PREPARATORY EXAMINATION

2019

MARKING GUIDELINES

MATHEMATICS (PAPER 1) (10611)

20 pages

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

MATHEMATICS
(Paper 1)

MARKING GUIDELINES

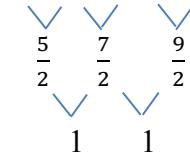| QUESTION 1		
1.1.1	$2x^2 + 3 = 8x$ $2x^2 - 8x + 3 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$ $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$ $x = 3,58 \quad$ **or** $\quad x = 0,42$ (4) **Penalise for rounding-off in this question ONLY.**	✓standard form ✓ correct substitution into correct formula ✓✓ answers (4)
1.1.2	$4x - 2x(x - 3) \le 0$ $4x - 2x^2 + 6x \le 0$ $2x^2 - 10x \ge 0$ $2x(x - 5) \ge 0$ $x \ge 5 \quad$ or $\quad x \le 0$ **Both inequalities MUST be correct for allocating the marks.**	✓standard form ✓factors ✓✓ answers (4)

1.1.3	$2^x - 5.2^{x+1} = -144$	✓factorise
	$2^x(1 - 5.2) = -144$	
	$2^x(1 - 10) = -144$	
	$2^x(-9) = -144$	✓simplification (-9)
	$2^x = 16$ **OR** $x = \log_2 16$	
	$2^x = 2^4$	✓simplification (2^4) **OR** correct use of logs
	$x = 4$	✓answer
		(4)
1.2	$f(x) = (x - 2)(x + 6)$	✓ both correct factors
	$f(x) = x^2 + 4x - 12$	✓ answer
		(2)
1.3	$2x + y = 17$ …………(1)	
	$\qquad xy = 8$ …… (2)	
	from (1)	
	$2x + y = 17$	
	$\qquad y = 17 - 2x$ ……(3)	✓ solve for y in equ. 1
	substitute (3) into (2)	
	$x(17 - 2x) = 8$	✓ substitute for y in equ. 2
	$17x - 2x^2 = 8$	
	$-2x^2 + 17x - 8 = 0$	
	$2x^2 - 17x + 8 = 0$	✓ standard form
	$(2x - 1)(x - 8) = 0$	✓ factors
	$x = \dfrac{1}{2}$ **or** $x = 8$	✓ both x answers
	substitute into (3)	

$$y = 17 - 2\left(\frac{1}{2}\right)$$

$$y = 16$$

or

$$y = 17 - 2(8)$$

$$y = 1$$

✓ both y answers

OR

OR

$2x + y = 17$ …………..(1)

$xy = 8$ ………..(2)

from (2)

$xy = 8$

$y = \dfrac{8}{x}$ ………….(3)

✓ solve for y in equ. 2

substitute (3) into (1)

$$2x + \frac{8}{x} = 17$$

$$2x^2 - 17x + 8 = 0$$

$$(2x - 1)(x - 8) = 0$$

$$x = \frac{1}{2} \quad \text{or} \quad x = 8$$

✓ substitute for y in equation 1

✓ standard form

✓ factors

✓ both x answers

substitute into (2)

$\dfrac{1}{2}y = 8$ **OR** $8y = 8$

$y = 16$ $y = 1$

✓ both y answers

		(6)
1.4	$2mx^2 = 3x - 8$ $2mx^2 - 3x + 8 = 0$ $\Delta = (-3)^2 - 4(2m)(8)$ $\Delta = 9 - 64m$ for non-real roots, $\Delta < 0$ $9 - 64m < 0$ $\quad 9 < 64m \qquad$ **OR** $\qquad -64m < -9$ $\quad \dfrac{9}{64} < m \qquad\qquad\qquad m > \dfrac{9}{64}$	✓ standard form ✓ correct substitution into correct formula ✓ $\Delta < 0$ ✓ answer (4)
		[24]

QUESTION 2

2.1	$-\dfrac{1}{2}$; 2 ; $\dfrac{11}{2}$; 10 $2a = 1 \qquad\qquad\qquad 3a + b = \dfrac{5}{2} \qquad\qquad\qquad a + b + c = -\dfrac{1}{2}$ $\quad a = \dfrac{1}{2} \qquad\qquad\quad 3(\dfrac{1}{2}) + b = \dfrac{5}{2} \qquad\qquad \dfrac{1}{2} + 1 + c = -\dfrac{1}{2}$ $\qquad\qquad\qquad\qquad\qquad\quad b = 1 \qquad\qquad\qquad\qquad\quad c = -2$ $T_n = \dfrac{1}{2}n^2 + n - 2$ $T_n = \dfrac{1}{2}(n^2 + 2n - 4)$	✓ $a = \dfrac{1}{2}$ ✓ $b = 1$ ✓ $c = -2$ ✓ $T_n = \dfrac{1}{2}n^2 + n - 2$ (4)
2.2	$T_{75} - T_{74} = [\dfrac{1}{2}\{(75)^2 + 75 - 2\}] - [\dfrac{1}{2}\{(74)^2 + 74 - 2\}]$ $T_{75} - T_{74} = \dfrac{151}{2}$	✓ correct substitution ✓ answer (2)

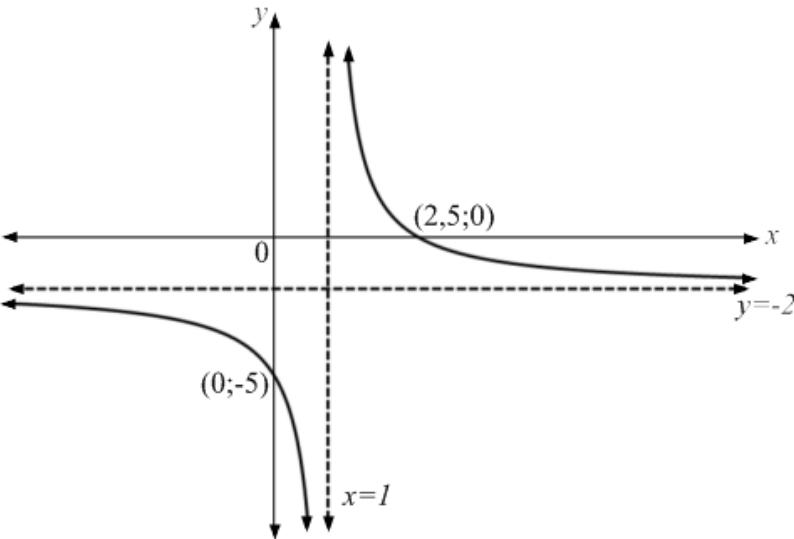| 2.3.1	Arithmetic Constant difference between successive terms	✓ arithmetic ✓ correct reason (2)
2.3.2	74th	✓ answer (1)
2.3.3	$T_n = a + (n-1)d$ $T_{30} = \dfrac{5}{2} + (30-1)(1)$ $T_{30} = \dfrac{63}{2}$	✓ correct substitution of a, n and d ✓ answer (2)
2.3.4	$Sn = \dfrac{n}{2}[2a + (n-1)d]$ $2176 = \dfrac{n}{2}[2(\dfrac{5}{2}) + (n-1)(1)]$ $0 = n^2 + 4n - 4352$ $0 = (n-64)(n+68)$ $n = 64$ $n \neq -68$ $64 + 1 = 65$ **OR** $-\dfrac{1}{2} + 2176 = \dfrac{1}{2}(n^2 + 2n - 4)$ $-1 + 4352 = n^2 + 2n - 4$ $n^2 + 2n - 4355 = 0$ $(n+67)(n-65) = 0$ $n \neq -67 \quad n = 65$	✓ substitute into correct formula ✓ standard form ✓ choice of $n = 64$ ✓ answer **OR** ✓✓ equating correctly ✓ standard form ✓ choosing correct n-value (4)
		[15]

QUESTION 3		
3.1	$r = \dfrac{2(3x-1)^2}{2(3x-1)}$ $r = 3x - 1$ $-1 < 3x - 1 < 1$ $0 < 3x < 2$ $0 < x < \dfrac{2}{3}$	✓ value of r ✓ $-1 < 3x - 1 < 1$ ✓ correct constraints (3)

3.2	$T_2 = ar$ $S_\infty = 25$
	 $6 = kr$
	 $r = \dfrac{6}{k}$ ... (1) $S_\infty = \dfrac{a}{1-r}$
	
	              $25 = \dfrac{k}{1-r}$ ... (2)
	 sub. (1) into (2)
	
	 $25 = \dfrac{k}{1 - \dfrac{6}{k}}$
	
	 $k = 25\left(1 - \dfrac{6}{k}\right)$
	
	 $k = 25 - \dfrac{150}{k}$
	 $0 = k^2 - 25k + 150$
	 $0 = (k-10)(k-15)$
	 $\therefore k = 10$ **or** k = 15
		✓ $r = \dfrac{6}{k}$
		 ✓ substitution
		     (a, r, S_∞)
		
		 ✓ standard form
		
		 ✓ factors
		 ✓ answers
		 (5)

3.3	$(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + ... + (81 \times 82)$
	
	 $(4n - 3) \times (4n - 2)$
	 $= 16n^2 - 20n + 6$
	
	 $4n - 3 = 81$ **OR** $4n - 2 = 82$
	      $4n = 84$ $4n = 84$
	        $n = 21$ $n = 21$
	
	 $\displaystyle\sum_{n=1}^{21} 16n^2 - 20n + 6$ **OR** $\displaystyle\sum_{n=1}^{21}(4n-3)(4n-2)$
		✓ $(4n - 3)$
		 ✓ $(4n - 2)$
		
		 ✓ $n = 21$
		
		 ✓ answer
		 (4)

		[12]

QUESTION 4		
4.1	$4500 = 3000\left(1 + \dfrac{0,08}{12}\right)^{n}$ $\dfrac{3}{2} = \left(1 + \dfrac{0,08}{12}\right)^{n}$ $\log_{\left(1+\frac{0,08}{12}\right)} \dfrac{3}{2} = n$ $n = 61,02$ months ⟋ (accept 62) $n = 5,09$ years ⟋ (accept 5,17) **NOTE: (5,08 is NOT accepted.)**	✓ correct substitution ✓ simplification ✓ correct use of logs ✓ correct answer in years (4)
4.2.1	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $40\,000 = \dfrac{x[1-(1+\frac{0,24}{12})^{-240}]}{\frac{0,24}{12}}$ $x = $ R806,96	✓ $n = 240$ ✓ $i = \dfrac{0,24}{12}$ ✓ correct sub. into correct formula ✓ correct answer (4)
4.2.2	$P_o = \dfrac{x[1-(1+i)^{-n}]}{i}$ $P_o = \dfrac{806,96\left[1-\left(1+\frac{0,24}{12}\right)^{-180}\right]}{\frac{0,24}{12}}$ $= R39205,67$ **OR** $P_o = P(1+i)^n - \dfrac{x[(1+i)^n -1]}{i}$ $P_o = 40\,000(1+\frac{0,24}{12})^{60} - \dfrac{806,96\left[\left(1+\frac{0,24}{12}\right)^{60}-1\right]}{\frac{0,24}{12}}$	✓ correct substitution into correct formula ✓ $n = 180$ ✓ $i = \dfrac{0,24}{12}$ ✓ answer **OR** ✓ correct substitution into correct formula ✓ $n = 60$ ✓ $i = \dfrac{0,24}{12}$ ✓ answer (4)

	$= R39\,206,20$	
4.2.3	$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$ $$39205,67 = \frac{x[1-(1+\frac{0,18}{12})^{-180}]}{\frac{0,18}{12}}$$ $$x = R631,38$$ **NOTE: If the candidate uses R39 206,20 then the answer is R631,39.**	✓ $n = 180$ ✓ $i = \frac{0,18}{12}$ ✓ correct substitution into correct formula ✓ correct answer (4)
		[16]

QUESTION 5		
5.1	$f(x) = \dfrac{3}{x-1} - 2$ $0 = \dfrac{3}{x-1} - 2$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 5$ $x = \dfrac{5}{2}$ $\therefore \left(\dfrac{5}{2} \; ; \; 0 \right)$ **NOTE: Must be in coordinate form.**	✓ equate to 0 ✓ answer (2)
5.2	$f(0) = \dfrac{3}{0-1} - 2$ $y = -5$ $\therefore (0 \; ; \; -5)$ **NOTE: Not necessarily in coordinate form.**	✓ answer (1)
5.3		✓ shape ✓ *x*- and *y*-intercepts ✓ correct asymptotes (3)

5.4	$f(x) = \dfrac{3}{x-1} - 2$ Point of intersection $(1 \; ; \; -2)$ $y = -x + q$ $-2 = -(1) + q$ $q = -1$ $\therefore y = -x - 1$	✓ $m = -1$ ✓ substitute point $(1;-2)$ ✓ answer (3)
		[9]

QUESTION 6			
6.1	$f(x) = -2x^2 - 5x + 3$ $x = \dfrac{-(-5)}{2(-2)}$ $x = -\dfrac{5}{4}$ $f(\dfrac{-5}{4}) = -2(\dfrac{-5}{4})^2 - 5(\dfrac{-5}{4}) + 3$ $f(\dfrac{-5}{4}) = \dfrac{49}{8}$ $\therefore TP(\dfrac{-5}{4}; \dfrac{49}{8})$ **OR** $f'(x) = 0$ $-4x - 5 = 0$ $x = \dfrac{-5}{4}$ $f(\dfrac{-5}{4}) = -2(\dfrac{-5}{4})^2 - 5(\dfrac{-5}{4}) + 3$ $f(\dfrac{-5}{4}) = \dfrac{49}{8}$ $\therefore TP(\dfrac{-5}{4}; \dfrac{49}{8})$ **OR** $f(x) = -2x^2 - 5x + 3$ $f(x) = -2[x^2 + \dfrac{5}{2}x - \dfrac{3}{2}]$ $f(x) = -2[x^2 + \dfrac{5}{2}x + (\dfrac{5}{4})^2 - (\dfrac{5}{4})^2 - \dfrac{3}{2}]$ $f(x) = -2[(x + \dfrac{5}{4})^2 - \dfrac{49}{16}]$ $f(x) = -2(x + \dfrac{5}{4})^2 + \dfrac{49}{8}$ $\therefore TP(\dfrac{-5}{4}; \dfrac{49}{8})$			✓ correct substitution into $x = \dfrac{-b}{2a}$ ✓ x-value ✓ y-value **OR** ✓ derivative ✓ equate to zero ✓ y-value **OR** ✓ determine $\dfrac{25}{16}$ or $\left(\dfrac{5}{4}\right)^2$ ✓ factorisation $\left(x + \dfrac{5}{4}\right)^2$ ✓ y-value (3)

6.2	$y \in (-\infty; \frac{49}{8}]$ **OR** $y \le \frac{49}{8}$	✓ answer (1)
6.3	$\tan 135^0 = -1$ $m = -1$ $f'(x) = -1$ $-4x - 5 = -1$ $\therefore x = -1$ $f(-1) = -2(-1)^2 - 5(-1) + 3$ $y = 6$ $\therefore P(-1 ; 6)$	✓ $m = -1$ ✓ $f'(x) = -1$ ✓ $x = -1$ ✓ $y = 6$ (4)
6.4	$k < -\dfrac{49}{8}$ **or** $k > -\dfrac{49}{8}$	✓ answer ✓ answer (2)
		[10]

QUESTION 7		
7.1	$f(x) = a^x$ $\dfrac{1}{4} = a^2$ $\sqrt{\dfrac{1}{4}} = a$ $\dfrac{1}{2} = a$	✓ correct substitution ✓ square root (2)
7.2	$y = \left(\dfrac{1}{2}\right)^x$ $x = \left(\dfrac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$	✓ interchange x and y ✓ answer (2)
7.3	$y = \left(\dfrac{1}{2}\right)^x$ $-y = \left(\dfrac{1}{2}\right)^x$ $h(x) = -\left(\dfrac{1}{2}\right)^x$ **OR** $h(x) = -a^x$ **OR** $h(x) = -f(x)$	✓ answer (1)
7.4	$x \le 0$ **OR** $x > 0$ **OR** $x < 0$ **OR** $x \ge 0$	✓✓ answer (2)
		[7]

QUESTION 8			
8.1	8.1.1	$x < 2$	✓✓ answer (2)
	8.1.2	$0 < x \le 1$	✓ critical values ✓ answer in correct notation (2)
8.2	8.2.1	$g(x) = \log_2 x$ $y = \log_2 x$ $\therefore x = 2^y$ $\therefore y = 2^x$ $\therefore g^{-1}(x) = 2^x$	✓ interchange x and y ✓ answer (2)
	8.2.2	$\log_2(3-x) = x$ $2^x = 3 - x$ therefore point of intersection of g^{-1} and f	✓ g^{-1} (inverse) ✓ point of intersection (2)
	8.2.3	$x = 1$	✓ answer (1)
			[9]

QUESTION 9			
9.1	9.1.1	$f(x) = 3x - x^2$ $f(x + h) = 3(x + h) - (x + h)^2$	✓ $f(x + h)$
		$f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \dfrac{3(x + h) - (x + h)^2 - (3x - x^2)}{h}$	✓correct substitution
		$f'(x) = \lim_{h \to 0} \dfrac{3x + 3h - x^2 - 2xh + h^2 - 3x + x^2}{h}$	
		$f'(x) = \lim_{h \to 0} \dfrac{3h - 2xh + h^2}{h}$	✓simplification
		$f'(x) = \lim_{h \to 0} \dfrac{h(3 - 2x + h)}{h}$	✓common factor h
		$f'(x) = 3 - 2x$	✓correct answer
		NOTE: Answer only $(\dfrac{0}{5})$	(5)
	9.1.2	$f(1) = 2$ $f(3) = 0$	✓ $f(1)$ and $f(3)$ both correct
		$\dfrac{f(3) - f(1)}{2}$	
		$= \dfrac{0 - 2}{2}$	✓ substitution
		$= -1$	✓ answer (3)
9.2	9.2.1	$y = \dfrac{8 - 3x^6}{8x^5}$	
		$y = x^{-5} - \dfrac{3}{8}x$	✓ simplification
		$\dfrac{dy}{dx} = -5x^{-6} - \dfrac{3}{8}$	✓ $-5x^{-6}$
			✓ $-\dfrac{3}{8}$ (3)

9.2.2	$D_x\left[\sqrt[3]{x^2} + \dfrac{1}{x} + 2x\right]$		
	$D_x\left[x^{\frac{2}{3}} + x^{-1} + 2x\right]$		✓ simplification
	$D_x\left[\dfrac{2}{3}x^{-\frac{1}{3}} - x^{-2} + 2\right]$		✓ $\dfrac{2}{3}x^{-\frac{1}{3}}$
			✓ $-x^{-2}$
			✓ 2
			(4)
			[15]

QUESTION 10		
10.1		✓ shape ✓ x and y-intercepts ✓ turning points (3)
10.2	$f(x) = a(x-1)^2(x-4)$ $\quad 8 = a(3-1)^2(3-4)$ $\quad 8 = a(-4)$ $\quad a = -2$ $f(x) = -2(x-1)^2(x-4)$ $f(x) = -2(x^2 - 2x + 1)(x-4)$ $f(x) = -2x^3 + 12x^2 - 18x + 8$	✓ substitution ✓ $a = -2$ ✓ substitution ✓ simplification (4)
10.3	$f(x) = -2x^3 + 12x^2 - 18x + 8$ $f'(x) = -6x^2 + 24x - 18$ $f''(x) = -12x + 24$ $f''(x) < 0$ $-12x + 24 < 0$ $-x < -2$ $x > 2$	 ✓second derivative ✓ $f''(x) < 0$ ✓ answer (3)
		[10]

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QUESTION 11		
11.1	$P = \pi r + 4x + 2l$ $32 = \pi.2x + 4x + 2l$ $16 = \pi x + 2x + l$ $l = 16 - \pi x - 2x$	✓ correct formula ✓ correct substitution ✓ answer (3)
11.2	$A = 4x.l + \dfrac{1}{2}\pi(2x)^2$ $A = 4x[16 - \pi x - 2x] + 2\pi x^2$ $= -8x^2 - 2\pi x^2 + 64x$	✓ correct formula ✓ correct substitution and simplification (2)
11.3	$A'(x) = 0$ $-16x - 4\pi x + 64 = 0$ $4x + \pi x - 16 = 0$ $x(4 + \pi) = 16$ $x = \dfrac{16}{4 + \pi}$ $x = 2,24m$ **OR** $x = \dfrac{-64}{2(-8 - 2\pi)}$ $x = 2,24$	 ✓ derivative ✓ derivative = 0 ✓ answer **OR** ✓✓ sub. *a* and *b* into A.o.S. formula ✓ answer (3)
		[8]

QUESTION 12			
12.1		P(A or B) = P(A) + P(B) – P(A and B) 0,6 = 0,3 + 0,4 - P(A and B) P(A and B) = 0,1	✓ substitution ✓ answer (2)
12.2	12.2.1	$A = 32$ $B = 48$	✓ answer ✓ answer (2)
	12.2.2	P(iPhone) $= \dfrac{101}{166}$ $= 0,61$	✓✓ $\dfrac{101}{166}$ (2)
	12.2.3	P(iPhone/Gr 12) $= \dfrac{48}{101}$ $= 0,48$	✓✓ $\dfrac{48}{101}$ (2)
			[8]

QUESTION 13		
13.1	9! = 362 880 **OR** 9×8×7×6×5×4×3×2×1 = 362 880	✓✓ 9! **OR** ✓✓ $9\times8\times7\times6\times5\times4\times3\times2\times1$ (2)
13.2	4! × 3! × 2! × 3! = 1 728	✓✓ 4!×3!×2!×3! ✓ answer (3)
13.3	From left to right and from right to left Therefore 2 ways **NOTE: If answer is 1 way, award 1 mark ONLY**	✓✓ answer (2)
		[7]

TOTAL: 150