**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2022

MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of ELEVEN questions. Answer ALL the questions.

2.   Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

3.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4.   Answers only will not necessarily be awarded full marks.

5.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

6.   Diagrams are NOT necessarily drawn to scale.

7.   Number the answers correctly according to the numbering system used in this question paper.

8.   An information sheet with formulae is included at the end of the question paper.

9.   Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

    1.1.1 $x^2 + 4x - 21 = 0$ (2)

    1.1.2 $x(2x - 7) = 3$ (correct to TWO decimal places)   (4)

    1.1.3 $(2x + 3)(x + 1) < 6$ (4)

    1.1.4 $2\sqrt{x} + x = 3$ (5)

1.2 Solve simultaneously for x and y:

    $2y + x + 3 = 0$ and $x^2 + y^2 + 2xy = 1$ (6)

1.3 It is given that $K^{\frac{1}{x}} = 3$, $K^{\frac{1}{y}} = 4$ and $K^{\frac{1}{w}} = 12$.

    Prove that $w = \dfrac{xy}{x + y}$.   (4)

    [25]

QUESTION 2

2.1   An arithmetic series has a common difference of 4. $(3x-1)$ and $(2x+8)$ are the fourth and the seventh terms of the series, respectively.

      2.1.1   Determine the value of x.   (3)

      2.1.2   Calculate the:

            (a)   First term of the series   (3)

            (b)   Sum of the first 42 terms of the series   (3)

2.2   The first term of a quadratic number pattern is 61. $T_k = 4k - 26$ forms the first differences of the quadratic number pattern.

      2.2.1   Write down the second and third terms of the quadratic number pattern.   (2)

      2.2.2   If the nth term of the quadratic number pattern is given by $T_n = 2n^2 - 28n + 87$, calculate the value of the smallest term.   (3)

      2.2.3   A constant k is added to T_n such that all the terms of the quadratic number pattern become positive. Determine the values of k.   (2)

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[16]

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QUESTION 3

3.1   Given that $p = 0,\dot{7} = 0,777777...$

      3.1.1   Write down p as a geometric series.   (1)

      3.1.2   Represent the series in sigma notation.   (3)

      3.1.3   Determine the sum to infinity of the geometric series as a proper fraction.   (2)

3.2   In a geometric sequence the sum of the 9th and 10th terms is 6 times the 8th term. Determine the value(s) of r, the common ratio of the sequence.   (4)

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[10]

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QUESTION 4

In the diagram below the graph of a hyperbolic function, $f(x) = \dfrac{x+k}{x+p}$, where k is a constant, is drawn. A(1 ; 0) and B are the x-intercept and y-intercept of f, respectively. The vertical asymptote goes through the x-axis at 3.
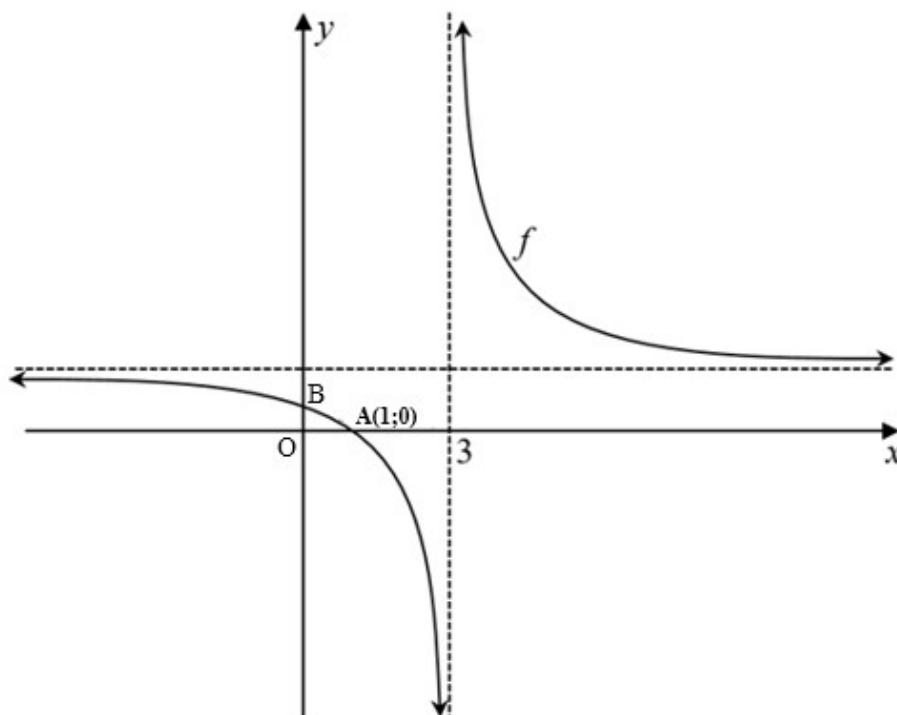
4.1   Write down the value of p.   (1)

4.2   Determine the value of k.   (2)

4.3   Calculate the coordinates of B.   (2)

4.4   Determine the values of x for which $x.f(x) \leq 0$.   (3)

4.5   Rewrite the equation of f in the form $f(x) = \dfrac{a}{x+p} + q$.   (2)

[10]

QUESTION 5

Given the function: $f(x) = -3^x + 1$

5.1	Draw the graph of f in your ANSWER BOOK. Clearly show the intercepts with the axes as well as the asymptote of the graph.	(3)
5.2	Write down the range of f.	(2)
5.3	Determine the equation of the asymptote of g, given that $g(x) = -f(x)$.	(2)
5.4	If g is shifted 1 unit upwards to give a new function h, determine the equation of h^{-1}, the inverse of h in the form $y = \ldots$	(3)
		[10]

QUESTION 6

The diagram below shows the graphs of $f(x) = x^2 - 4x - 11$ and $g(x) = f^{/}(x)$. A and B are the x-intercepts of f and C the x-intercept of g. D is the turning point of f. f and g intersect at M(–1 ; t) and N(7 ; 10).
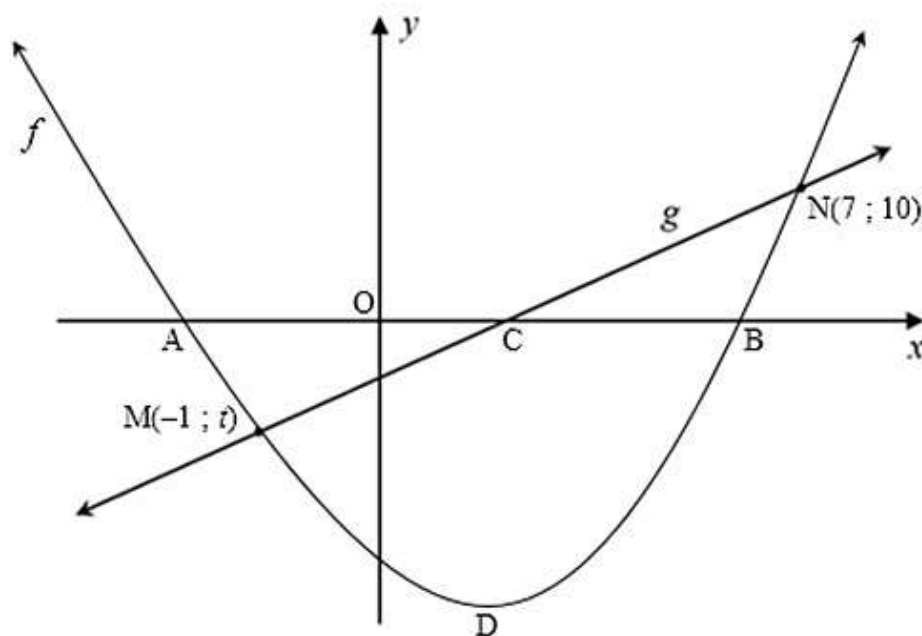
6.1   Calculate the:

        6.1.1   Coordinates of D   (3)

        6.1.2   Distance CN   (4)

6.2   For which value(s) of x, is:

        6.2.1 $f(x) < g(x)$?   (2)

        6.2.2 $g(x) - f(x)$ a maximum?   (4)

                                                                    [13]

QUESTION 7

7.1   Corniel bought an ice cream machine that depreciated at 17% p.a. on the reducing balance method. The value of the machine depreciated to a book value of R27 763,12 over a period of 4 years. What was the original price of the machine?   (2)

7.2   After completing his studies, Lubabalo decides to save money to buy himself a car for cash. He wants to save R300 000 by making equal monthly deposits into a savings account that pays interest of 8,6% p.a. compounded monthly, over a period of 7 years. How much must he deposit per month if he wants to achieve his goal?   (3)

7.3   Yolanda acquired a mortgage loan to buy a house. She was required to pay R8 901,96 monthly and she was charged 10,4% interest per annum compounded monthly. Her payment period was 25 years and her first payment was made at the end of the first month after she took out the loan.

      7.3.1   Calculate the total value of the mortgage loan, (to the nearest rand), Yolanda needed.   (3)

      7.3.2   After 204 payments, Yolanda could only afford to pay R7 500 per month, going forward.

          (a)   Determine the outstanding balance after the 204$^{\text{th}}$ payment.   (3)

          (b)   How long did it take for Yolanda to pay up the outstanding balance, if she was allowed to pay the new instalment?   (4)

[15]

QUESTION 8

8.1   Determine $f^{/}(x)$ from first principles if $f(x) = x - 3x^2$.   (5)

8.2   Determine:

8.2.1 $D_x\left[3x^4 - \dfrac{4}{x^2}\right]$ (3)

8.2.2 $\dfrac{dy}{dx}$ if $y = a^2 x + 6\sqrt{x}$ (3)

[11]

QUESTION 9

Given: $f(x) = x^3 - 3x + 2$

9.1   Calculate the coordinates of the turning points of f.   (4)

9.2   Calculate the x-intercepts of f.   (3)

9.3   Determine the values of x for which f:
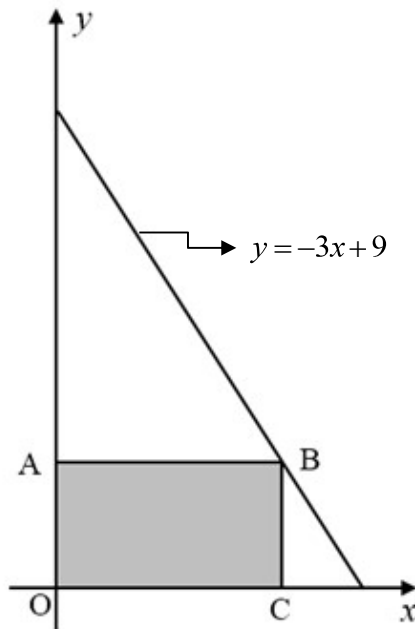
9.3.1   Is decreasing   (2)

9.3.2   Will be concaved down   (3)

9.4   Draw the graph of $g(x) = (x-3)^3 - 3(x-3) + 2$, clearly indicating the intercepts with the axes and the turning points.   (4)

9.5   Determine the value(s) of k such that $g(x) = k$ always has 3 distinct roots.   (2)

[18]

QUESTION 10

The diagram below shows a rectangle OABC, where B lies on the straight line $y = -3x + 9$.   C lies on the *x*-axis and A lies on the *y*-axis as shown.


10.1   If B(x ; y), write down the lengths of OC and OA in terms of x.   (2)

10.2   Determine the coordinates of B for which rectangle OABC has a maximum area.   (4)

                                                                                                                                                    [6]

QUESTION 11

11.1 During a survey at a certain school, 900 learners were asked to indicate what sport they would like to play as a winter sport code. Learners could choose at most three sport codes. The sport codes indicated by learners were Rugby (R), Hockey (H) and Tennis (T). There will be boys and girls teams in all three sport codes.
The data collected is shown in the Venn diagram below.
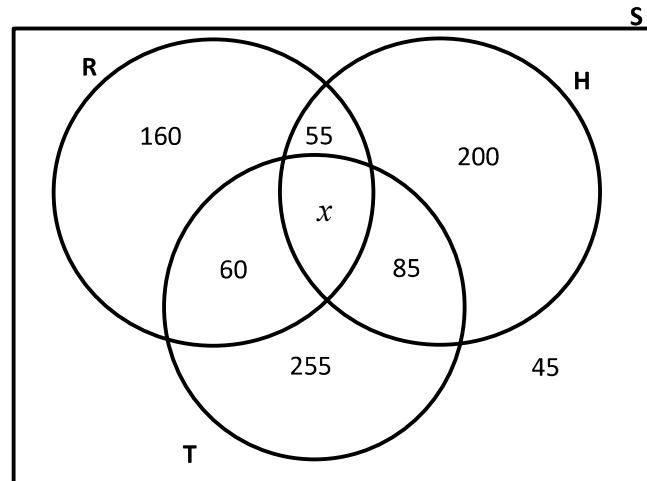
11.1.1 Determine how many learners want to play all three sport codes. (2)

11.1.2 If a learner is randomly chosen, what is the probability that he/she prefers to play hockey only? (2)

11.1.3 Determine the percentage of learners who are likely to play at least 2 of the sport codes. (2)

11.2 Consider the word SPECTRUM.

11.2.1 How many ways can the 8 letters be arranged:

(a) In any order? (1)

(b) Such that the first letter is a vowel? (2)

11.2.2 Calculate the probability that in a particular arrangement of the 8 letters, the letters T, P and R will be next to each other, in any order. (2)

11.3 A bag contains only two colours of tennis balls, red and green, in the ratio 1 : 3. Two balls are picked at random, one after the other, without replacement. Calculate the number of balls in the bag given that the probability of picking first a red ball and second a green ball is equal to $\frac{1}{5}$. (5)

[16]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad\qquad A = P(1 - ni) \qquad\qquad A = P(1 - i)^n \qquad\qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad\qquad\qquad S_n = \frac{n}{2}\big(2a + (n-1)d\big)$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \; r \neq 1 \qquad\qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad\qquad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$: $\qquad\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad\qquad\qquad \sigma^2 = \frac{\sum\limits_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$