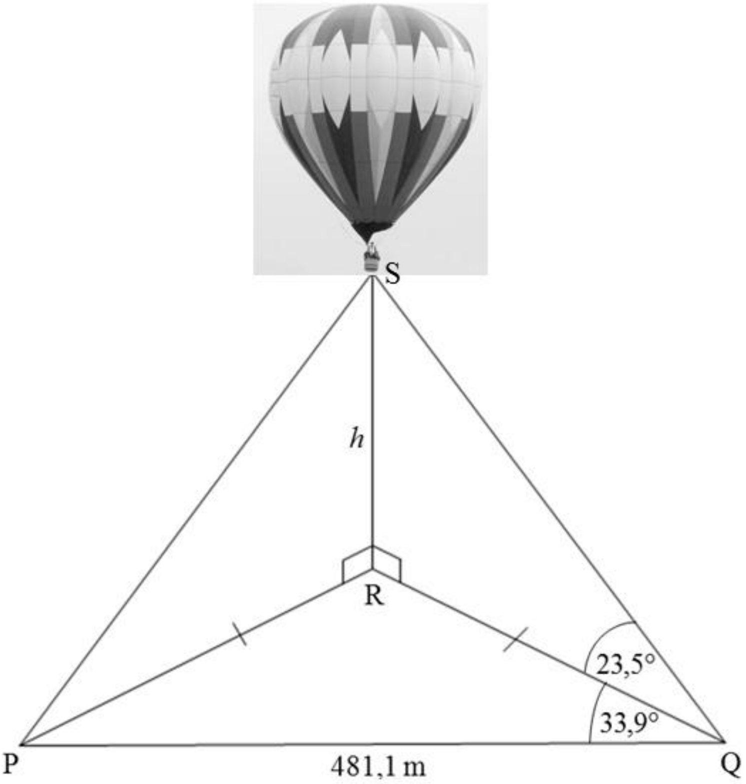
5.2 Draw sketch graphs of f and g on the same set of axes on the grid provided in the ANSWER BOOK. Clearly indicate ALL turning points, end points and intercepts with the axes. (6)

[7]

QUESTION 6

The diagram below represents two observers at P and Q who are equidistant from point R.
The two observers are 481,1 m apart.
The observers sight an air balloon at S, which is h metres above R.
The angle of elevation of S from Q is 23,5°
P, Q and R lie on the same horizontal plane.

$P\hat{Q}R = 33{,}9°$



Determine:

6.1 The size of $P\hat{R}Q$ (2)

6.2 RQ, the distance between the observer at Q and point R (3)

6.3 The value of h, to the nearest metre (3)

6.4 The area of ΔQPR (2)

[10]

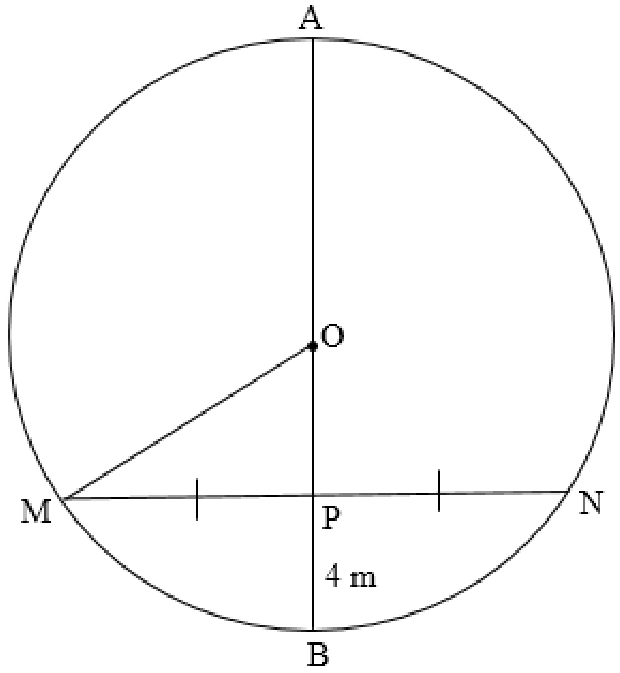
Give reasons for your statements in QUESTIONS 7, 8 and 9.

QUESTION 7

7.1 Complete the following theorem statement:

The line drawn from the centre of a circle to the midpoint of a chord is … (1)

7.2 The diagram below shows a circle with centre O.
AB bisects MN at P.
AP = 16 m and PB = 4 m



7.2.1 Determine the length of OM. (1)

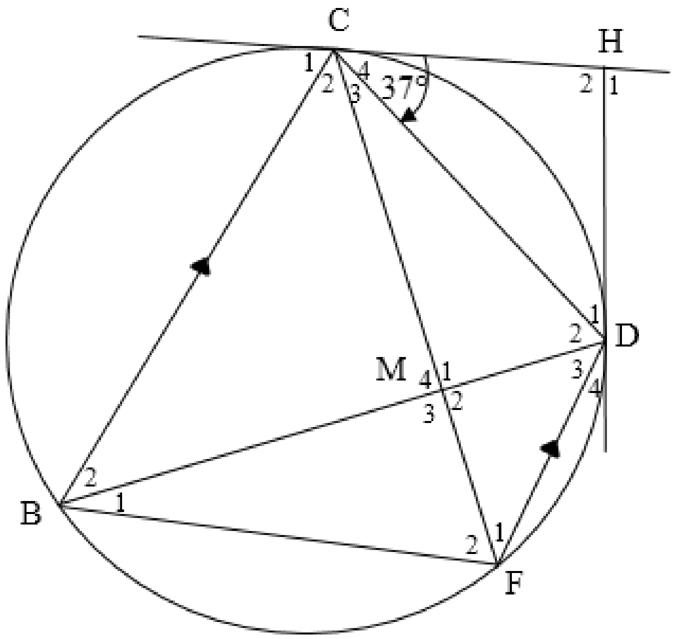
7.2.2 Determine, stating reasons, the length of MP. (5)

[7]

QUESTION 8

In the diagram below, CBFD is a circle such that BC||FD.
CH and DH are tangents at C and D respectively. Tangents CH and DH intersect at H.
CF and BD intersect at M.

$\hat{C}_4 = 37°$



8.1 Determine, giving reasons, the size of $\hat{H}_1$ (4)

8.2 Determine, stating reasons, the size of $\hat{C}_2$ (4)

8.3 Show that MD = MF (3)

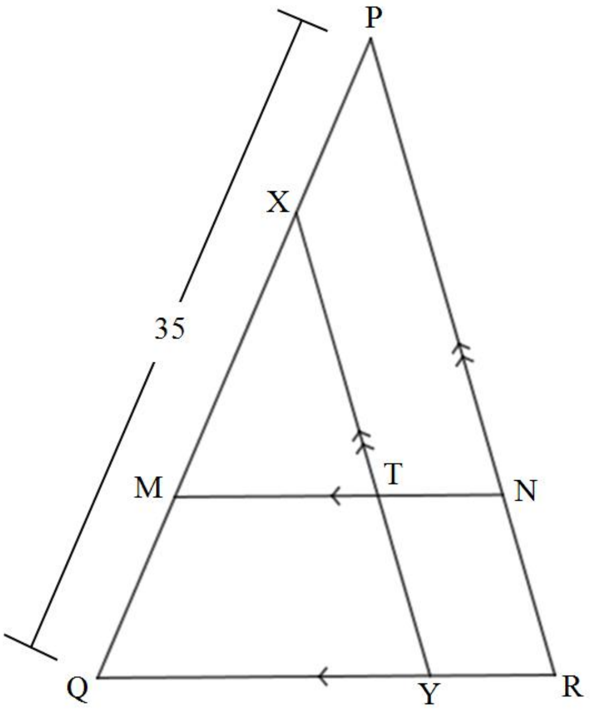
8.4 Prove that CHDM is a cyclic quadrilateral. (4)
[15]

QUESTION 9

9.1 Complete the following theorem statement:

A line drawn parallel to one side of a triangle … (1)

9.2 In ΔPQR below, $XY \parallel PR$ and $MN \parallel QR$.
XY and MN intersect at T.
Furthermore, PQ = 35 units, PN : NR = 5 : 2 and QY = 3YR.

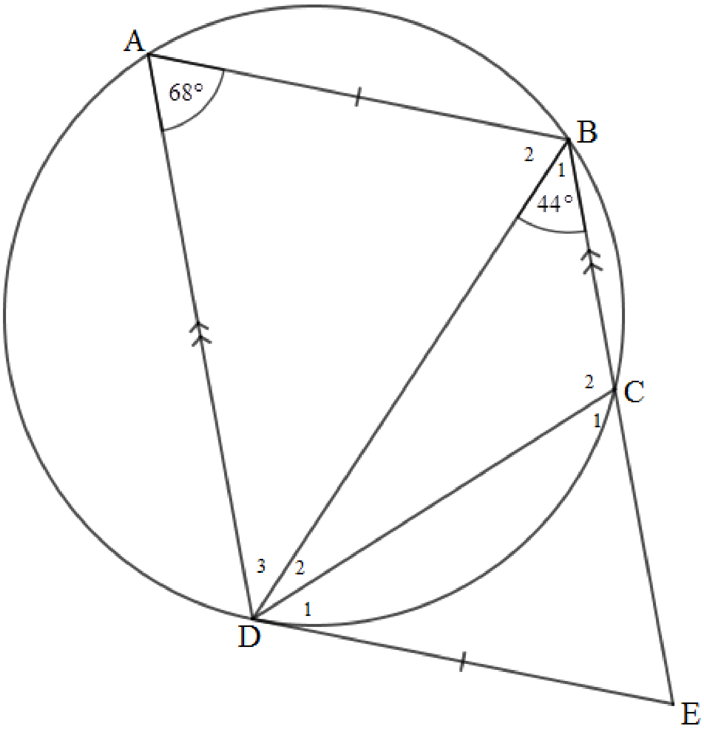


Determine with reasons:

9.2.1 PM (4)

9.2.2 XM (5)

9.3 In the diagram below, ABCD is a cyclic quadrilateral with BC || AD.
ED is a tangent to the circle at D and BC is produced to E.
AB = DE
$\hat{A} = 68°$ and $\hat{B}_1 = 44°$



9.3.1 Write down, stating reasons, TWO other angles, each equal to $44°$ (4)

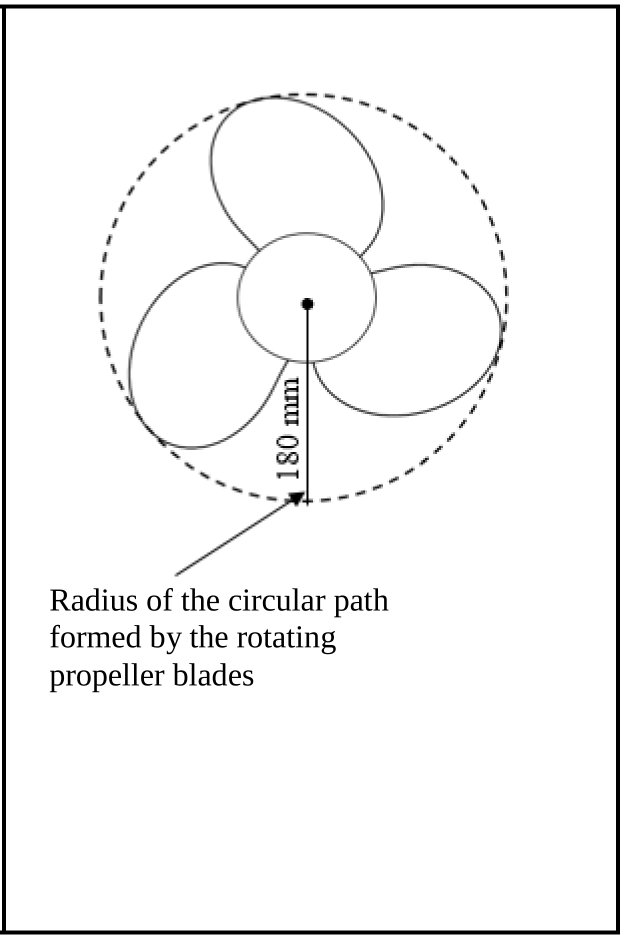
9.3.2 Determine, giving reasons, the size of $\hat{C}_2$ (2)

9.3.3 Prove, giving reasons, that $\Delta ABD \parallel\!| \; \Delta CED$ (3)

[19]

QUESTION 10

10.1 The outboard motor (pictured below) is used to propel boats through water and has a 4-stroke engine. At cruising speed, the engine causes the tips of the propeller blades to rotate at a circumferential velocity of 30 km/h. The diagram alongside depicts the circular path formed by the rotating propeller blades and the radius of the circular path is 180 mm.

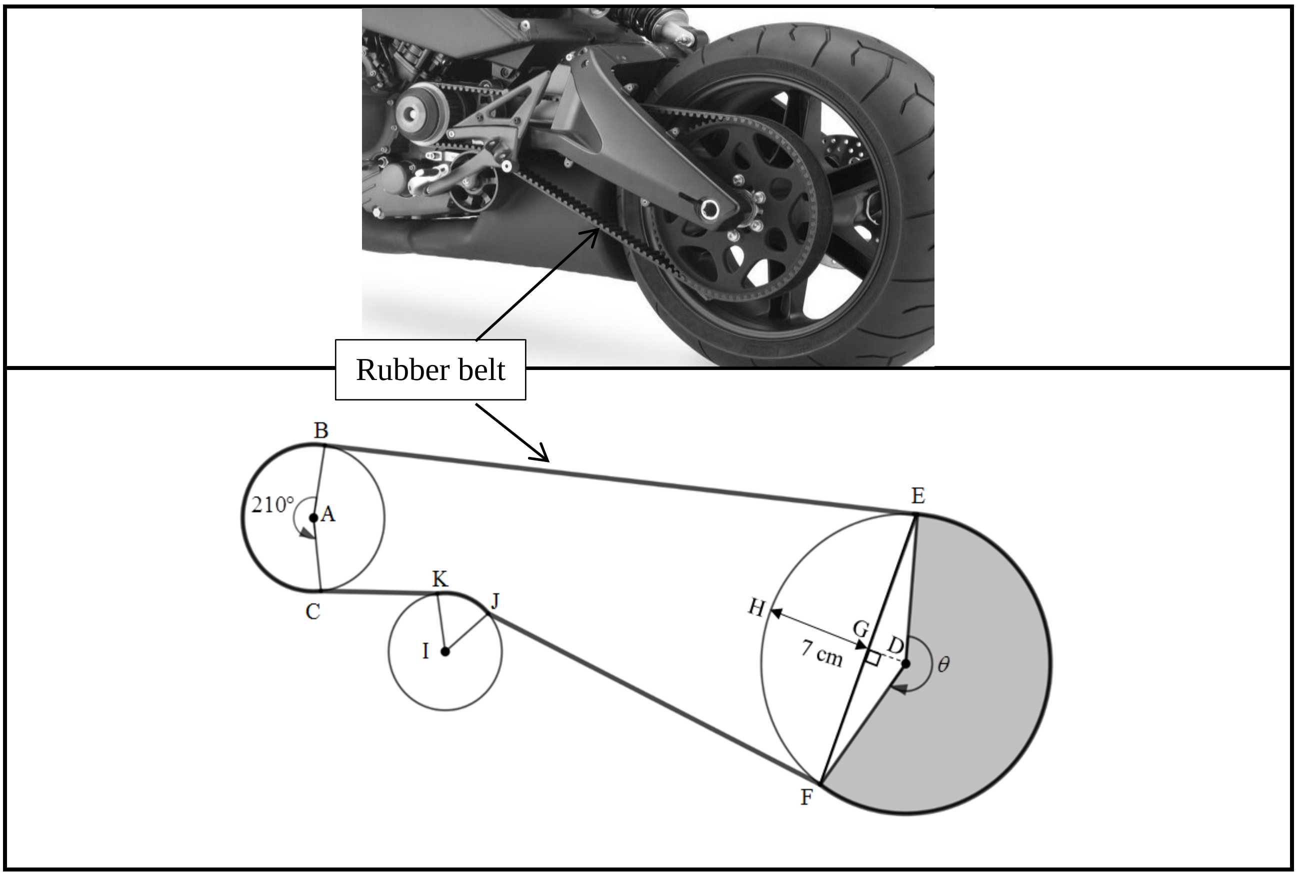


10.1.1 Convert 30 km/h, the circumferential velocity, to metres per second. (2)

10.1.2 Hence, determine the angular velocity of the rotating blades in radians per second. (4)

10.2 The picture below shows a motorcycle drive belt that uses a three-pulley system. The diagram below the picture models the system.

- The radius of the circle (centre A) is 5 cm and major arc BC subtends a central reflex angle $B\hat{A}C$ equal to 210°.
- The area of the shaded sector EFD of the largest circle (centre D) is 54π cm^2.
- The largest circle has a radius of 9 cm and major arc EF subtends a central reflex angle of θ .
- The height of minor segment GH in relation to chord EF is 7 cm.
- The rubber belt in contact with the smallest circle (centre I) is represented by minor arc JK.
- The length of the rubber belt around the entire system is 140 cm.



10.2.1 Convert 210° to radians. (1)

10.2.2 Hence, determine the length of major arc BC. (3)

10.2.3 Calculate the size of θ in the largest circle with centre D. (3)

10.2.4 Determine the length of chord EF. (3)

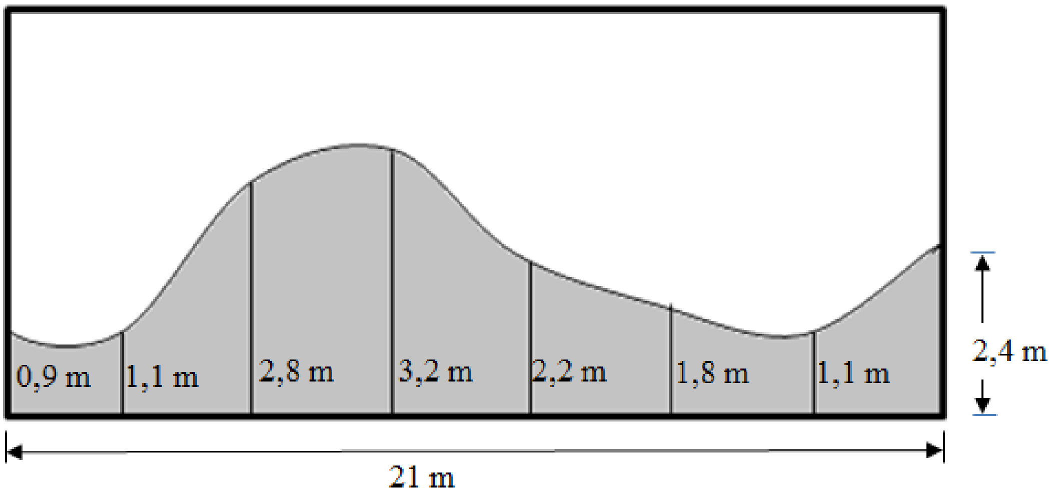
10.2.5 If the length of minor arc JK is 4,19 cm, calculate the length of the rubber belt that is NOT in contact with the three pulleys. (5)

[21]

QUESTION 11

11.1 The diagram below shows rectangular wall art with a partially shaded irregular shape. The shaded irregular shape has a horizontal straight side, 21 m long, which is divided into seven equal parts.

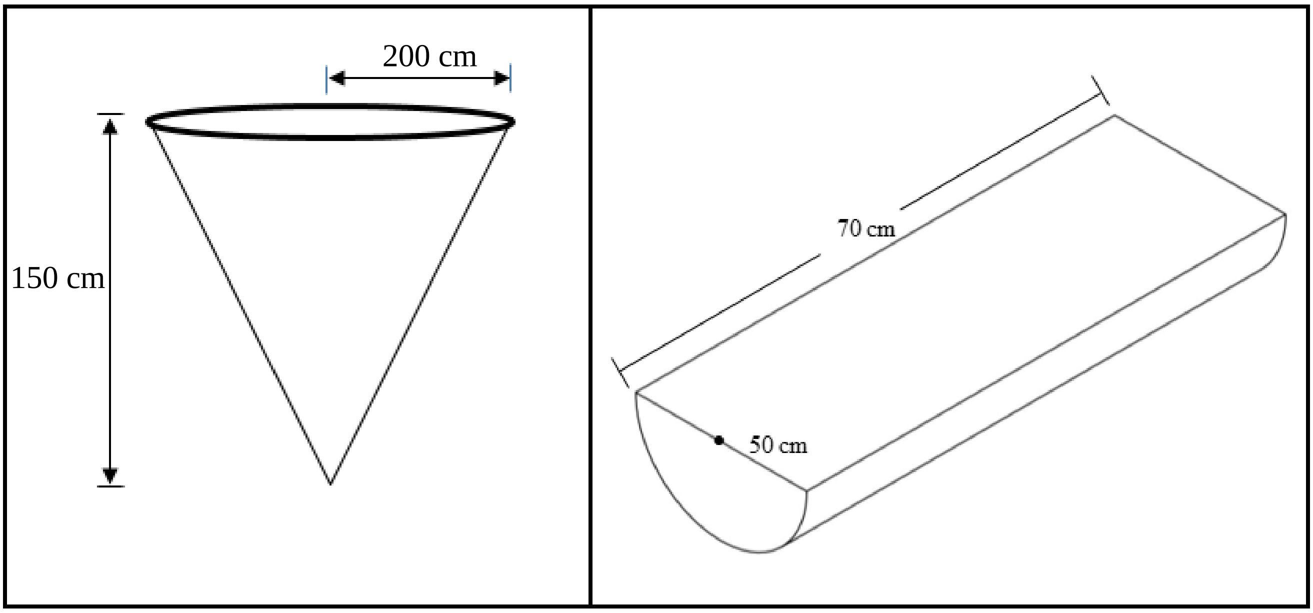
The ordinates dividing the parts are:
0,9 m ; 1,1 m ; 2,8 m ; 3,2 m ; 2,2 m ; 1,8 m ; 1,1 m ; 2,4 m



11.1.1 Determine the area of the shaded irregular shape by using the mid-ordinate rule. (4)

11.1.2 Determine the new shaded area if the horizontal straight side is increased by 80%, while the number of equal parts and the heights of the ordinates remain the same. (2)

11.2 A cone-shaped water tank is used to fill a water container in the shape of a half-cylinder.
The cone has a height of 150 cm and a radius of 200 cm.
The half-cylindrical container has a radius of 50 cm and a length of 70 cm.



The following formulae may be used:

Surface area of a cone = $\pi r s + \pi r^2$
Total surface area of a cylinder = $2\pi r^2 + 2\pi r h$
Volume of a cone = $\frac{1}{3}\pi r^2 h$
Volume of a cylinder = $\pi r^2 h$

11.2.1 Calculate the exterior surface area of the half-cylindrical container. (3)

11.2.2 How many times will it be possible to fully fill the half-cylindrical tank from the cone-shaped tank? (5)

[14]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1 \text{ and } b > 0$$

$$\mathrm{A} = \mathrm{P}(1 + ni) \qquad \mathrm{A} = \mathrm{P}(1 - ni) \qquad \mathrm{A} = \mathrm{P}(1 + i)^n \qquad \mathrm{A} = \mathrm{P}(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \qquad \int k x^n\,dx = k.\frac{x^{n+1}}{n+1} + C \quad, n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln x + C, \quad x > 0 \qquad \int \frac{k}{x}\,dx = k.\ln x + C, \quad x > 0$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C, \quad a > 0 \qquad \int k a^{nx}\,dx = k.\frac{a^{nx}}{n \ln a} + C \quad, a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta\mathrm{ABC}: \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}} \qquad a^2 = b^2 + c^2 - 2bc.\cos \mathrm{A}$$

$$\textit{area of } \Delta\ \mathrm{ABC} = \frac{1}{2} ab.\sin \mathrm{C}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$\pi \, rad = 180°$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate
and n = number of ordinates