LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF

EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

AUGUST 2022

PRETRIAL EXAMINATION

MARKS: 150

TIME: 3 HOUR

This question paper consists of 10 pages and 1 formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 Questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams and graphs that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number your answers correctly according to the numbering system used in this question paper.
9. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1

1.1 Solve for x

1.1.1 $(x-2)(x-7)=0$ (2)

1.1.2 $4x+\frac{4}{x}+11=0; x\neq 0$ (Correct to TWO decimal places.) (4)

1.1.3 $6x<3x^2$ (3)

1.1.4 $\sqrt{x-1}=x-3$ (4)

1.2 Solve for x and y simultaneously:

$2x-y+1=0$

and

$x^2+xy+2=3x+y$ (6)

1.3 Determine the sum of the digits of $5^{2009}2^{2010}.24$ (4)

[23]

QUESTION 2

2.1 The first four terms of a quadratic number pattern are 13; 20; 29; 40.

2.1.1 Write down the value of the 5^{th} term. (1)

2.1.2 Determine the general term of the number pattern in the form $T_n = an^2 + bn + c$. (5)

2.1.3 Michael stated that one of the terms of the sequence is 4 493. Is Michaels claim correct? Use calculations to motivate your verdict. (3)

2.2 Consider the finite arithmetic series 5; 8; 11; …. 173

2.2.1 Calculate the number of terms of the series. (3)

2.2.2 Determine the sum of the even terms of the series. (4)

[16]

QUESTION 3

Consider the infinite geometric series 90; 30; 10; …

3.1 Write the series in sigma notation. (1)

3.2 Write down the 10th term of the series as a simple fraction. (1)

3.3 Explain why the series converges. (1)

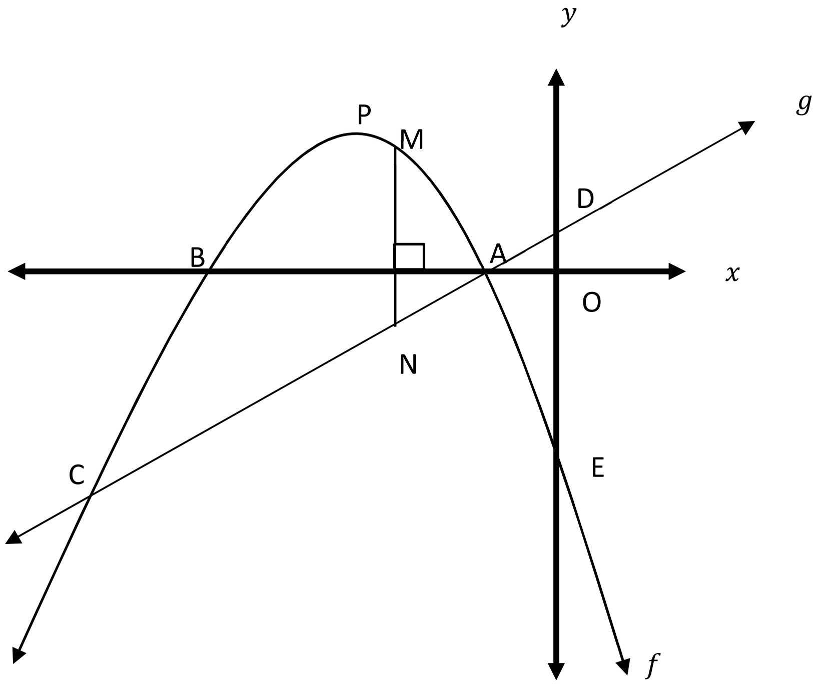
3.4 Determine the largest value of n for which $S_\infty - S_n > 1$. (6)

[09]

QUESTION 4

The figure below shows the graphs of $f(x) = -(x+7)^2 + 25$ and $g(x) = x + 2$

MN is parallel to the $y-$axis. A and B are the x-intercepts of (x) . P is the turning point of f. Points A and C lie on both graphs. D and E are the y-intercepts of the graphs.



4.1 Determine:

4.1.1 The coordinates of point P, the turning point of the parabola. (2)

4.1.2 The length of DE. (2)

4.1.3 The coordinates of C. (6)

4.2 The graph of g is shifted up to pass at B. Determine the equation of the shifted graph of g. (5)

4.3 The value of x for which the length of MN is at a maximum in the domain where $f > g$. (4)

[19]

QUESTION 5

Given the function

$$f(x) = \frac{-2}{(x+3)}$$

5.1 Determine the value of k if the graph passes through the point $\left(k; -\frac{1}{4}\right)$. (2)

5.2 Write down the equation(s) of the asymptote(s). (2)

5.3 Write down the equation of the line of symmetry that intersects the graph defined by the above function. (2)

5.4 Determine the range of h if $h(x) = f(x) + 2$ (2)

[08]

QUESTION 6

Given $f(x) = \frac{1}{4}x^2, x \geq 0$

6.1 Sketch the graph of f^{-1} showing any intercepts with the axes and ONE other point on the graph. (3)

6.2 State the domain of f^{-1}. (2)

6.3 When the graph of f is shifted to $f(x+3)$, point A on f, shifted to point $(4; y)$ on $f(x+3)$. Determine the coordinates of the image of point A on f^{-1}. (3)

[08]

QUESTION 7

7.1 Exactly six years ago Lesego bought a new car which is worth one third of its original price now. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the annual rate of depreciation. (4)

7.2 Bonang is granted a home loan of R 650 000 to be repaid over a period of 15 years. The bank charges interest at 11, 5% per annum compounded monthly. She repays her loan by equal monthly installments starting one month after the loan was granted.

7.2.1 Calculate Bonang's monthly installment. (4)

7.2.2 How long will it take her to finish the repayment of the loan, if she decides to increase her monthly installment to R 9 000 per month after 5 years? (5)

7.2.3 Calculate her final/last month installment (2)

[15]

QUESTION 8

8.1 If $f(x) = 2x - x^2$, determine $f'(x)$ from FIRST PRINCIPLES. (5)

8.2 8.2.1 Determine $\frac{dy}{dx}$ if $xy = 3$. (2)

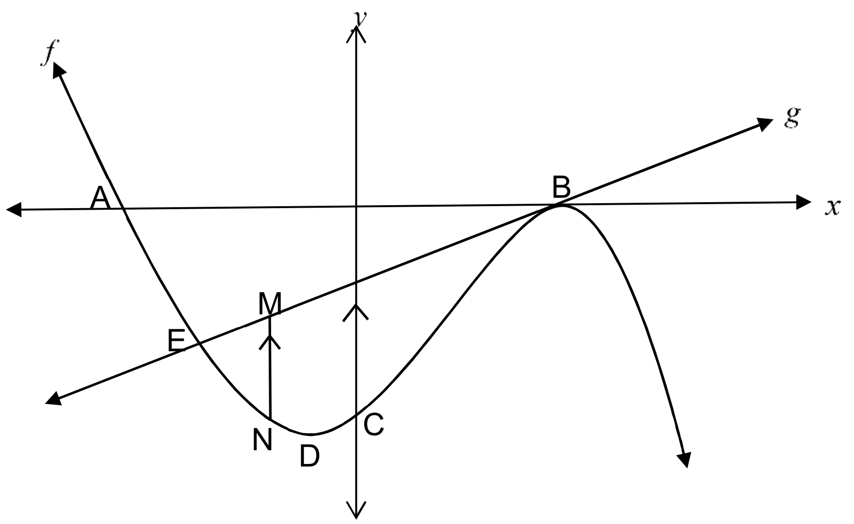
8.2.3 Determine $D_x\left[\frac{\sqrt[3]{x^2} - 2x + 1}{x^2}\right]$ (4)

[11]

QUESTION 9

The graphs of $f(x) = -x^3 + 3x^2 + 9x - 27$ and $g(x) = tx + q$ are sketched below.

A and B are the $x-$intercepts and C, the $y-$intercept of f. The turning points of f are $D(-1;-32)$ and B. g and f intersect at E $(-2;-10)$ and B.



9.1 Show that the equation of g is $y = 2x - 6$ (4)

9.2 For what value(s) of x is

9.2.1 f increasing? (1)

9.2.2 the derivative of f decreasing? (2)

9.2.3 $x.f'(x) < 0$? (2)

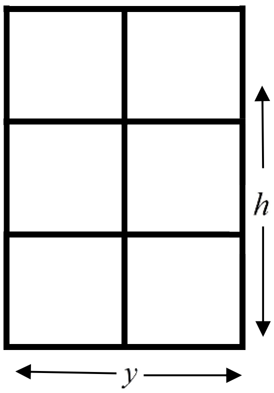
9.2.4 MN a maximum, if M is a point on g between E and B, N is a point on f and MN is parallel to the $y-$axis. (5)

[14]

QUESTION 10

A window frame with dimensions y by h is illustrated below.

The frame consists of six smaller frames.



10.1 If 12m of material is used to make the entire frame, show that

$y = \frac{1}{4}(12 - 3h)$. (2)

10.2 Show that the area of the window is given by $A = 3h - \frac{3}{4}h^2$. (3)

10.3 Find $\frac{dA}{dh}$ and hence the dimensions, h and y, of the frame so that the area of the window is a maximum. (5)

[10]

QUESTION 11

Consider the word "**CALCULATOR**".

11.1 How many different word arrangements can be made from all the letters of the word CALCULATOR? (2)

11.2 What is the probability of making a word arrangement that will start and end with the letter L? (2)

11.3 In how many ways can all the letters be arranged if no similar letters should be close to each other? (3)

[7]

QUESTION 12

12.1 Given $P(A) = 0{,}5$; $P(B) = x$ and $P(A \text{ or } B) = 0{,}88$.

Calculate the value(s) of x, if

1.1.1 Events A and B are mutually exclusive. (2)

1.1.2 Events A and B are independent. (3)

12.2 South African National Women Football Team known as "Banyana Banyana" are to play a friendly football match against their Nigerian counterparts known as "Super Falcons". There should be a winner on the day of the match. Penalties would decide the winner in case the match is drawn after regulation time. On the day that the match takes place, there is a 30% chance that it could rain(R), 45%

chance that it could be sunny(S) or it could be cloudy(C). Banyana Banyana has a 24% chance of winning on a rainy day, a 65% of winning on a sunny day or a 33% chance of winning on a cloudy day.

12.2.1 Draw a tree diagram to represent all outcomes of the above scenario. (3)

12.2.2 What is the probability of Banyana Banyana winning the match? (3)

[11]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$ $\bar{x} = \frac{\sum fx}{n}$

$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$ $P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

