Metro North Education District

Grade 12

Mathematics P1
September 2019

MARKS: 150
TIME: 3 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included at the end of the question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write legibly and present your work neatly.

This question paper consists of 10 pages and 1 information sheet.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(3x+2)=0$ (2)

1.1.2 $2x(2x-1)=3$ (correct to TWO decimal places) (4)

1.1.3 $3^{x+1}-3^{x-1}-7=2^{-x}.2^{x}$ (4)

1.1.4 $\sqrt{2-x}=\frac{x^2-x-2}{\sqrt{2-x}}$ (5)

1.2 Given $f(x)=-x^2+7x+8$ and $g(x)=-3x+24$

Calculate the coordinates for $f(x)=g(x)$ (6)

1.3 The roots of a quadratic equation are given by the following:

$$x=\frac{-4\pm\sqrt{k(3-k)}}{2}$$

1.3.1 Determine the nature of the roots if $k=2$ (2)

1.3.2 For which values of k will the roots be non-real? (3)
[26]

QUESTION 2

2.1 The sequence 6 ; 8 ; 10 ; 12 ; …; 164 is an arithmetic sequence.

2.1.1 Determine the n^{th} term of the sequence. (2)

2.1.2 How many terms are in the sequence? (2)

2.1.3 It is further given that the above sequence forms the row of the first differences for a quadratic sequence $T_n = an^2 + bn + c$ where $T_5 = 39$. Determine the values of a, b and c. (4)

2.2 The first three terms of a geometric sequence are: $p - 1\ ;\ p\ \ and\ p + 2$. Calculate the value of p. (3)

2.3 If $n \epsilon$ N, calculate the smallest value of n for which:

$$\sum_{k=1}^{n} \frac{1}{32}(2)^{k-1} > 900$$

(5)

2.4 Given:

$$\sum_{p=1}^{n}(2p - 5) = m \quad and \quad \sum_{p=1}^{14}(2p - 5) = 140$$

Determine, in terms of m, the value of

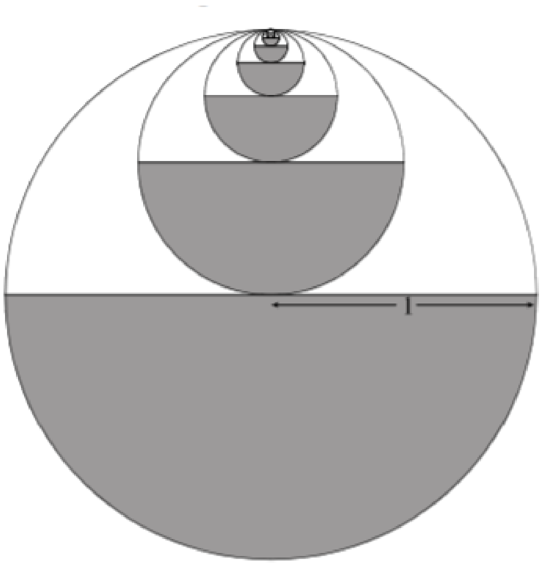
$$\sum_{p=15}^{n}(2p - 5)$$

(2)

[18]

QUESTION 3

The diagram represents a sketch of circles. Half of the area of each circle is shaded.
The radius of the largest circle is 1 $unit$. The diameter of each smaller circle is equal to the radius of the previous circle.
Determine the sum of the shaded areas of all the circles, if this process is going on forever.
(Give your answer in terms of π.)



(4)
[4]

QUESTION 4

Given: $f(x) = \dfrac{3}{x+1} + 1$

4.1 Determine the equations of the asymptotes of f. (2)

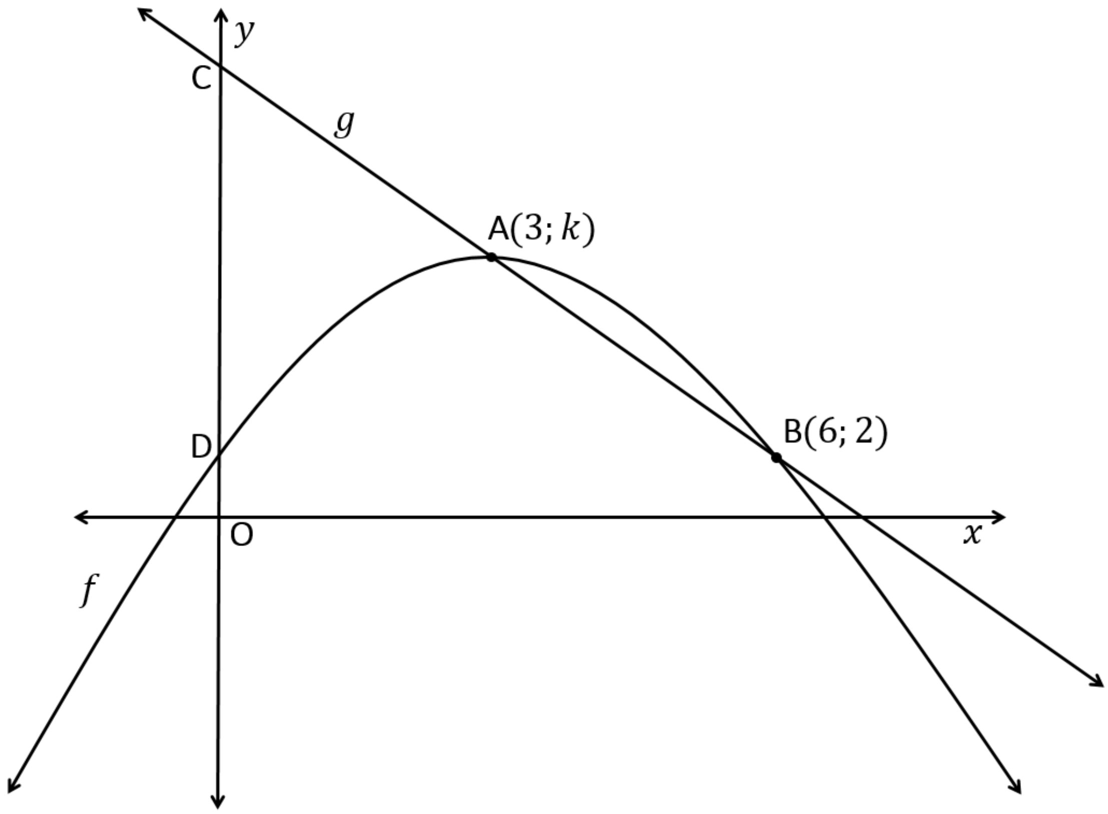
4.2 Calculate the $y-$intercept of f. (2)

4.3 Sketch the graph of f for $x > -1$.
Clearly show the intercept(s) with the axis and any asymptotes. (4)

4.4 Determine the value(s) of x for which $f(x) \geq 4$. (2)
[10]

QUESTION 5

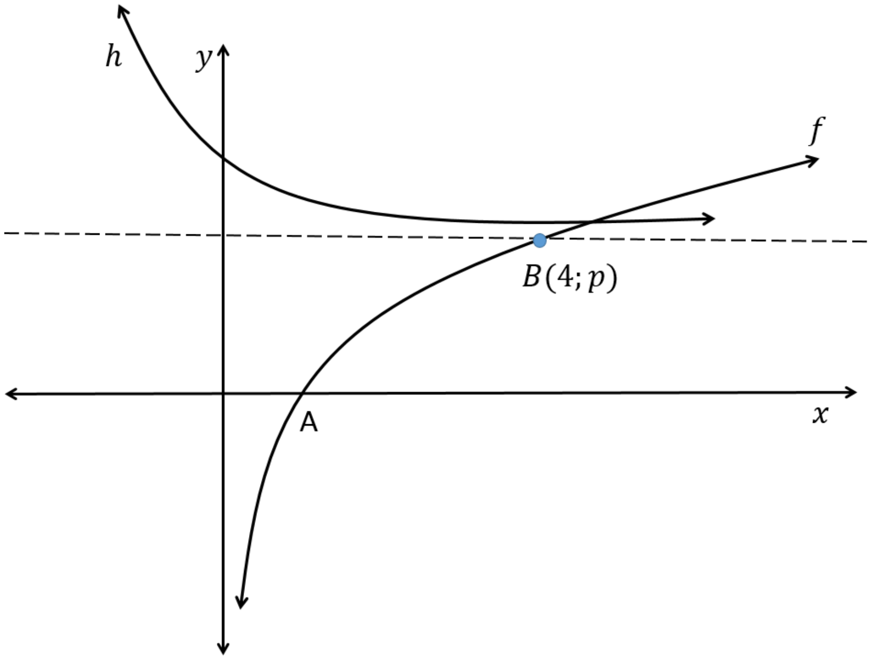
Sketched below are the graphs of $g(x) = -3x + 20$ and $f(x) = ax^2 + bx + c$.
Graph f has a turning point at A$(3; k)$. Graph f and g intersects at A and B$(6; 2)$.



5.1	Calculate the numerical value of k, the y-coordinate of A.	(2)
5.2	Determine the range of $y = -f(x)$.	(2)
5.3	Calculate the numerical values of a, b and c.	(6)
5.4	Determine the value(s) of x for which $f(x) > g(x)$.	(2)
5.5	Describe the nature of the roots for $f(x) - 11$.	(2)
5.6	Determine the value(s) of x for which $f'(x) \cdot g'(x) > 0$.	(2)
		[16]

QUESTION 6

Sketched below are the graphs of $h(x) = \left(\frac{1}{2}\right)^x + q$ and $f(x) = \log_2 x$.
Graph f and the asymptote of h intersect at $\mathrm{B}(4; p)$.



6.1	Write down the coordinates of A, the x −intercept of f.	(1)
6.2	Determine the domain of f.	(1)
6.3	Determine the equation of f^{-1} in the form $y = \ldots$	(2)
6.4	Sketch the graph of f^{-1}. Clearly labelling the intercept(s) with the axes as well as the coordinates of any one other point on the graph.	(3)
6.5	Determine the equation of the asymptote of h.	(2)
6.6	Describe, in words, the transformation of h to f^{-1}.	(2)
		[11]

QUESTION 7

7.1 A car cost x rand. The value of the car decreased by $r\%$ annually on the reducing-balance method. After 42 months the book value of the car is half of what it was worth when it was bought. Calculate r. (4)

7.2 A man plans to buy a car for his family. The car will cost R90 000.
He has two options:

Option 1:
He applies for a car loan.
The bank charges interest at 14% per annum, compounded monthly. He can afford a monthly instalment of R3 000. He will make the first monthly repayment one month after the loan is granted.

Option 2:
He deposits a fixed amount at the end of each month to buy the car at R90 000 over three years. (assuming the price of the car will still be R90 000 over 3 years.) The bank pays interest at 10% per annum, compounded monthly. He will make the first deposit at the end of the first month.

7.2.1 Calculate, to the nearest year, how long it will take the man to pay back the loan, if he chooses Option 1. (5)

7.2.2 Suppose he chooses Option 1. Calculate the outstanding balance immediately after the 24$^{\text{th}}$ payment. (3)

7.2.3 Calculate the monthly deposit if he chooses Option 2. (4)

[16]

QUESTION 8

8.1 Given: $f(x) = 1 - 3x^2$.

Determine $f'(x)$ from first principles. (5)

8.2 Given: $g(x) = \dfrac{x^4 - x}{x}$

Determine the gradient of the tangent to g drawn at $x = -2$. (4)

8.3 Determine $\dfrac{dy}{dx}$ if:

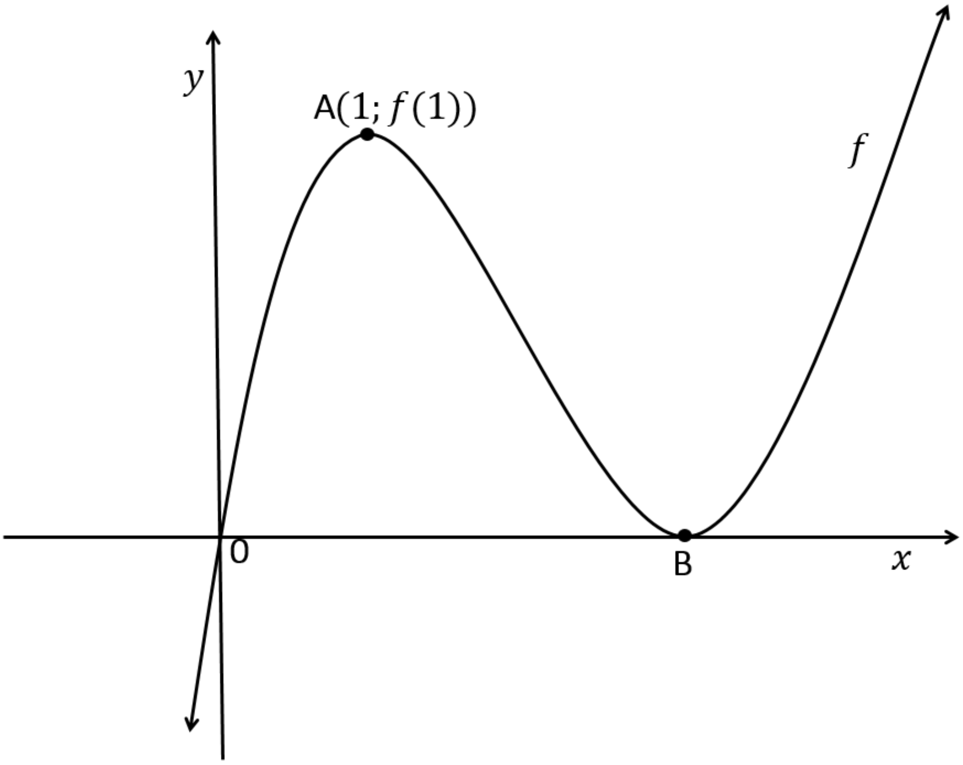
8.3.1 $y = \sqrt[3]{x} + \dfrac{5}{x^2}$ (4)

8.3.2 $x = \sqrt{y} + \dfrac{1}{x}$ (4)

[17]

QUESTION 9

Sketched below are the graph of $f(x) = 3x^3 + bx^2 + 27x + d$. A$(1; f(1))$ and B are the turning points of f. The graph of f intersects the x −axis at $(0; 0)$ and at B.



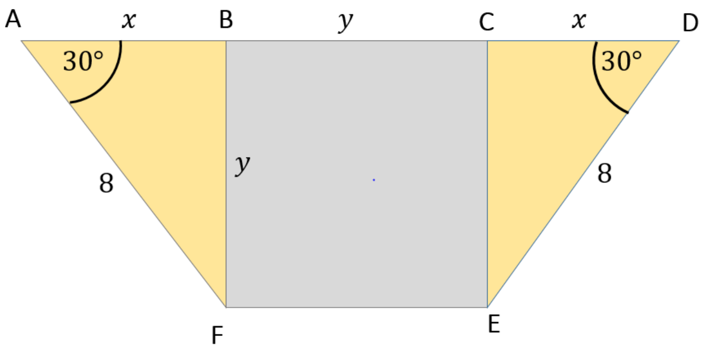
9.1	Give the value of d.	(1)
9.2	Show that the numerical value of $b = -18$.	(4)
9.3	Calculate the coordinates of B.	(4)
9.4	Determine the value(s) of x for which $f'(x) > 0$.	(2)
9.5	Calculate the x −coordinate of the point of inflection of f.	(3)
9.6	For which values of x will the graph of f be concave down?	(2)

[16]

QUESTION 10

A certain high school wants to build a stage as shown in the diagram below:

- The stage floor consists of a square (BCEF) and two congruent triangles (ΔABF and ΔDCE)
- Each side of the square is equal to y $units$.
- The length of the hypotenuse of the congruent triangles is equal to 8 units each.
- AB = CD = x $units$.
- $\hat{A} = \hat{D} = 30°$



10.1 Show that the area of the stage floor can be written as:
A= $8x \sin 30° + 64 - x^2$ (3)

10.2 Calculate the value of x, for which the area of the floor will be a maximum. (3)
[6]

QUESTION 11

11.1

A gift basket is made up with one book, one slab of chocolate, one packet of nuts and one bottle of fruit juice.

The person who makes up the gift basket can choose from:

- five different books,
- three different slabs of chocolates,
- four kinds of nuts and
- six flavours of fruit juice.

Calculate the number of different gift baskets that can possibly be produced. (2)

11.2 In a class, the probability of the students studying Mathematics is 60% while the probability of the students studying Economics is 30%. The probability of the students studying Mathematics or Economics is 70%.
Calculate the probability that a student will study both Mathematics and Economics. (2)

11.3 A photograph needs to be taken of the Representative Council of Learners (RCL) at a school. There are three girls and two boys in the RCL and all of them need to sit in one row for the photograph.
The President of the RCL is a boy and the vice-president is a girl.

11.3.1 In how many ways can the RCL be arranged in a row, if there were no restriction on the order in which the RCL members could sit? (2)

11.3.2 If the members of the RCL are seated randomly, calculate the probability that the President and the Vice-President of the RCL are seated next to each other. (4)

[10]

TOTAL = 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\qquad A = P(1-ni)$ $\qquad A = P(1-i)^n$ $\qquad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\qquad S_n = \frac{a(r^n - 1)}{r-1}$ **;** $r \neq 1$ $\qquad S_\infty = \frac{a}{1-r}$ **;** $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$