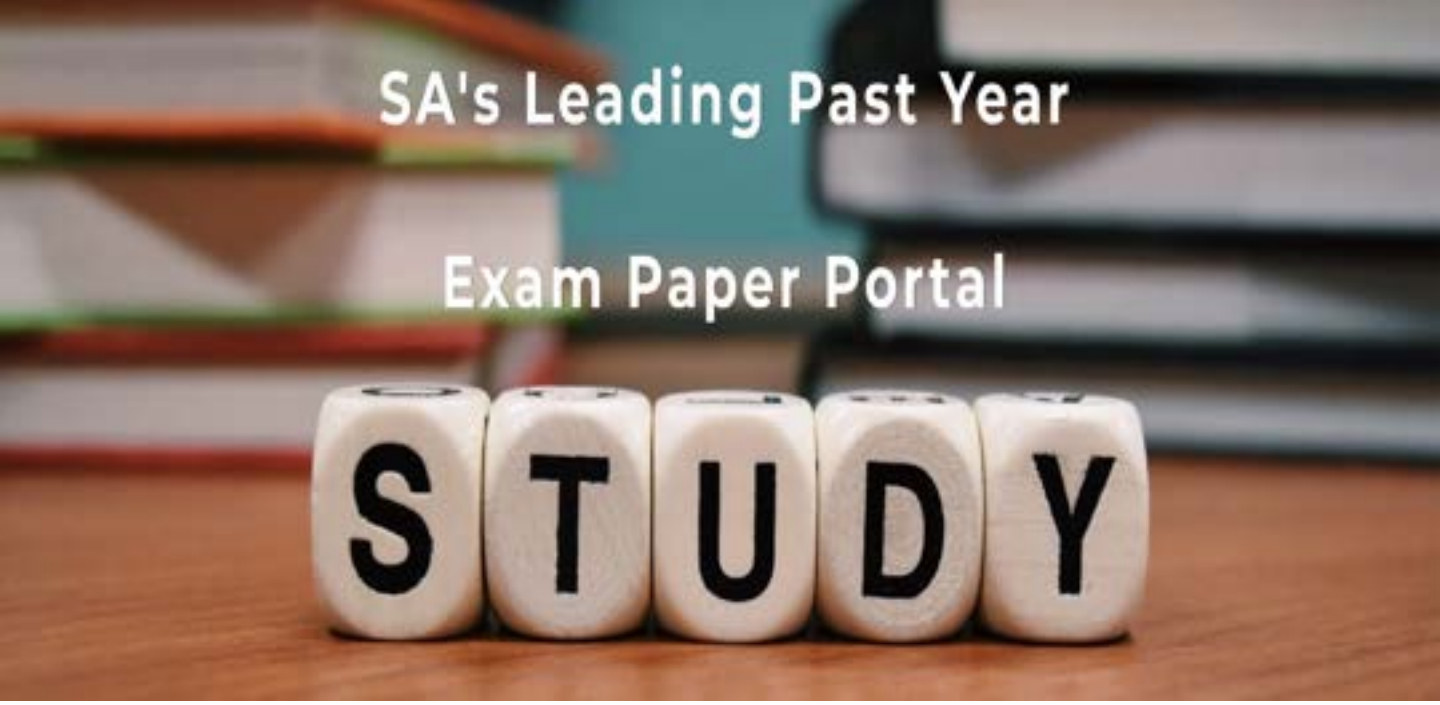

education

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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE/ NASIONALE SENIOR SERTIFIKAAT

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

SEPTEMBER 2022

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

These marking guidelines consist of 18 pages with 3 pages containing the cognitive grid/
Hierdie nasienriglyne bestaan uit 18 bladsye met 3 bladsye wat die kognitiewe tabel bevat.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and has not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As *'n kandidaat* 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek* en nie oordoen nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar *om 'n pr*obleem op te los, word NIE toegelaat nie.

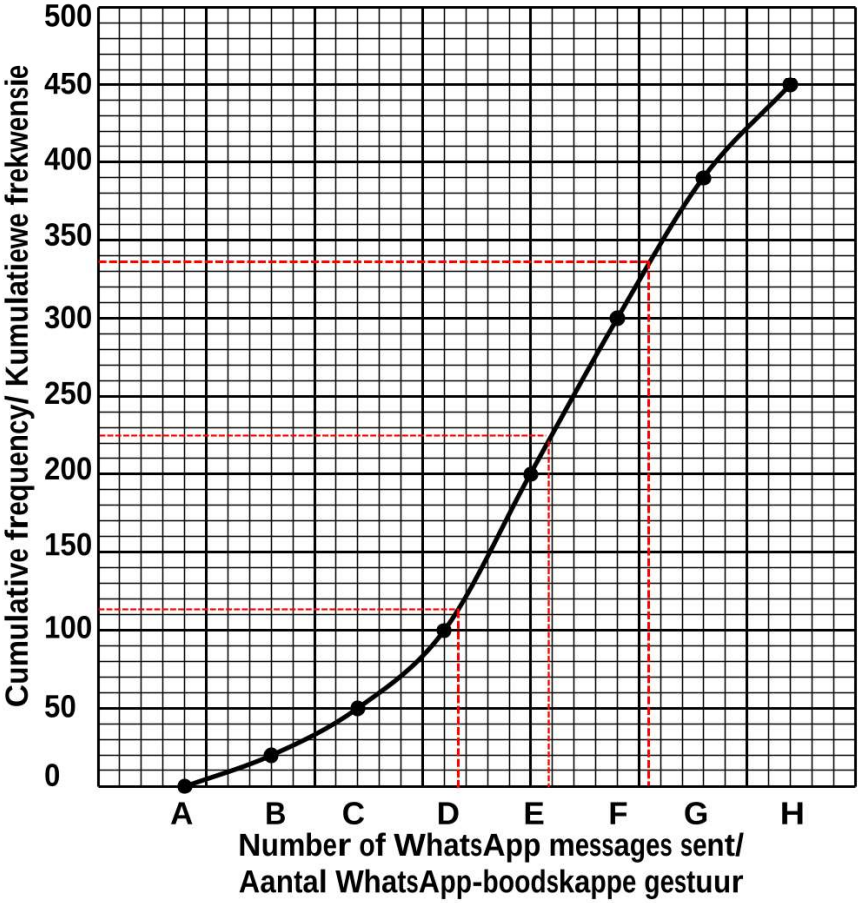
QUESTION/VRAAG 1

Additional rest days given/ Addisionele rusdae gegee	5	2	9	1	3	10	4	6
Productivity of the employee(in %)/ Produktiwiteit van werknemer (in %)	85	63	90	56	70	88	72	62

1.1	r = 0,81	✓ r = 0,81 (1)
1.2	C	✓ C (1)
1.3	$\hat{y} = a + bx$ $a = 56,79$ and / en $b = 3,29$ $\therefore\ \hat{y} = 56,79 + 3,29x$	✓ a = 56,79 ✓ b = 3,29 ✓ equation/ vergelyking (3)
1.4	$\hat{y} = 56,79 + 3,29(8)$ $\hat{y} = 83,11\%$ productivity / produktiwiteit **OR/OF** $\hat{y} \approx 83,13\%$ (calculator / sakrekenaar)	✓substitute 8 into eq. / vervang 8 in vgl. ✓83,11 ✓✓ answer/antwoord (2)
1.5	No/Nee The line of regression is only valid for /Die regressielyn is slegs geldig vir $1 \leq x \leq 10$	✓No/Nee ✓explanation/ verduideliking (2)
		[9]

QUESTION/VRAAG 2

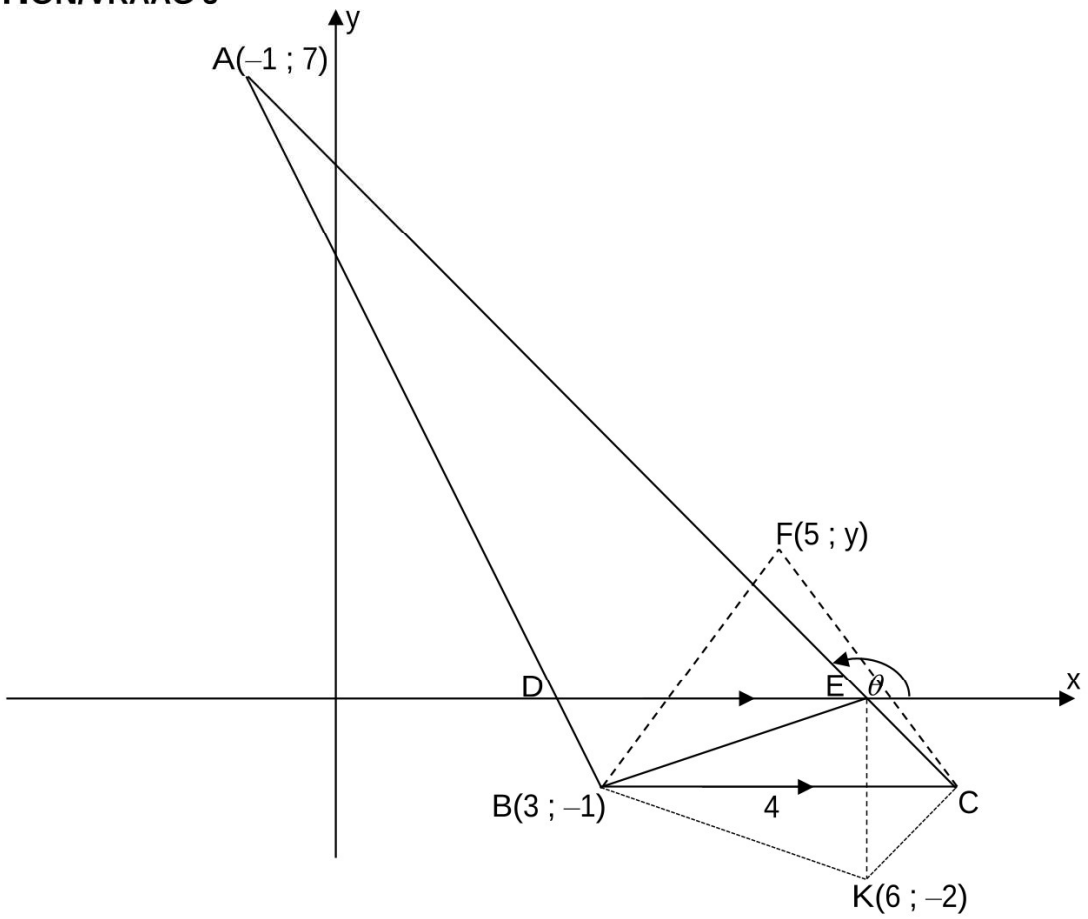
Cumulative Frequency Graph (Ogive)/ Kumulatiewefrekwensie-grafiek (Ogief)



WhatsApp messages sent / WhatsApp-boodskappe gestuur	Frequency/ Frekwensie
$40 \le x < 80$	20
$80 \le x < 120$	30
$120 \le x < 160$	Q
$160 \le x < 200$	R
$200 \le x < 240$	100
$240 \le x < 280$	90
$280 \le x < 320$	60

2.1	40	✓ 40 (1)
2.2	450	✓ 450 (1)
2.3	Q = 100 – 50 = 50 (calculating from/bereken vanaf interval) R = 100	✓✓Q = 50 ✓ R = 100 (3)
2.4	209	✓✓ 209 (2)
2.5	$Q_1 = 168$ $Q_3 = 255$ $\therefore$ IQR / IKO $= 255 - 168$ $= 87$ Also accept/Aanvaar ook $Q_1 \in [167;169]$ $Q_3 \in [254;256]$	✓ $Q_1 = 168$ ✓ $Q_3 = 255$ ✓ answer/antwoord (3)
2.6	The standard deviation will be smaller/Die standaardafwyking sal kleiner wees. Because three intervals are taken away, the data becomes more closely grouped together/Omdat drie intervalle verwyder word, sal die data nader aan mekaar gegroepeer wees.	✓smaller/kleiner ✓explanation/ verduideliking (2)
		[12]

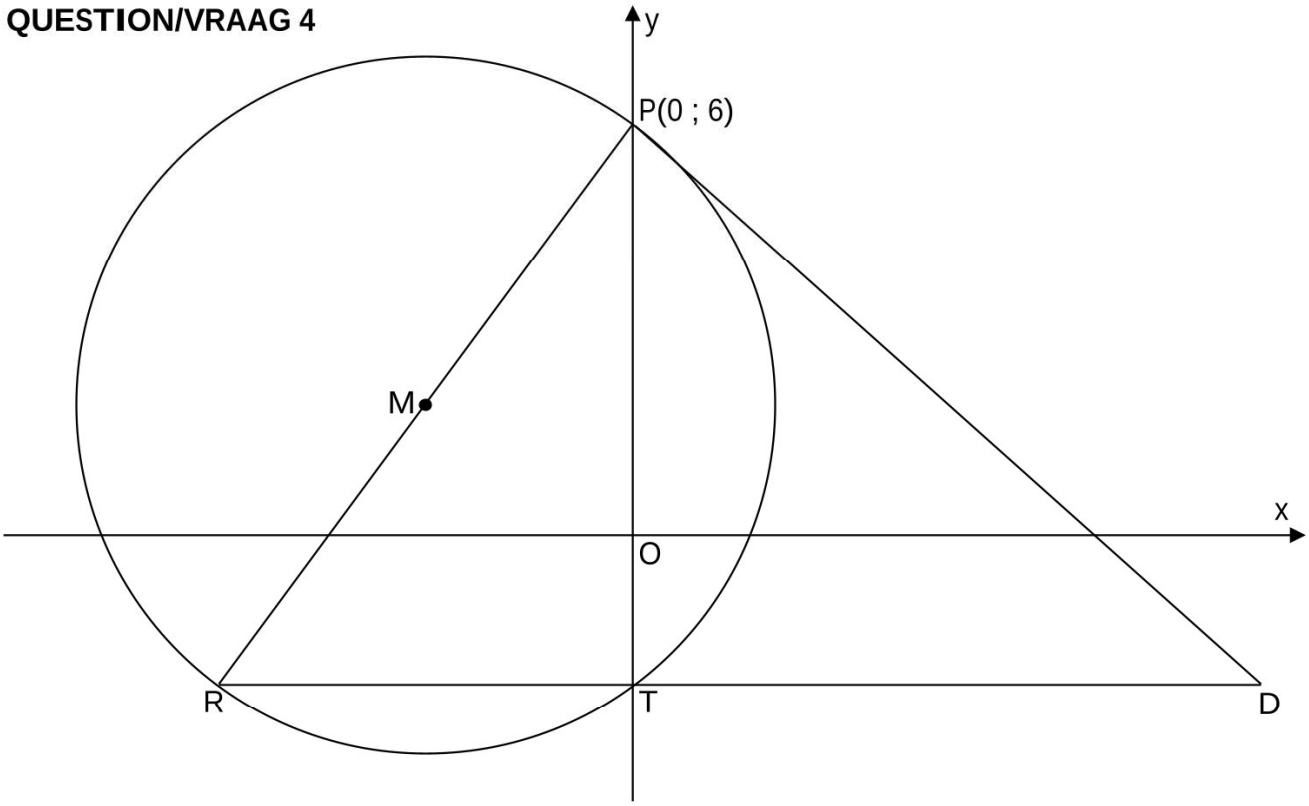
QUESTION/VRAAG 3



3.1	$y = -1$	✓ $y = -1$ (1)
3.2	$x_C = 3 + 4 = 7$	✓ 7 (1)
3.3	$C(7;-1)$ $m_{AC} = \frac{7-(-1)}{-1-7}$ $= -1$ Through / Deur $(-1;7)$ **OR / OF** $(7;-1)$ $y-7 = -1(x-(-1))$ $\quad$ $y-(-1) = -1(x-7)$ $y - 7 = -1(x+1)$ $\quad$ $y+1 = -x+7$ $\therefore y = -x + 6$	✓method gradient/metode gradiënt ✓ $m_{AC} = -1$ ✓Subst./Vervang $(-1;7)$ **OR / OF** $(7;-1)$ ✓ $y = -x + 6$ (4)
3.4	$\tan\theta = m_{AC}$ $\tan\theta = -1$ $\tan^{-1}(-1) = \theta$ $\theta = -45° + 180°$ $\therefore \theta = 135°$ $A\hat{E}D = 45°$ [∠s on straight line / ∠e op reguit lyn] $\therefore A\hat{C}B = 45°$ [corresp.∠s / oreenk.∠e; DE ∥ BC]	✓ $\tan\theta = -1$ ✓135° ✓ $A\hat{E}D = 45°$ ✓ $A\hat{C}B = 45°$ (4)

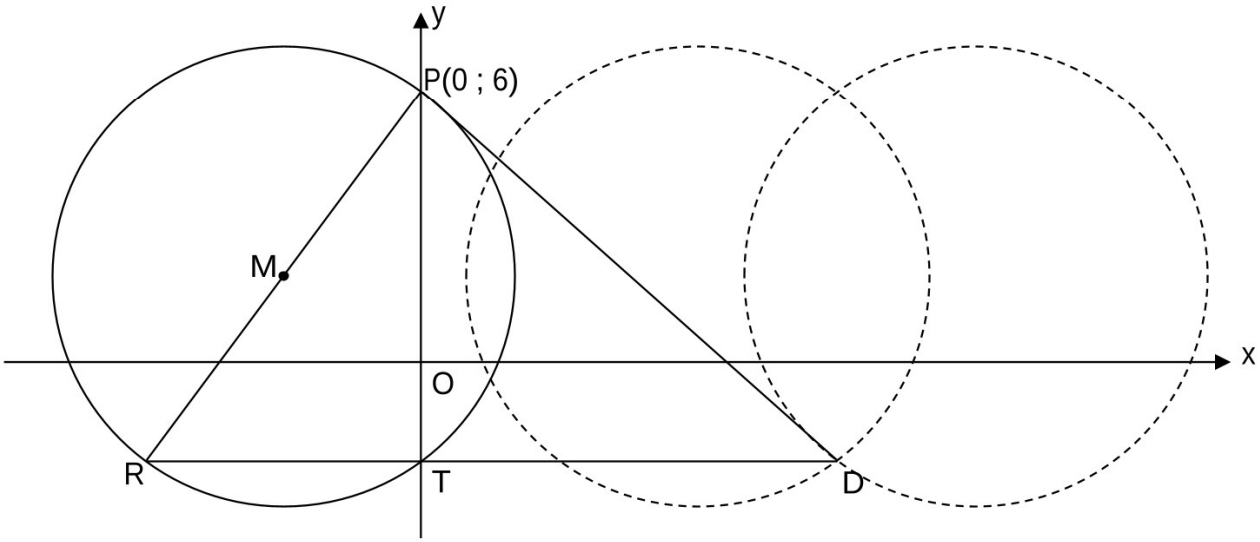
3.5.1	E (6 ; 0) K(6 ; –2)	✓ $x_E = 6$ ✓ $x_K = 6$ ✓ $y_K = -2$ (3)
3.5.2	EK = 2 units/eenhede $\text{Area BECK} = \frac{BC \times EK}{2}$ $= \frac{4 \times 2}{2}$ $= 4\,\text{units}^2$ / eenhede^2 **OR/OF** $\text{Area } \Delta BEC = \frac{1}{2}(4)(1)$ $= 2$ $\text{Area } \Delta BKC = \frac{1}{2}(4)(1)$ $= 2$ $\text{Area BECK} = 2 + 2 = 4\ \text{units}^2$ / eenhede^2	✓ EK = 2 ✓method/metode ✓answer/antwoord (3) ✓ Area $\Delta BEC = 2$ ✓ Area $\Delta BKC = 2$ ✓answer/antwoord (3)
3.6	$x_F = \frac{x_B + x_C}{2}$ $= \frac{3 + 7}{2}$ $= 5$ $\therefore$ F(5; y) $BF = \sqrt{(5-3)^2 + (y+1)^2}$ **OR / OF** $FC = \sqrt{(5-7)^2 + (y+1)^2}$ BF = FC = BC = 4 $16 = 4 + (y+1)^2$ $(y+1)^2 = 12$ $y = \pm\sqrt{12} - 1$ $\therefore y = \sqrt{12} - 1$ (first quadr. / eerste kwadr.)	✓method/metode ✓ $x_F = 5$ ✓distance form./ afstandsformule ✓ $(y+1)^2 = 12$ ✓ antwoord/answer (5)
		[21]

QUESTION/VRAAG 4

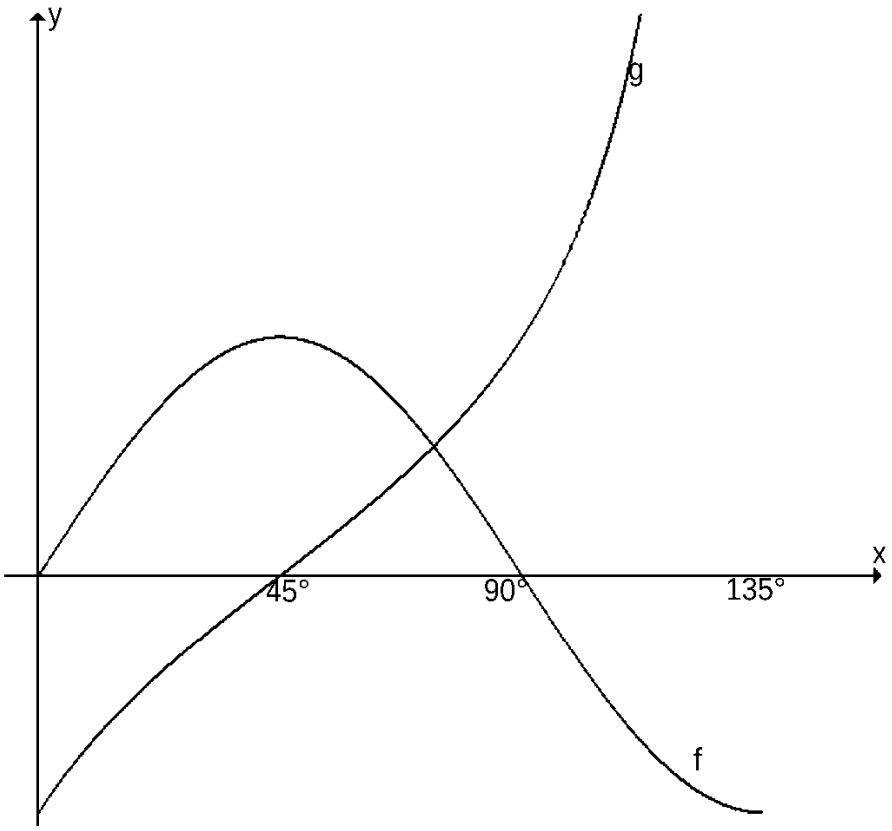


4.1	$x^2 + 6x + (3)^2 + y^2 - 4y + (-2)^2 = 12 + 9 + 4$ $(x+3)^2 + (y-2)^2 = 25$ $\therefore M(-3; 2)$ Answer only: Full Marks	✓method/metode ✓$x = -3$ ✓$y = 2$ (3)
4.2	$r^2 = 12 + (3)^2 + (-2)^2 = 25$ $\therefore r = 5$ $\therefore PR = 10$	✓$r^2 = 25$ (can also be shown in 4.1/ kan ook in 4.1 gewys word) ✓ PR = 10 (2)
4.3	$m_{MP} = \frac{6-2}{0-(-3)}$ $= \frac{4}{3}$ $\therefore m_{PD} = -\frac{3}{4}$ [rad ⊥ tangent / rad ⊥ raaklyn] $y = -\frac{3}{4}x + c$ $\therefore y = -\frac{3}{4}x + 6$	✓ $m_{MP} = \frac{4}{3}$ ✓ $m_{PD} = -\frac{3}{4}$ ✓equation / vergelyking (3)

4.4	$\frac{y_P + y_R}{2} = y_M$ $\frac{6 + y_R}{2} = 2$ $y_R = -2$ $D(x\ ;-2)$ $-2 = -\frac{3}{4}x + 6$ $x = \frac{32}{3}$ $\therefore D(\frac{32}{3};-2)$	✓method/metode ✓ $y_R = -2$ ✓subst. y = –2 into equation/vervang y = –2 in vergelyking ✓ $D(\frac{32}{3};-2)$ (4)
4.5	$T_x \rightarrow D_x = \frac{32}{3}$ units / eenhede new centre / nuwe middelpunt $\left(\frac{23}{3}\ ;\ 2\right)$ $R_x \rightarrow D_x = \frac{50}{3}$ units / eenhede new centre / nuwe middelpunt $\left(\frac{41}{3}\ ;\ 2\right)$	✓ $\frac{32}{3}$ ✓ $\left(\frac{23}{3}\ ;\ 2\right)$ ✓ $\frac{50}{3}$ ✓ $\left(\frac{41}{3}\ ;\ 2\right)$ (4)
		[16]

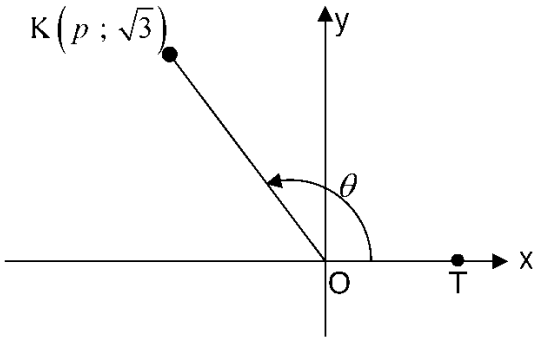


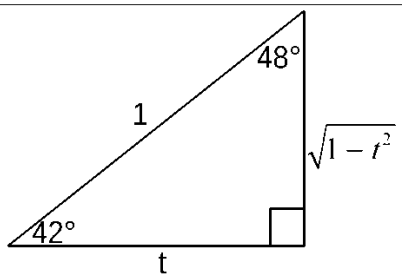
QUESTION/VRAAG 5



5.1	a = 2 and/en b = – 45°	✓a = 2 ✓b = – 45° (2)
5.2	$-1 \leq y \leq 1$ **OR/OF** $y \in [-1 ; 1]$	✓critical values/kritiese waardes ✓ notation/notasie (2)
5.3	180°	✓ 180° (1)
5.4	x = 135°	✓ 135° (1)
5.5	x = 0° or/of x = 45° or/of x = 90°	✓ ✓any 2 correct solutions/ enige 2 korrekte oplossings (one mark per solution/ een punt per oplossing) (2)
5.6	$0° \leq x < 45°$ **OR/OF** $x \in [0° ; 45°)$	✓ 0° and/en 45° ✓ notation/notasie (2)
		[10]

QUESTION/VRAAG 6

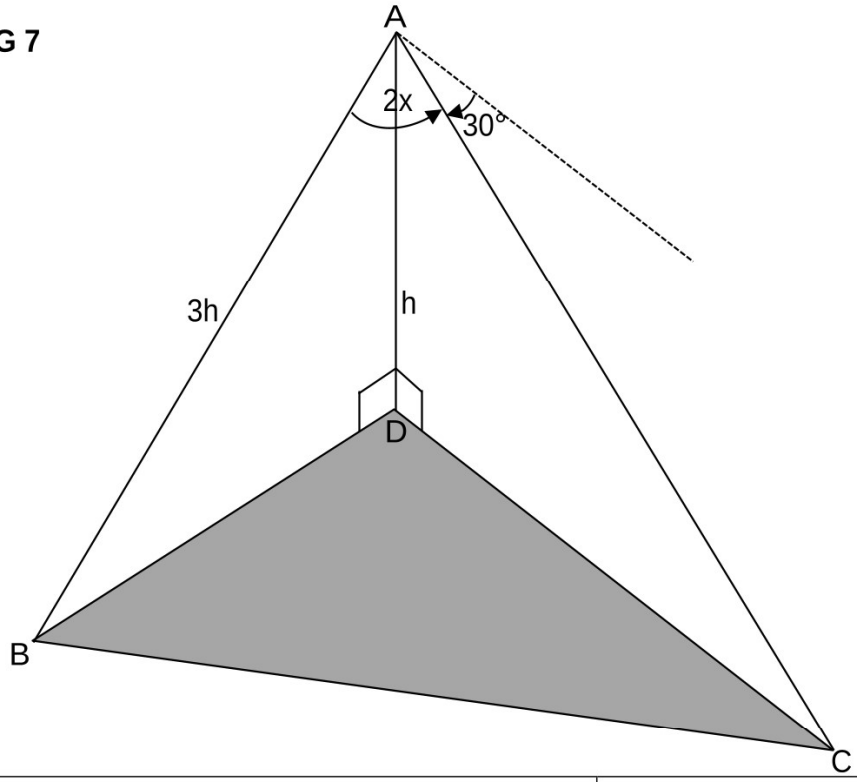


6.1.1	$\tan\theta = \dfrac{\sqrt{3}}{p}$	✓ $\dfrac{\sqrt{3}}{p}$ (1)
6.1.2	$\tan 120° = \dfrac{\sqrt{3}}{p}$ $\tan(180° - 60°) = \dfrac{\sqrt{3}}{p}$ $-\tan 60° = \dfrac{\sqrt{3}}{p}$ $-\sqrt{3} = \dfrac{\sqrt{3}}{p}$ $p = -1$	✓ $-\tan 60°$ ✓ $-\sqrt{3} = \dfrac{\sqrt{3}}{p}$ ✓ $p = -1$ (3)
6.2.1	$\cos 42°$ $= \sin 48°$ $= t$ Answer only: Full Marks  	✓$\sin 48° = \cos 42°$ ✓t (2)
6.2.2	$\cos 2(42°)$ $= 2\cos^2 42° - 1$ $= 2t^2 - 1$ **OR/OF** $\cos 2(42°)$ $= 1 - 2\sin^2 42°$ $= 1 - 2\left(\sqrt{1-t^2}\right)^2$ $= 2t^2 - 1$	✓cos2(42°) ✓$2\cos^2 42° - 1$ ✓ $2t^2 - 1$ ✓cos2(42°) ✓$1 - 2\sin^2 42°$ ✓$2t^2 - 1$

6.2.2	**OR/OF** $\cos 2(42°)$ $= \cos^2 42° - \sin^2 42°$ $= t^2 - \left(\sqrt{1-t^2}\right)^2$ $= 2t^2 - 1$	✓ $\cos 2(42°)$ ✓ $\cos^2 42° - \sin^2 42°$ ✓ $2t^2 - 1$ (3)
6.2.3	$\cos(42° + 30°)$ $= \cos 42°.\cos 30° - \sin 42°.\sin 30°$ $= t.\frac{\sqrt{3}}{2} - \sqrt{1-t^2}.\frac{1}{2}$ $= \frac{\sqrt{3}t}{2} - \frac{\sqrt{1-t^2}}{2}$ $= \frac{\sqrt{3}t - \sqrt{1-t^2}}{2}$	✓ manipulation / manipulasie ✓ expansion of identity/ uitbreiding van identiteit ✓ $\frac{\sqrt{3}}{2}$ & $\frac{1}{2}$ ✓ t & $\sqrt{1-t^2}$ ✓ answer/antwoord (5)
6.3	$\frac{\sin x}{\sin 2x} + \frac{\sin(180° - x)}{\cos x} - \frac{\sin^2 x + \cos^2 x}{2\sin(90° + x)}$ $= \frac{\sin x}{2\sin x.\cos x} + \frac{\sin x}{\cos x} - \frac{1}{2\cos x}$ $= \frac{1}{2\cos x} + \frac{\sin x}{\cos x} - \frac{1}{2\cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	✓ $\sin 2x = 2\sin x.\cos x$ ✓ $\sin(180° - x) = \sin x$ ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $2\sin(90° + x) = 2\cos x$ ✓ $\frac{\sin x}{\cos x}$ ✓ $\tan x$ (6)

6.4	$f(x) = \sin\left[(2\theta - 15°) + (\theta - 30°)\right]$ $f(x) = \sin[3\theta - 45°]$ $\therefore \sin[3\theta - 45°] = 0,8$ $3\theta - 45° = 53,1301...° + k.360°$, $k \in Z$ $3\theta = 98,1301...° + k.360°$ $\theta = 32,71° + k.120°$ or / of $3\theta - 45° = 180° - 53,1301...° + k.360°$, $k \in Z$ $3\theta = 171,8699...° + k.360°$ $\theta = 57,29° + k.120°$	✓ $f(x) = \sin[3\theta - 45°]$ ✓ $\sin[3\theta - 45°] = 0,8$ ✓ $3\theta - 45° = 53,1301...°$ ✓ $\theta = 32,71° + k.120°$ ✓ $3\theta - 45° = 180° - 53,1301...°$ ✓ $\theta = 57,29° + k.120°$ ✓ $+ k.120°$, $k \in Z$ (7)
6.5	$4^{\sin x} = 2^{2\sin x} = 6$ $6^{\cos x} = \left(2^{2\sin x}\right)^{\cos x} = 2^{2\sin x.\cos x} = 8$ $8 = 2^{2\sin x.\cos x} = 32^{1 - 2\sin^2 x}$ $\therefore 2^{2\sin x.\cos x} = 2^{5(1 - 2\sin^2 x)}$ $\sin 2x = 5\cos 2x$ $\tan 2x = 5$	✓ $2^{2\sin x} = 6$ ✓ $2^{2\sin x.\cos x} = 8$ ✓ $2^{2\sin x.\cos x} = 2^{5(1 - 2\sin^2 x)}$ ✓ $\sin 2x = 5\cos 2x$ ✓ $\tan 2x = 5$ (5)
		[32]

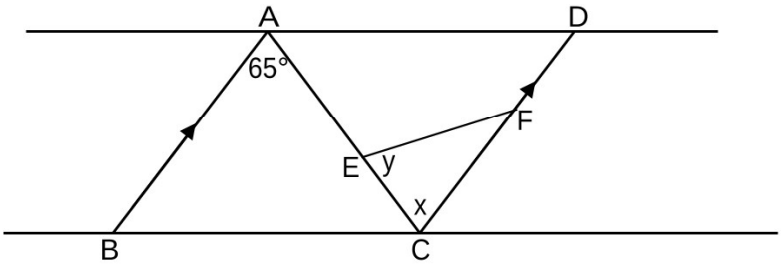
QUESTION/VRAAG 7



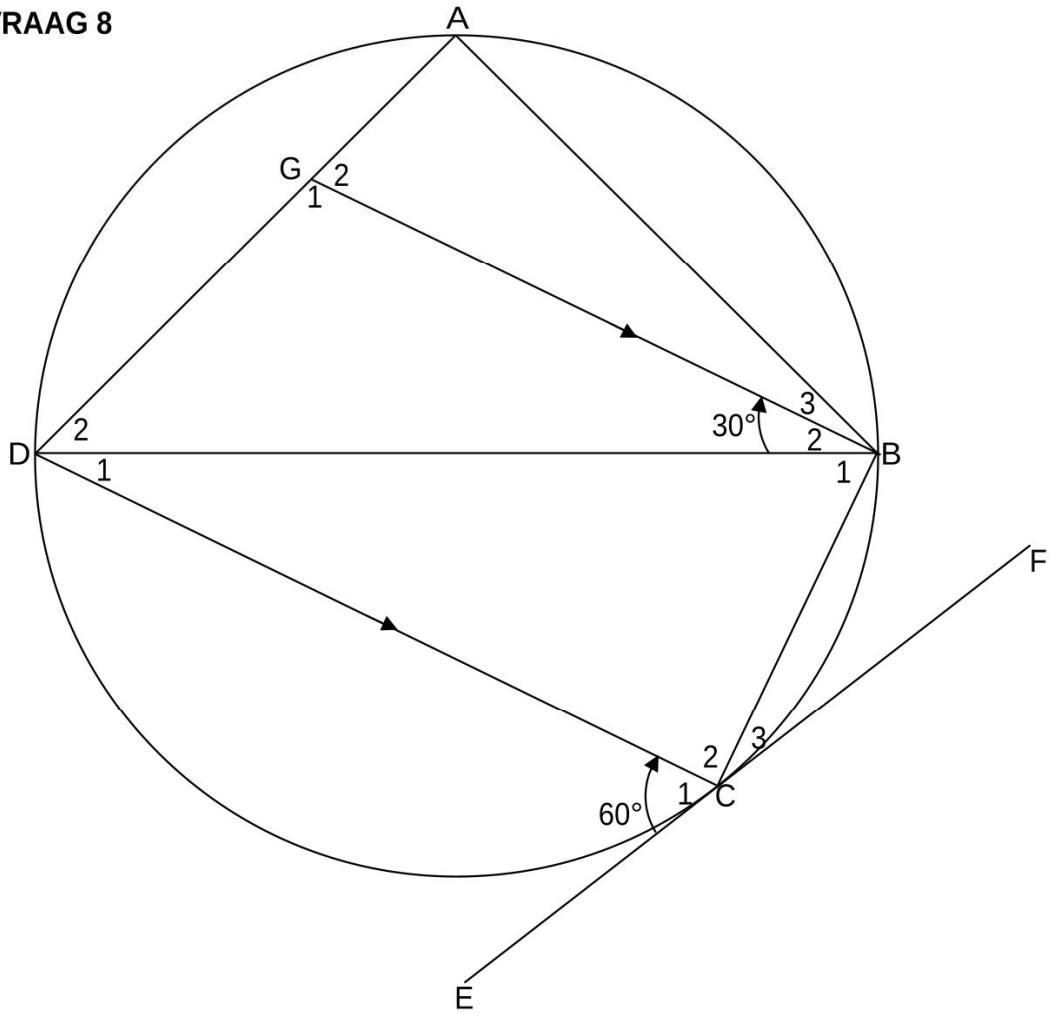
7.1	$A\hat{C}D = 30°$	✓30° (1)
7.2	$\sin 30° = \frac{h}{AC}$ $\therefore AC = 2h$	✓ $\sin 30° = \frac{h}{AC}$ ✓ $AC = 2h$ (2)
7.3	$\sin A\hat{B}D = \frac{h}{3h} = \frac{1}{3}$ $A\hat{B}D = \sin^{-1}\left(\frac{1}{3}\right)$ $A\hat{B}D = 19{,}47°$	✓ $\sin A\hat{B}D = \frac{1}{3}$ ✓ $A\hat{B}D = 19{,}47°$ (2)
7.4	$BC^2 = AB^2 + AC^2 - 2AB.AC.\cos 2x$ $\left(\sqrt{7}h\right)^2 = (3h)^2 + (2h)^2 - 2(3h)(2h)\cos 2x$ $7h^2 = 9h^2 + 4h^2 - 12h^2 \cos 2x$ $-6h^2 = -12h^2 \cos 2x$ $\cos 2x = \frac{1}{2}$ $2x = 60°$ $x = 30°$	✓correct use of cosine-rule/korrekte gebruik van cos-reël ✓subst/vervanging ✓ $\cos 2x = \frac{1}{2}$ ✓ $2x = 60°$ ✓ $x = 30°$ (5)
		[10]

GEOMETRY/MEETKUNDE

Please read carefully through the following table before marking **QUESTION 8–10** /
Lees asseblief sorgvuldig deur die volgende tabel alvorens **VRAAG 8–10** nagesien word.

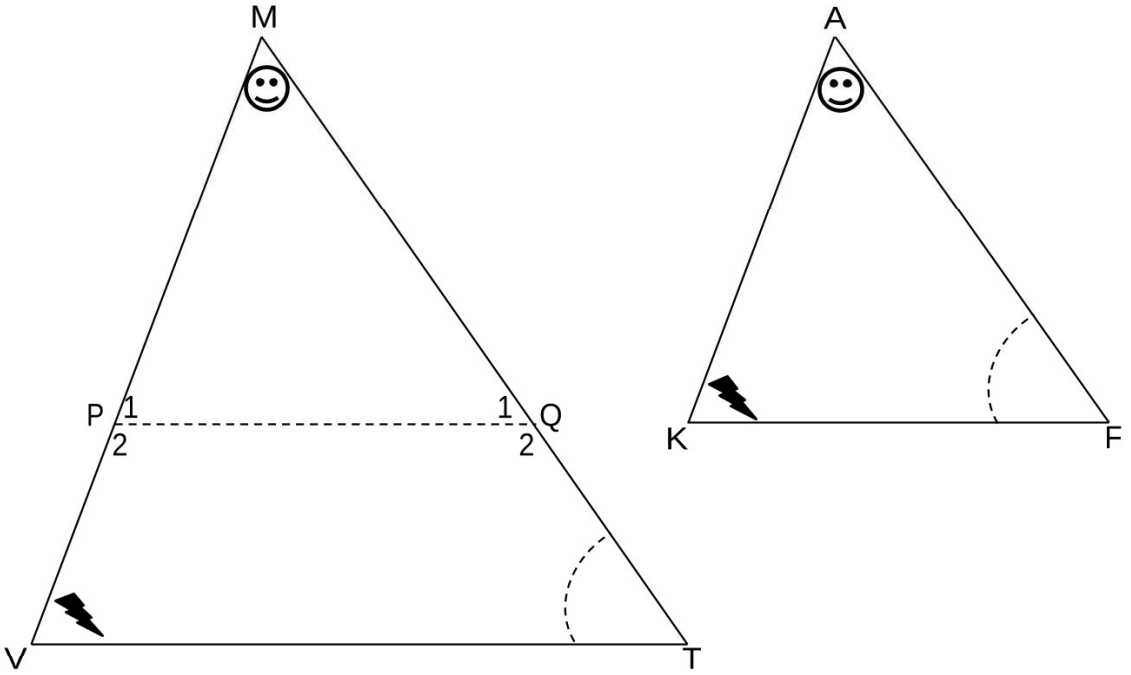
	The order in which the candidate answers a geometry question must follow logically/ Die *volgorde waarin 'n kandidaat 'n meetkundevraag beantwoord moet* logies volg. **Example/Voorbeeld** Given/Gegee AB ∥ CD and/en $E\hat{F}D = 115°$  The candidate first need to calculate x BEFORE he/she can calculate y/Die kandidaat moet eerste vir x bereken VOORDAT hy/sy vir y kan bereken.
S	A mark for a correct statement (A statement mark is independent of a reason)
	'n Punt vir 'n korrekte bewering *('n Punt vir 'n bewering is* onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	'n Punt vir 'n korrekte rede *('n Punt word slegs vir die rede toegeken as die bewering korrek is)*
S/R	Award a mark if the statement AND reason are both correct (Both MUST be correct to get one mark)
	Ken 'n punt toe as die bewering EN rede beide korrek is (Beide MOET korrek wees om een punt te kry)

QUESTION/VRAAG 8



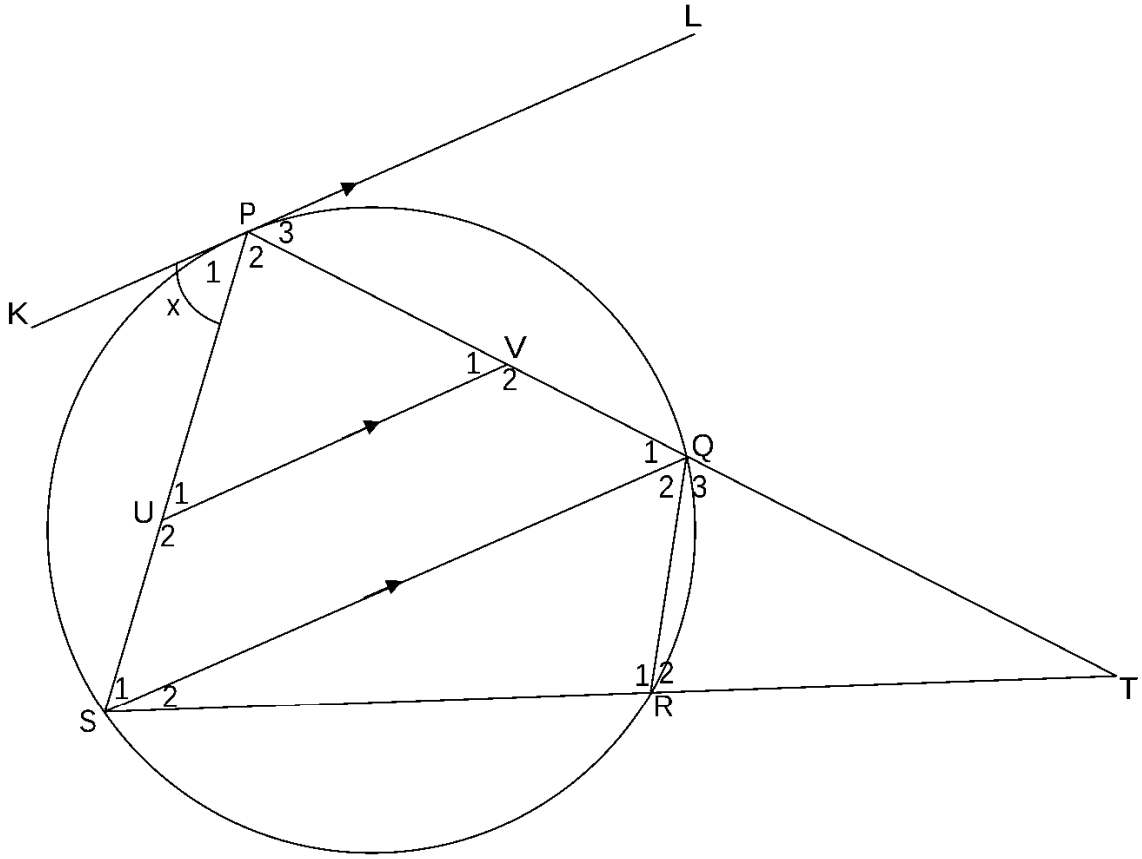
8.1.1	$\hat{D}_1 = 30°$ [alt ∠s / verwiss ∠e; BG ∥ DC]	✓S/R (1)
8.1.2	$\hat{B}_1 = 60°$ [tan-chord th./raaklyn - koordst.]	✓S✓R (2)
8.1.3	$\hat{C}_2 = 90°$ [∠s of Δ / binne.∠e van Δ]	✓S/R (1)
8.1.4	$D\hat{A}B = 90°$ [opp.∠ of cq / teenoorst.∠e van kvh]	✓S✓R (2)
8.2	Yes / Ja BD is a diameter/BD is 'n middellyn [converse / omgekeerde ∠ in $\frac{1}{2}$ □]	✓yes/ja ✓S/R (2)
		[8]

QUESTION/VRAAG 9



9.1	Construction: Mark P and Q such that MP = AK and MQ = AF. Draw PQ Konstruksie: Merk P en Q sodat MP = AK en MQ = AF. Trek PQ	✓ constr/ konstr
	In ΔMPQ and/en ΔAKF	
	$\hat{M} = \hat{A}$ [given / gegee]	
	$MP = AK$ [construction / konstruksie]	
	$MQ = AF$ [construction / konstruksie]	
	$\therefore \Delta MPQ \equiv \Delta AKF$ [S, ∠, S]	✓ S/R
	$\hat{P}_1 = \hat{K}$ [from / vanuit ≡]	✓ S
	but / maar $\hat{V} = \hat{K}$ [given / gege]	
	$\therefore \hat{P}_1 = \hat{V}$	✓ S
	$\therefore PQ \parallel VT$ [corresp.∠s = /ooreenk.∠e =]	✓ S/R
	$\frac{MV}{MP} = \frac{MT}{MQ}$ [prop.th / eweredigheidst.; $PQ \parallel VT$]	✓ S ✓R
	$MP = AK$ and / en $MQ = AF$	
	$\therefore \frac{MV}{AK} = \frac{MT}{AF}$	(7)

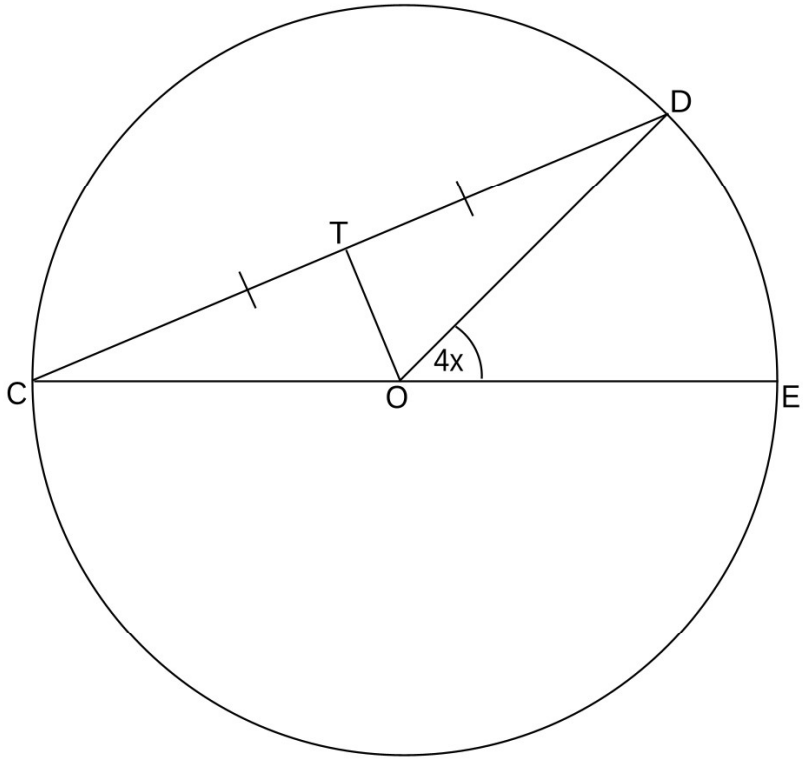
9.2



9.2.1	$K\hat{P}S = P\hat{Q}S = x$ [tan-chord th./raaklyn - koordst.]	✓S/R
	$K\hat{P}S = P\hat{S}Q = x$ [alt.∠s/verwissel.∠e; KL ∥ SQ]	✓S/R
	$K\hat{P}S = P\hat{U}V = x$ [alt.∠s/verwissel.∠e; KL ∥ UV]	✓S/R
	$P\hat{V}U = P\hat{Q}S = x$ [corresp.∠s/ooreenk.∠e; UV ∥ SQ]	✓S/R
	$L\hat{P}Q = P\hat{S}Q = x$ [tan-chord th./raaklyn - koordst.]	✓S/R (5)
9.2.2 (a)	$P\hat{S}Q = P\hat{Q}S = x$	
	∴ PQ = PS [sides opp. = ∠s / sye teenoor = ∠e]	✓R (1)
9.2.2 (b)	$K\hat{P}U = P\hat{V}U = x$	
	∴ KP is a tangent / 'n raaklyn	
	[converse tan-chord th. / omgekeerde raaklyn – koordst.]	✓R (1)
9.2.3	Prop.th / eweredigheidst. ; UV ∥ SQ	✓R (1)

9.2.4 (a)	In ΔPTS and / en ΔRTQ $\hat{T} = \hat{T}$ [common / gemeenskaplik] $T\hat{P}S = T\hat{R}Q$ [ext.$\angle$ of cq/buite$\angle$ van kvh] $P\hat{S}T = T\hat{Q}R$ [sum of $\angle$s of Δ / som van $\angle$e van Δ] $\therefore \Delta TPS \parallel\parallel \Delta TRQ$ [$\angle\ \angle\ \angle$]	✓S/R ✓S✓R ✓S/R (4)
9.2.4 (b)	$\frac{TP}{TR} = \frac{PS}{RQ}$ [from / vanuit $\parallel\parallel\Delta$] $\therefore PT.RQ = PS.TR$	✓S✓R (2)
9.2.4 (c)	$\frac{PS}{RQ} = \frac{TS}{TQ}$ [from / vanuit $\parallel\parallel\Delta$] $PS.TQ = TS.RQ$ but / maar $TQ = PT - PQ$ $PS(PT - PQ) = TS.RQ$ but/maar $PQ = PS$ $PS.PT - PQ^2 = TS.RQ$ $PQ^2 = PS.PT - TS.RQ$ $\therefore PQ = \sqrt{PS.PT - TS.RQ}$	✓S ✓S ✓S ✓S (4)
		[25]

QUESTION/VRAAG 10



	$D\hat{C}E = 2x$ $\left[\begin{array}{l}\angle \text{ at centre} = 2\times\angle\text{at circumference/}\\ \text{midpts.}\angle = 2\times\text{omtreks.}\angle\end{array}\right]$	✓ S ✓ R
	$O\hat{T}C = 90°$ $\left[\begin{array}{l}\text{line from centre to midpt of chord/}\\ \text{midpt.}\square\ ;\ \text{midpt.koord}\end{array}\right]$	✓ S ✓ R
	In Δ OTC	
	$\sin 2x = \frac{OT}{OC}$	✓ S
	$2\sin x\cos x = \frac{OT}{OC}$	✓2sinxcosx
	$\sin x\cos x = \frac{OT}{2\times OC}$	✓ $\frac{OT}{2\times OC}$
	$\sin x\cos x = \frac{OT}{CE}$	(7)
		[7]

TOTAL/TOTAAL: 150