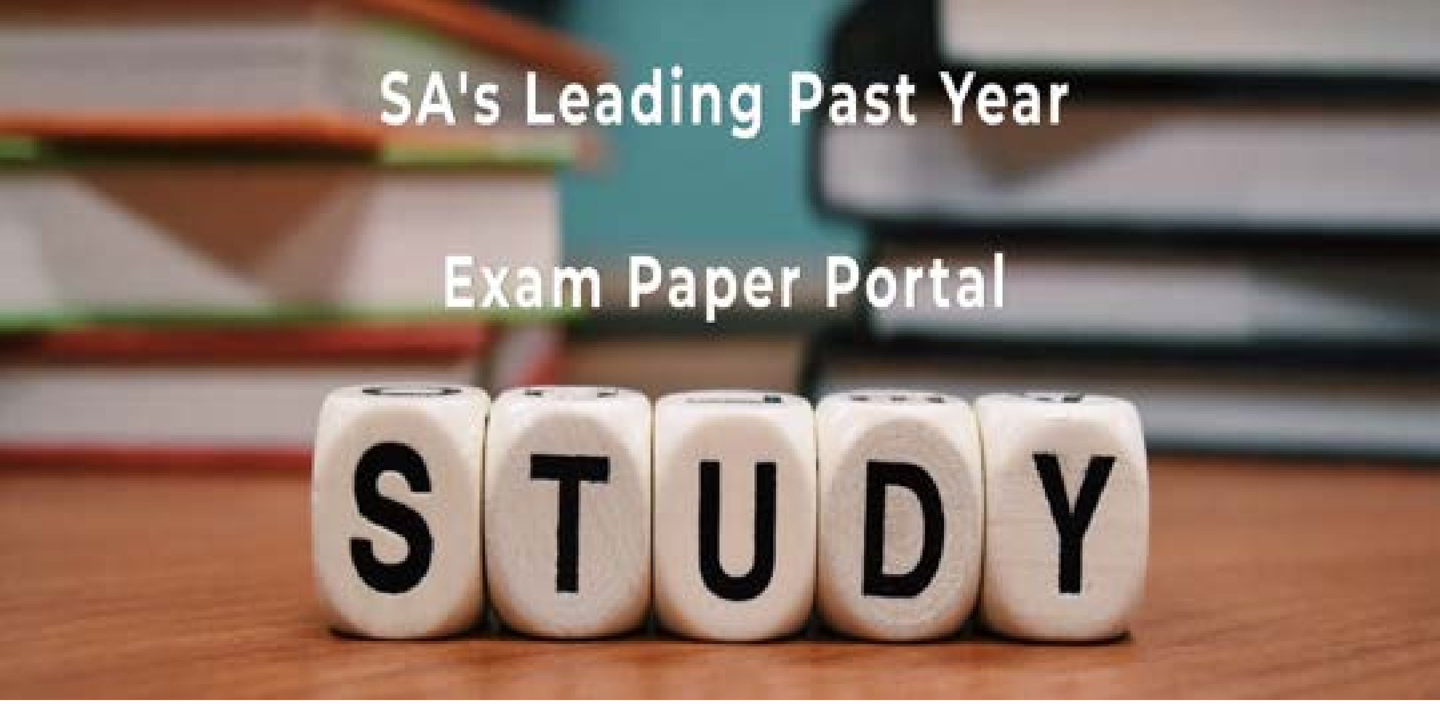
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PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P1

SEPTEMBER 2021

FMATHP1

TIME: 3 HOURS

MARKS: 150

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of this question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 4x - 21 = 0$ (3)

1.1.2 $x(5x-1) = 3$ (correct to TWO decimal places) (4)

1.1.3 $2x^2 - 9x + 4 \geq 0$ (3)

1.1.4 $3^{x+1} - 3^{x-1} - 24 = 0$ (4)

1.2 Solve simultaneously for x and y:

$y + 2x = 2$ and $y^2 - 3yx = -2x^2$ (5)

1.3 Simplify, **without using a calculator**:

$$\left(\sqrt[4]{\sqrt{20} - \sqrt{D_x(4x)}}\right)\left(\sqrt[4]{\sqrt{20} + \sqrt{D_x(4x)}}\right)$$ (4)

[23]

QUESTION 2

2.1 Given the quadratic pattern: $-22;\ -12;\ -6;\ -4;\ \ldots$

2.1.1 Write down the value of the next two terms of the pattern. (2)

2.1.2 Determine an expression for the n^{th} term of the pattern. (4)

2.1.3 Which term of the pattern will have the maximum value? (2)

2.2 The n^{th} term of the arithmetic sequence is $T_n = 27 - 6(n+1)$

2.2.1 Write down the value of the common difference. (1)

2.2.2 Which term of the sequence is equal to -117? (2)

2.3 The series $5+9+13+\ldots$ consists of n terms. The sum of the last six terms of this series is 906.

2.3.1 Determine, in terms of n, the sum to n terms of the series. (2)

2.3.2 If the last six terms of the series are excluded, determine, in terms of n, the sum of the remaining terms. (2)

2.3.3 Hence, determine the value of n. (4)

[19]

QUESTION 3

3.1 Given the geometric series: $3; \frac{3}{2}; \frac{3}{4}; \cdots$

Determine the largest value of n for which $T_n > \frac{3}{16384}$ (4)

3.2 Calculate the value of p if $\sum_{k=1}^{\infty} 27p^k = \sum_{n=1}^{30} 3\left(\frac{1}{2}\right)^{n-1}$ (5)

[9]

QUESTION 4

Given the function defined by $f(x) = -2x^2 - 4x + 16$

4.1 Write down the y-intercept of f. (1)

4.2 Calculate the x-intercepts of f. (2)

4.3 Determine the coordinates of the turning point of f. (3)

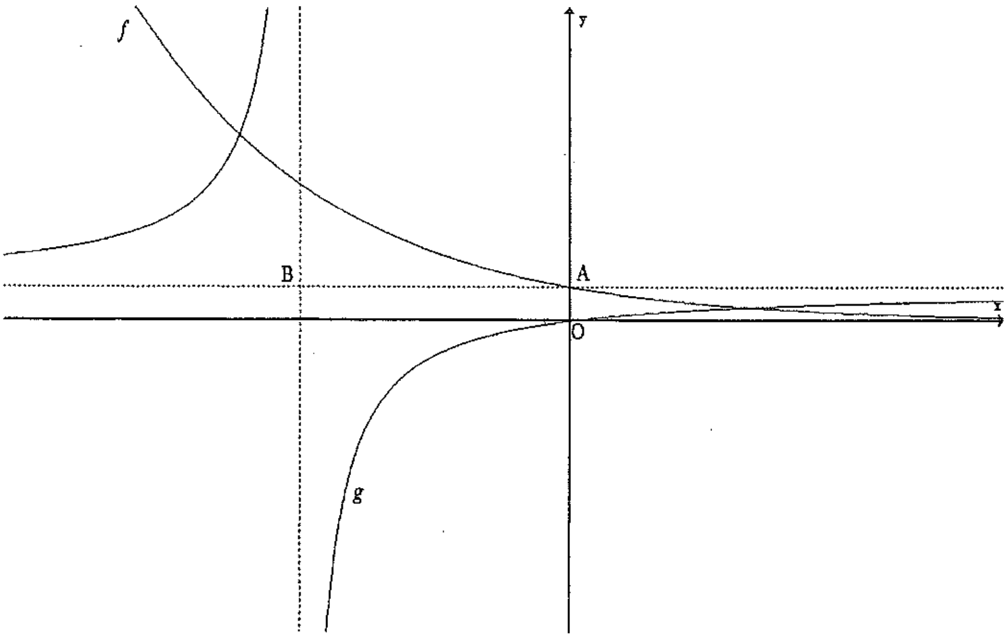
4.4 Sketch the graph of f, indicating all intercepts with the axes and the turning point. (3)

4.5 Write down the range of f. (2)

[11]

QUESTION 5

Sketched below are the graphs of $f(x)=\left(\frac{1}{2}\right)^x$ and $g(x)=\frac{a}{x+p}+q$. B is the point of intersection of the asymptotes of g. A is the y-intercept of f. The graph g passes through the origin. AB is parallel to the x axis.



5.1 Write down the equation of f^{-1}, the inverse of f in the form $y=...$ (2)

5.2 Write down the domain of f^{-1}. (1)

5.3 Sketch the graph of $y=f^{-1}(x)$ on your answer book. Clearly indicate the intercept with the axis and the asymptote on your graph. (2)

5.4 If $h(x)=x+3$ is the equation of one of the axes of symmetry of g, determine the coordinates of B. (2)

5.5 Hence, determine the equation of g in the form $g(x)=\frac{a}{x+p}+q$. (4)

5.6 Write down the equation of k if $k(x)=g(x-3)+1$ in the form

$$k(x)=\frac{a}{x+p}+q$$ (2)

5.7 Determine for which value(s) of x is:

5.7.1 $f(x)\times g(x)\leq 0$? (2)

5.7.2 $g'(x)>0$? (2)

[17]

QUESTION 6

6.1 Given: $p(x) = -3x^2$

6.1.1 Give a reason why p^{-1} is not a function (2)

6.1.2 Place the restrictions on the domain of p so that p^{-1} is a function. (2)

6.2 In a test, Peter was required to determine the derivative of a function $f(x)$. However, by mistake, he determined the inverse of $f(x)$ and his answer was $f^{-1}(x) = \sqrt{\frac{x}{2}}$.

Determine the correct answer to the question given in a test. (3)

[7]

QUESTION 7

7.1 An investment earns interest at 12% per annum, compounded every six months. Calculate the effective annual interest rate of this investment. (2)

7.2 A company bought equipment that depreciates annually at 7% on the reducing balance. How many years will it take for this equipment to be worth half the original purchase price? (3)

7.3 Kenneth invested R250 000 in an account that paid interest at 9,5% p.a. compounded quarterly. Twelve years after making the initial investment, Kenneth bought a house that cost R2 920 000. He used all the money that had accumulated in his investment as a deposit and the bank granted him a loan for the balance. The bank charged interest on the loan at 10,3% p.a., compounded monthly. The loan was for a 20-years period and the first monthly instalment was made one month after the loan was granted.

7.3.1 Calculate the value of Kenneth's investment after 12 years. (3)

7.3.2 Determine the value of the loan that Kenneth obtained. (1)

7.3.3 Calculate the monthly repayment on the loan. (3)

7.3.4 Kenneth paid off the loan at the end of 20 years. How much interest did he pay on the loan? (2)

[14]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -\frac{3}{x}$. (4)

8.2 Determine:

8.2.1 $D_x\left[(3x^3 - 2)^2\right]$ (3)

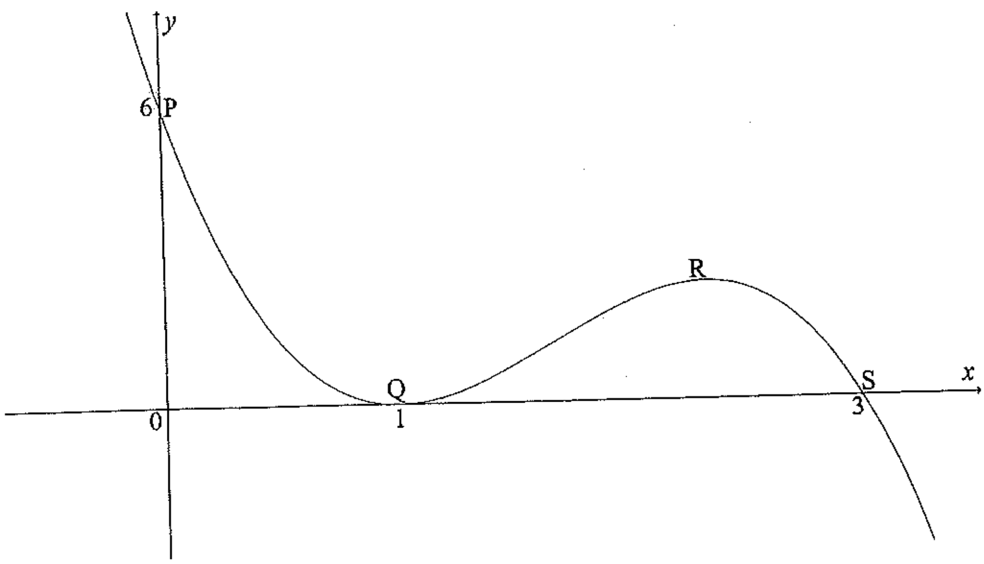
8.2.2 $\frac{dy}{dx}$ if $y = 2x^3 - \frac{4}{x} + 4\sqrt[3]{x}$ (4)

8.3 Determine the coordinates of the point on the curve of $y = 2(1-x)^2$ where the tangent at this point is perpendicular to $y = -2x + \frac{4}{3}$. (4)

[15]

QUESTION 9

The graph of a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, is sketched below.
$P(0;6); Q(1;0);$ R and $S(3;0)$ are points on the graph.
Q and R are the turning (or stationary) points of f.



9.1 Show that $a = -2; b = 10; c = -14$ and $d = 6$. (5)

9.2 Determine the x-coordinate of the point of inflection of f. (3)

9.3 For which value(s) of:

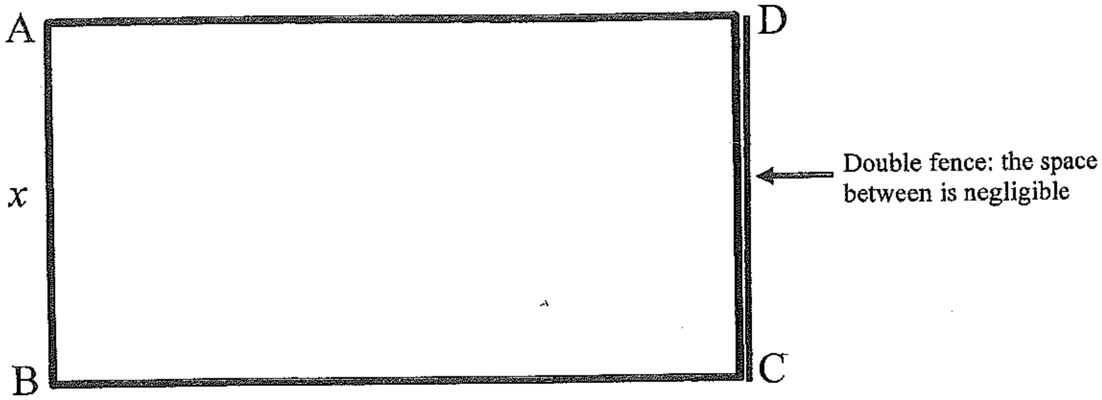
9.3.1 x is $f(x)$ increasing? (5)

9.3.2 k will $f(x) = k$ have exactly three different real roots? (3)

[16]

QUESTION 10

A farmer makes a rectangular enclosure for his sheep using 100 metres of fencing. He puts a double layer of fencing along one side to stop the sheep breaking through to his crops in the next field. A plan of his fence is shown below.



The length of AB is x metres.

10.1 Show that the area of the land which can be enclosed, in this way is given by the formula: $A(x)=50x-\frac{3}{2}x^2$ (2)

10.2 Determine the length of AB if the enclosed area is to be a maximum. (3)
[5]

QUESTION 11

11.1 A group of 540 people with green or blue eyes were randomly selected to determine whether green or blue eyes are gender dependent. The results are tabulated below:

	Male	Female	Total
Green eyes	183	147	330
Blue eyes	117	93	210
Total	300	240	540

11.1.1 If a person from this group is selected at random, write down the probability that the person is a female and has green eyes. (1)

11.1.2 After analysing the results, a learner concludes that the probability of having green eyes is independent of gender. Is she correct? Support your answer with relevant calculations. (4)

11.2 E and F are two independent events. $P(E) = x$ and $P(F) = y$.

$P(E \text{ and } F) = \frac{1}{3}$ and $P(E \text{ or } F) = \frac{9}{10}$. Show that $30y^2 + 10 = 37y$ (3)

11.3 Thirteen learners must be seated on a bus where the seats are arranged as shown below.

X X X X X ← back row

X X   X X

X X   X X ← front row

11.3.1 If the learners are arranged at random, how many ways can they be seated? (2)

11.3.2 Five friends insist on sitting in the back row and one learner who gets sick needs to sit next to the door in the seat marked D.

X X X X X

X X   X X

X X   X D

In how many ways can the learners now be arranged? (4)

[14]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$