KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Read the questions carefully.
3. Answer ALL the questions.
4. Number your answers exactly as the questions are numbered.
5. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x+5)(2x-1)=0$ (2)

1.1.2 $-3x^2-7x=-8$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x+5}+1=x$ (5)

1.1.4 $(2x-3)(x+5)\leq 0$ (3)

1.2 Solve for x and y simultaneously if:

$x+3y=5$ and $xy+y^2-3=0$ (6)

1.3 Simplify fully, without the use of a calculator:

$$\sqrt[n]{\frac{10^n+2^{n+2}}{5^{2n}+4.(5^n)}} \quad \text{where} \quad n\neq 0$$ (4)

[24]

QUESTION 2

Given the quadratic number pattern: 5 ; 9 ; 17 ; 29 ; ...

2.1 Write down the 5^{th} and 6^{th} terms of the pattern. (2)

2.2 Show that the n^{th} term of the quadratic pattern is given by $T_n=2n^2-2n+5$ (4)

2.3 Is 2023 a term in the pattern? Motivate your answer with relevant calculations. (4)

[10]

QUESTION 3

Evaluate: $\sum_{k=1}^{50}(30-4k)$ **[4]**

QUESTION 4

4.1 Given the geometric series $a + ar + ar^2 + ar^3 + \ldots$, where a is the first term and r is the common ratio. Prove that the sum to n terms of this series is given by

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}; r \neq 1$$ (4)

4.2 The first two terms of a geometric sequence with constant ratio r, and an arithmetic sequence with constant first difference, d, is the same. The first term is 12.

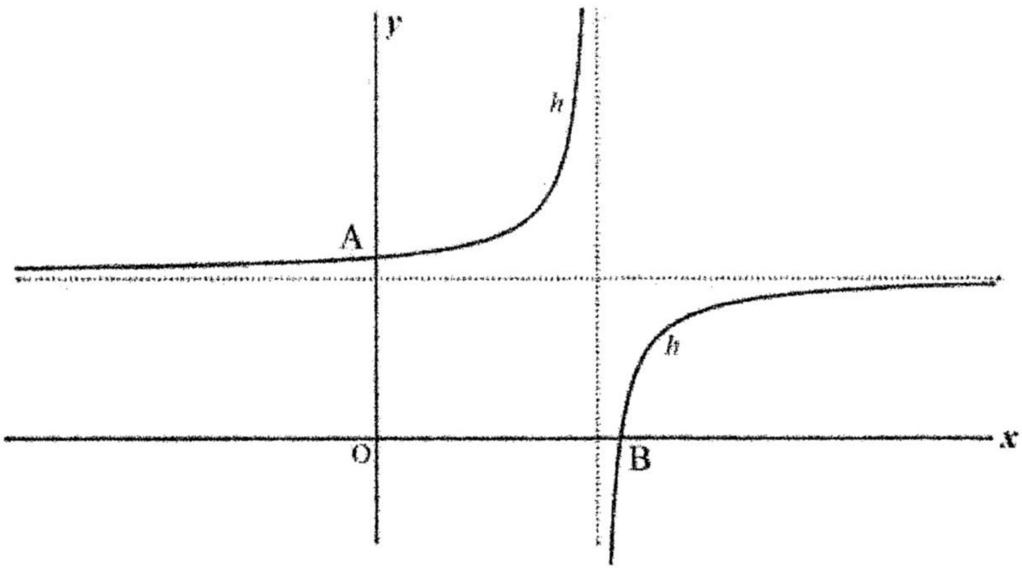
4.2.1 Write down the second and third terms of **EACH** sequence in terms of d and r. (2)

4.2.2 If it is further given that the sum of the first three terms of the geometric sequence is three more than the sum of the first three terms of the arithmetic sequence. Determine two possible values of the common ratio, r, of the geometric sequence. (5)

[7]

QUESTION 5

Sketched below is the graph of $h(x) = 1 - \frac{1}{x-2}$. A is y-intercept and B is the x-intercept of h.



5.1 Write down the equations of the asymptotes of h. (2)

5.2 Calculate the coordinates of A and B. (3)

5.3 Write down the equation of the line of symmetry of h with positive gradient. (2)

5.4 Write down the range of h. (1)

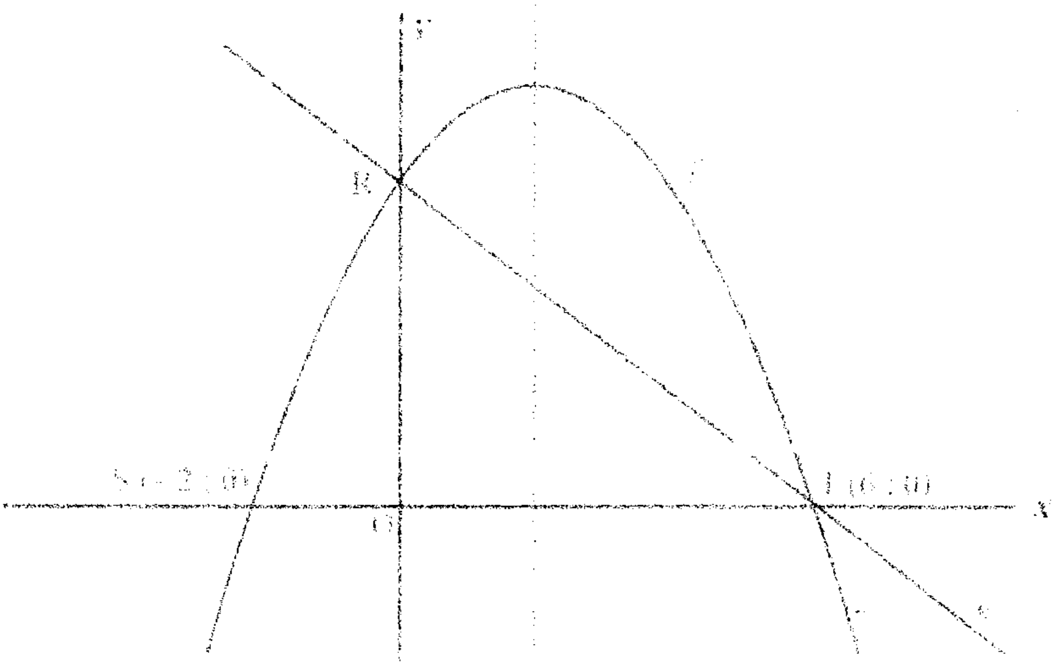
[8]

QUESTION 6

S(– 2; 0) and T(6 ; 0) are the x–intercepts of the graph of $f(x)=ax^2+bx+c;\ a\neq 0$.

R is the y–intercept of f and g.

The straight line through R and T has the equation $g(x)=-2x+d$.



6.1 Calculate the value of d. (2)

6.2 Show that $f(x)=-x^2+4x+12$. (4)

6.3 Calculate the coordinates of the turning point of f. (3)

6.4 Determine for which values of x will:

6.4.1 $f(x)-g(x)\geq 0$ (2)

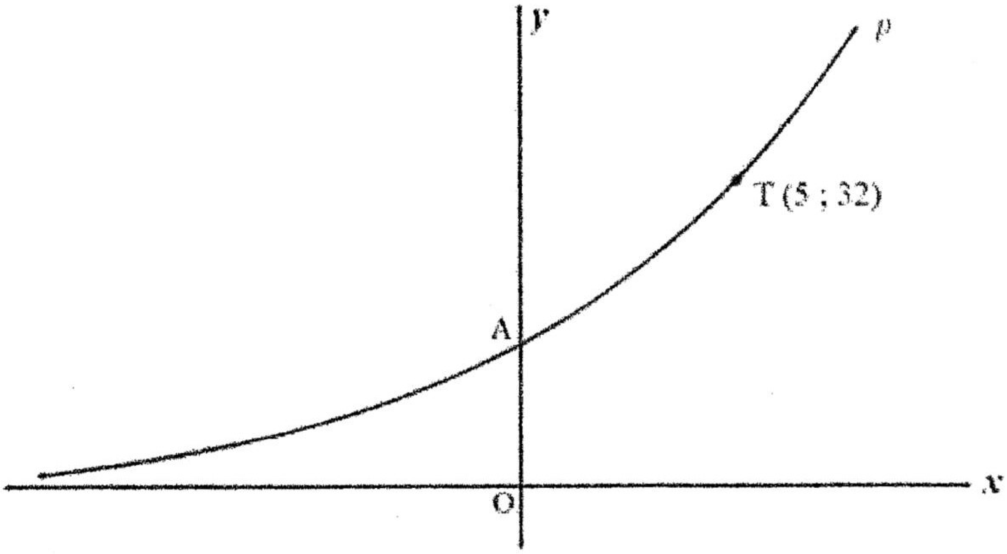
6.4.2 $x.f(x)<0$ (3)

6.5 Determine the coordinates of R', the image of R on p if $p(x)=-f(x-2)$. (2)

[16]

QUESTION 7

Sketched below is the graph of $p(x)=a^x$; $a>0$; $a\neq 1$. The graph intersects the y – axis at A. The point T (5 ; 32) lies on p.



7.1	Write down the coordinates of the point A.	(2)
7.2	Calculate the value of a.	(2)
7.3	Write down the domain of p.	(1)
7.4	Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$	(2)
7.5	Determine the values of x if $p^{-1}(x)\leq 5$.	(2)
		[9]

QUESTION 8

8.1 Dipinda opened an account with an amount of R5000 on 1 June 2021. She then makes monthly deposits of R600 at the end of every month. Her first deposit is made on the 30 June 2021 and her last deposit on 30 April 2023. The account earns interest of 14,25% per annum compounded monthly. Calculate the amount that is in the account directly after her last deposit is made into the account. (6)

8.2 Molly wants to buy a house for her family for R800 000. She agreed to pay monthly instalments of R10 000 on the loan which incurred interest at a rate of 13,35 % per annum compounded monthly. The first payment was made at the end of the first month after the loan was granted.

8.2.1 Show that the loan will be paid back in full after in 200 months. (4)

8.2.2 Suppose Molly encountered unexpected expenses and was unable to pay any instalment at the end of the 120th, 121st, 122nd and 123rd months. At the end of the 124th month she increased her payment to still pay off the loan in 200 months by making 77 equal monthly payments.

a) Calculate the balance on the loan after the 119th payment was made. (3)

b) Calculate the new monthly instalment Molly must pay from the 124th month to settle the loan in 200 months. (4)

[17]

QUESTION 9

9.1 Determine $f'(x)$ from first principles if $f(x)=\dfrac{2}{3x}$. (5)

9.2 Determine:

9.2.1 $g'(x)$ if $g(x)=(x+7)^3$ (5)

9.2.2 $\dfrac{dy}{dx}$ if $y=\sqrt{x^5}-\dfrac{4}{9x^2}$ (4)

[14]

QUESTION 10

10.1 Given: $f(x)=x^3-12x-16$

10.1.1 Calculate the x–intercepts of the graph of f. (5)

10.1.2 Determine the coordinates of the turning points of f. (4)

10.1.3 Sketch the graph of f, indicating the intercepts with the axes and the coordinates of the turning points. (4)

10.1.4 Determine the values of x for which the graph of f is concave up. (2)

10.2 Given: $p(x)=-x^3-8x$

Is it possible to draw a tangent with positive gradient to the graph of p? Show all calculations to justify your answer. (3)

[18]

QUESTION 11

The lead, L, in metres, of a runner in the comrades' marathon in the last t minutes of the race, where $t\in[0;75]$ is given by equation:

$$L=1000+6t-\frac{t^2}{4}$$

11.1 Determine $\frac{dL}{dt}$. (2)

11.2 Calculate the time at which the runner has the greatest lead. (2)

11.3 At what rate is the runner's lead decreasing when $t=60$ minutes? (2)

[6]

QUESTION 12

Given the word "**BRACKET**". The letters of this word are randomly arranged to form new arrangements of the letters.

12.1 How many unique arrangements of the letters can be made? (2)

12.2 Determine the number of unique arrangements of the letters that are randomly made if the letters R and A must be together. (3)
[5]

QUESTION 13

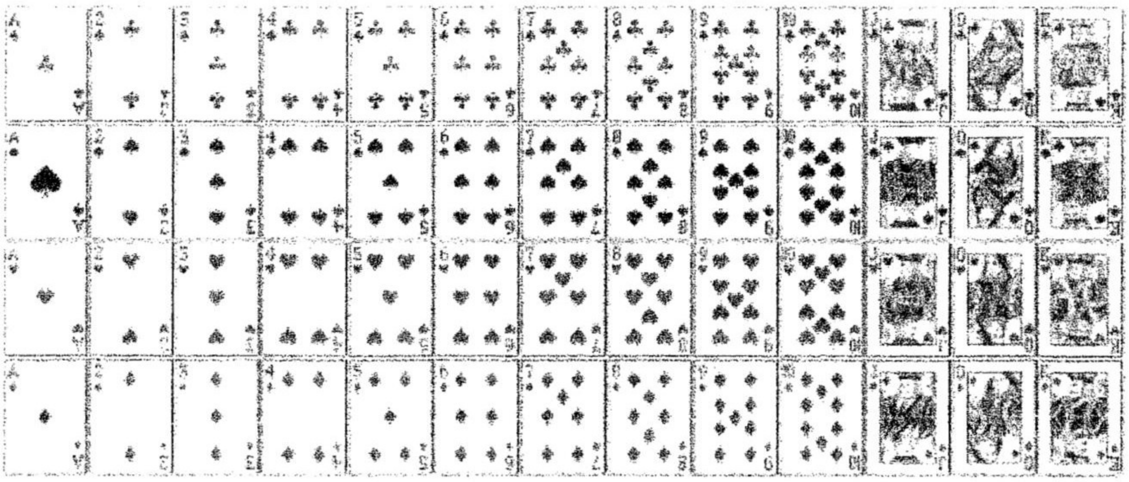
13.1 Two different events A and B are mutually exclusive.

It is further given that:

- 3P(B) = P(A)
- P (A or B) = 0,63

Calculate P(B). (3)

13.2 Two cards from a regular pack of 52 playing cards (shown below) , are drawn at random one after the other, without replacement.



Calculate the probability that:

13.2.1 both cards are picture cards. (2)

13.2.2 at least one of the cards is a picture card. (3)
[8]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$ $\quad$ $m = \tan\theta$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area}\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$