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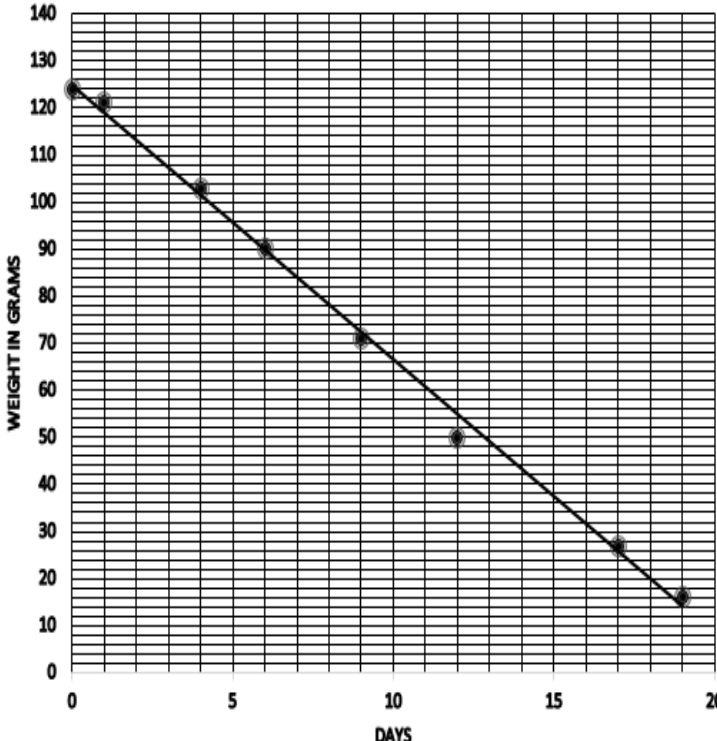
GRADE/GRADE 12

SEPTEMBER 2016

**MATHEMATICS P2 / *WISKUNDE V2*
MEMORANDUM**

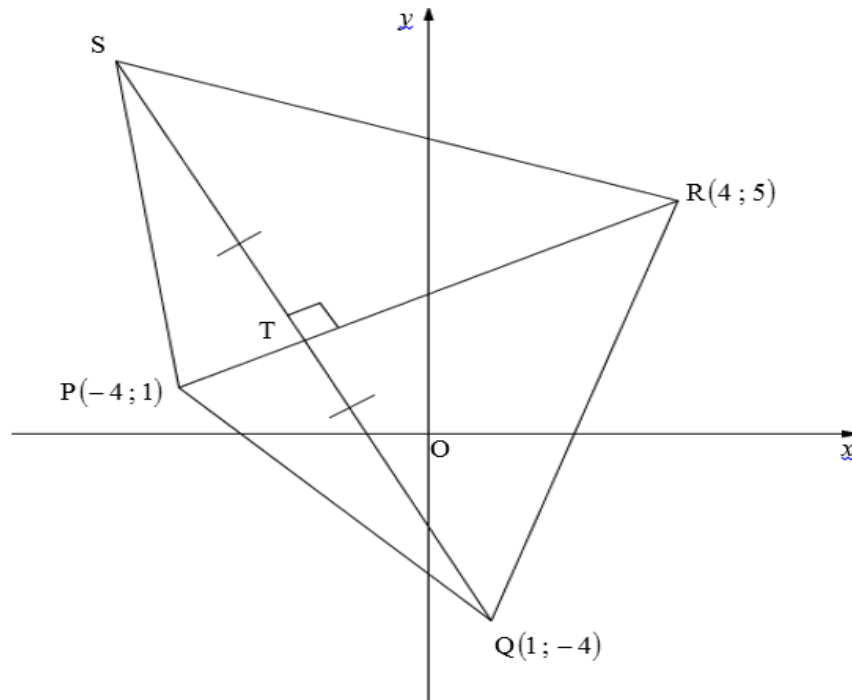
MARKS / PUNTE: 150

This memorandum consists of 16 pages.
Hierdie memorandum bestaan uit 16 bladsye.

QUESTION 1 / VRAAG 1										
	Day/Dag	0	1	4	6	9	12	17	19	
	Weight / Gewig	124	121	103	90	71	50	27	16	
1.1							✓ 2-4 correct points ✓ 5-7 correct points ✓ plotting all points ✓ 2 – 4 punte korrek ✓ 5 – 7 punte korrek ✓ alle punte afgesteek			(3)
1.2	$a = 124,84$ $b = -5,83$ $y = 124,84 - 5,83x$					✓ A ✓ B ✓ equation / vergelyking			(3)	
1.3	$(x; y) = (8; 5; 75,25)$ y-int 124,84					✓ $(8; 5; 75,25)$ and/en y-int / y-afsnit 124,84 ✓ regression line / regressielyn			(2)	
1.4	$124,84 - 5,83x = 80$ $5,83x = 44,84, 69$ $x = 7,69$ On the morning of the 8 th day the bar of soap will be less than 80 grams. <i>Op die oggend van die 8^{ste} dag sal steen seep minder as 80 gram wees.</i>					✓ substitution / vervanging ✓ answer / antwoord			(2)	
1.5	$r = -0,998$						✓ answer / antwoord			(1)
1.6	Very strong negative correlation. <i>Baie sterk negatiewe korrelasie.</i>						✓ answer / antwoord			(1)
										[12]

QUESTION 2 / VRAAG 2			
5	7		
4	3 4 4		
3	1 1 1 4 6 8		
2	2 4 8 9 9		
1	5 5 6 7		
0	3 5		
2.1	21 learners / <i>leerders</i>	✓ answer / <i>antwoord</i>	(1)
2.2	3 pages / <i>bladsye</i>	✓ answer / <i>antwoord</i>	(1)
2.3	$\bar{x} = 28,19$	✓✓ answer / <i>antwoord</i>	(2)
2.4	$\sigma = 13,12$	✓ answer / <i>antwoord</i>	(1)
2.5	$(28,19 - 13,12; 28,19 + 13,12)$ $(15,07; 41,31)$ $\therefore$ 8 learners are outside one standard deviation <i>8 leerders is buite een standaardafwyking</i> $\therefore \frac{8}{21} \times 100 = 38,10\%$	✓ interval / <i>interval</i> ✓ 8 learners / <i>leerders</i> ✓ 28,57%	(3)
			[8]

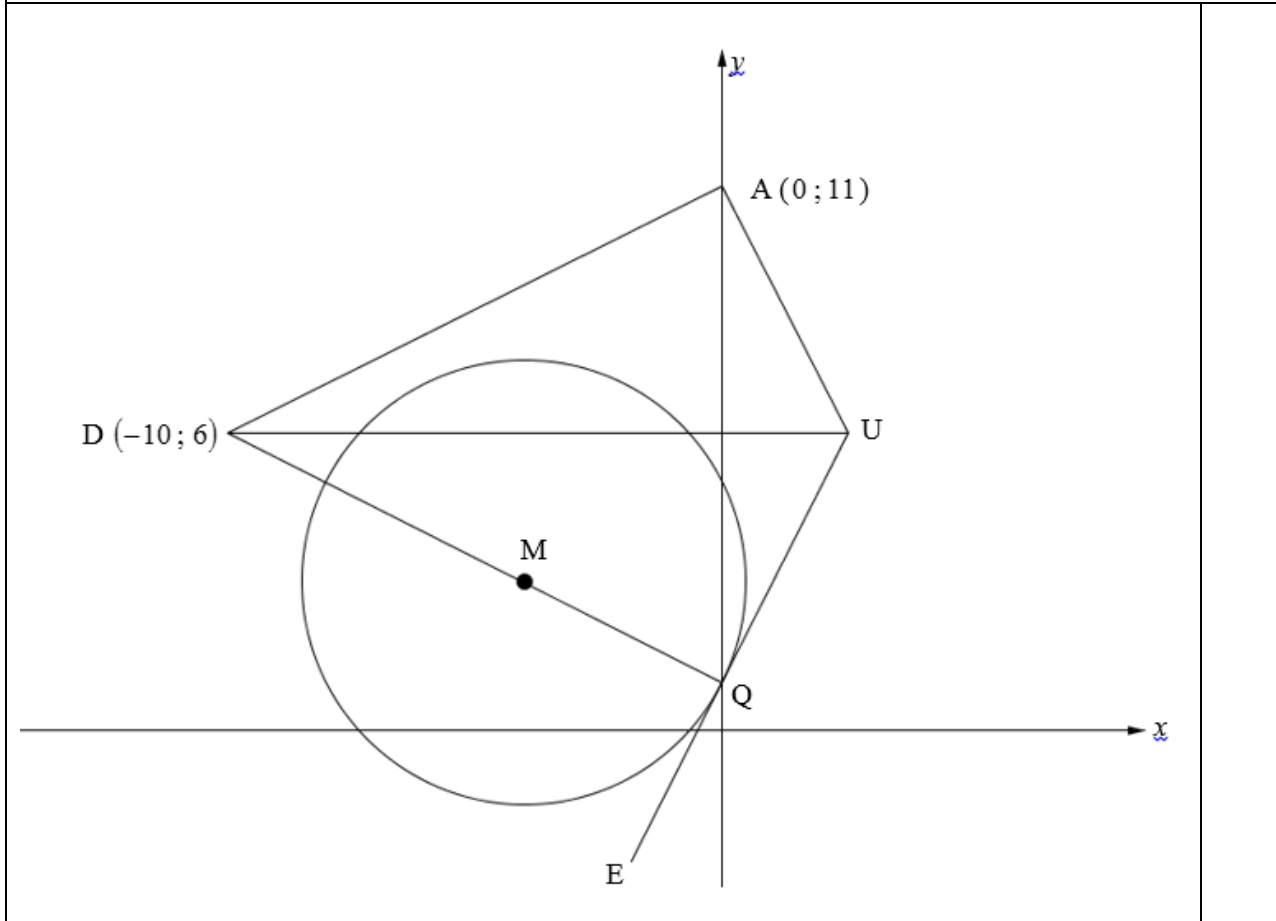
QUESTION 3 / VRAAG 3



3.1	$m_{PR} = \frac{5-1}{4+4}$ $= \frac{1}{2}$	✓ subst. P and R into correct formula verv. van P en R in korrekte formule ✓ $m_{PR} = \frac{1}{2}$	(2)
3.2	$m_{SQ} = -2$ $SP \perp PT$ $y + 4 = -2(x - 1)$ $y = -2x - 2$	✓ $m_{SQ} = -2$ ✓ subst. m and Q into correct formula verv. van m en Q in korrekte formule ✓ $y = -2x - 2$	(3)
3.3	Equation of PR / Vergelyking van PR $y - 1 = \frac{1}{2}(x + 4)$ $y = \frac{1}{2}x + 3$ $-2x - 2 = \frac{1}{2}x + 3$ $-4x - 4 = x + 6$ $-5x = 10$ $\therefore x = -2$ $y = -2(-2) - 2 = 2$ OR/OF	✓ substituting m and P into equation of a str. line / vervanging van m en P in die vergelyking van reguitlyn ✓ equation of PR / vergelyking van PR ✓ equating PR and SQ / gelykstel van PR en SQ ✓ x-value / x-waarde ✓ y-value / y-waarde ✓ substituting m and R into equation of a	

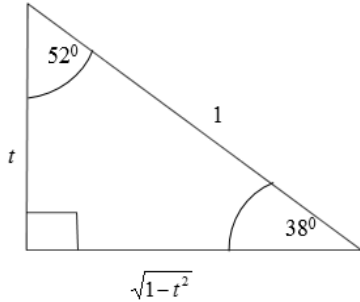
	$y - 5 = \frac{1}{2}(x - 4)$ $2y - 10 = x - 4$ $2y = x + 6$ $y = \frac{1}{2}x + 3$ $-2x - 2 = \frac{1}{2}x + 3$ $-4x - 4 = x + 6$ $-5x = 10$ $\therefore x = -2$ $y = -2(-2) - 2 = 2$	str. line / <i>vervanging van m en R in die vergelyking van reguitlyn</i> ✓ equation of PR / <i>vergelyking van PR</i> ✓ equating PR and SQ / <i>gelykstel van PR en SQ</i> ✓ x-value / <i>x-waarde</i> ✓ y-value / <i>y-waarde</i>	(5)
3.4	$-2 = \frac{x+1}{2} \quad 2 = \frac{y-4}{2}$ $\therefore x = -5 \quad \text{and/en} \quad y = 8$ $S(-5; 8)$	✓ substituting into correct formula / <i>vervanging in korrekte formule</i> ✓ x-value / <i>x-waarde</i> ✓ y-value / <i>y-waarde</i>	(3)
3.5	$SQ = \sqrt{(-5-1)^2 + (8+4)^2}$ $= 6\sqrt{5}$ $PT = \sqrt{(-2+4)^2 + (2-1)^2}$ $= \sqrt{5}$ $\text{Area } \Delta PQS = \frac{1}{2} \times 6\sqrt{5} \times \sqrt{5}$ $= 15 \text{ unit}^2 / \text{eenhede}^2$ OR/OF $\therefore \text{area of } \Delta PQS = ST \times PT$ $= 3\sqrt{5} \times \sqrt{5}$ $= 15 \text{ unit}^2 / \text{eenhede}^2$	✓ subst. into correct form / <i>verv. in korrekte formule</i> ✓ $SQ = 6\sqrt{5}$ ✓ $PT = \sqrt{5}$ ✓ Subt into correct form. / <i>verv. in korrekte formule</i> ✓ $15 \text{ units}^2 / \text{eenhede}^2$ ✓ $SQ = 6\sqrt{5}$ ✓ $ST = 3\sqrt{5}$ ✓ $PT = \sqrt{5}$ ✓ subst into form / <i>verv. in formule</i> ✓ $15 \text{ units}^2 / \text{eenhede}^2$	(5)
			[18]

QUESTION 4 / VRAAG 4



4.1	$x^2 + 8x + 16 + y^2 - 6y + 9 = 20$ $(x + 4)^2 + (y - 3)^2 = 20$ $\therefore M(-4; 3)$	✓ completing square <i>kwadraatsvoltooing</i> ✓ $(x + 4)^2 + (y - 3)^2 = 20$ ✓ x-coordinate / x-koördinaat ✓ y-coordinate / y-koördinaat	(4)
4.2	$(0 + 4)^2 + (y - 3)^2 = 20$ $(y - 3)^2 = 4$ $y = 3 \pm 2$ $\therefore y = 1$ $Q(0; 1)$	✓ subst. $x = 0$ into circle equation <i>verv. $x = 0$ in sirkel vergelyking</i> ✓ $(y - 3)^2 = 4$ ✓ $y = 1$	(3)
4.3	$m_{\text{radius}} = \frac{3 - 1}{-4 - 0} = -\frac{1}{2}$ $m_{\text{tan/rkl}} = 2$ $y - 1 = 2(x - 0) \quad [\text{tangent } \perp \text{ radius}]$ $\therefore y = 2x + 1 \quad [\text{raaklyn } \perp]$	✓ $m_{\text{rad}} = -\frac{1}{2}$ ✓ $m_{\text{tan}} = 2$ / $m_{\text{raaklyn}} = 2$ ✓ subst. $m_{\text{tan/rkl}} = 2$ and Q into correct form. / <i>verv. $m_{\text{tan/rkl}} = 2$ en Q in korrekte formule.</i> ✓ equation / <i>vergelyking</i>	(4)
4.4	$y = 6$	✓ answer / <i>antwoord</i>	(1)

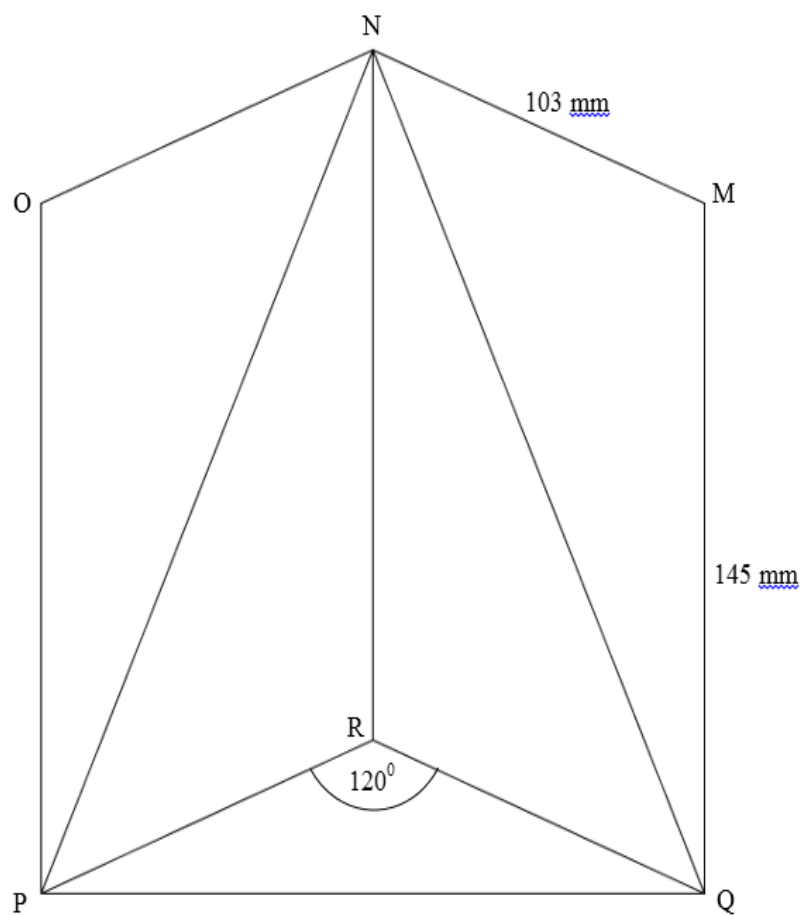
4.5	$6 = 2x + 1$ $x = \frac{5}{2}$ $U\left(\frac{5}{2}; 6\right)$	$\checkmark \quad 6 = 2x + 1$ $\checkmark \quad x = \frac{5}{2}$	(2)
4.6	$m_{AU} = \frac{11-6}{0-\frac{5}{2}}$ $= -2$ $m_{AD} = \frac{6-11}{-10-0}$ $= \frac{1}{2}$ $m_{AU} \times m_{DA} = -2 \times \frac{1}{2}$ $= -1$ $\therefore AU \perp DA$ $\therefore \hat{A} = 90^\circ$ $D\hat{Q}U = 90^\circ$ [tangent $\perp$ radius] / [raaklyn $\perp$ radius] $\therefore$ QUAD is a cyclic quad. [opp. $\angle^s$ add up to 180°] <i>QUAD is 'n koordevierhoek [teenoorst. $\angle^e$ se som is 180°]</i>	$\checkmark \quad m_{AU} = -2$ $\checkmark \quad m_{AD} = \frac{1}{2}$ $\checkmark \quad m_{AU} \times m_{DA} = -1$ $\checkmark \quad \hat{A} = 90^\circ$ $\checkmark \quad D\hat{Q}U = 90^\circ$ $\checkmark \quad R$	(6)
			[20]

QUESTION 5 / VRAAG 5			
			
5.1.1	$\sin(-52^\circ) = -\sin 52^\circ$ $= \sqrt{1-t^2}$	✓ $-\sin 52^\circ$ ✓ ✓ answer	(3)
5.1.2	$\cos(2 \cdot 19^\circ) = \cos 38^\circ$ $2 \cos^2 19^\circ - 1 = \cos 38^\circ$ $\cos 19^\circ = \sqrt{\frac{\cos 38^\circ + 1}{2}}$ $\therefore \cos 19^\circ = \sqrt{\frac{\sqrt{1-t^2} + 1}{2}}$	✓ $\cos(2 \cdot 19^\circ) = \cos 38^\circ$ ✓ $\cos(2 \cdot 19^\circ) = 2 \cos^2 19^\circ - 1$ ✓ simplification / vereenvoudiging ✓ answer / antwoord	(4)
5.2	$\frac{2 \cos(180^\circ + x) \cdot \sin(180^\circ - x) \cdot \sin 74^\circ}{\sin(x + 360^\circ) \cdot \sin 37^\circ \cdot \sin 53^\circ \cdot \sin(x - 90^\circ)}$ $= \frac{2(-\cos x) \cdot \sin x \cdot \sin 74^\circ}{\sin x \cdot \sin 37^\circ \cdot \cos 37^\circ \cdot (-\cos x)}$ $= \frac{2 \sin 74^\circ}{\sin 37^\circ \cdot \cos 37^\circ}$ $= \frac{1}{2} \sin 74^\circ$ $= 4$ OR/OF $\frac{2 \cos(180^\circ + x) \cdot \sin(180^\circ - x) \cdot \sin 74^\circ}{\sin(x + 360^\circ) \cdot \sin 37^\circ \cdot \sin 53^\circ \cdot \sin(x - 90^\circ)}$ $= \frac{2(-\cos x) \cdot \sin x \cdot \sin(2 \times 37^\circ)}{\sin x \cdot \sin 37^\circ \cdot \cos 37^\circ \cdot (-\cos x)}$ $= \frac{2 \sin(2 \times 37^\circ)}{\sin 37^\circ \cdot \cos 37^\circ}$ $= \frac{4 \sin 37^\circ \cdot \cos 37^\circ}{\sin 37^\circ \cdot \cos 37^\circ}$ $= 4$	✓ $-\cos x$ ✓ $\sin x$ ✓ $\sin x$ ✓ $\cos 37^\circ$ ✓ $-\cos x$ ✓ $\frac{1}{2} \sin 74^\circ$ ✓ answer / antwoord ✓ $-\cos x$ ✓ $\sin x$ ✓ $\sin x$ ✓ $\cos 37^\circ$ ✓ $-\cos x$ ✓ $2 \sin(2 \times 37^\circ)$ ✓ answer / antwoord	(7)

5.3.1	$1 - \cos 2x = 0$ $\cos 2x = 1$ $\therefore 2x = 0^\circ + k \cdot 360^\circ$ $x = 0^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$	$\checkmark 1 - \cos 2x = 0$ $\checkmark 2x = 0^\circ + 360^\circ \cdot k$ $\checkmark x = 0^\circ + 180^\circ \cdot k;$ $\checkmark k \in \mathbb{Z}$	(4)
5.3.2	$\begin{aligned} \text{L.H.S/LK} &= \frac{2\sin x}{2(1 - \cos 2x)} \\ &= \frac{2\sin x}{2 - 2\cos 2x} \\ &= \frac{2\sin x}{2 - 2(1 - 2\sin^2 x)} \\ &= \frac{2\sin x}{2 - 2 + 2\sin^2 x} \\ &= \frac{2\sin x}{2\sin^2 x} \\ &= \frac{1}{\sin x} \end{aligned}$	$\checkmark 1 - 2\sin^2 x$ $\checkmark$ removing brackets <i>verwyder hakies</i> $\checkmark 2\sin^2 x$	(3)
			[21]

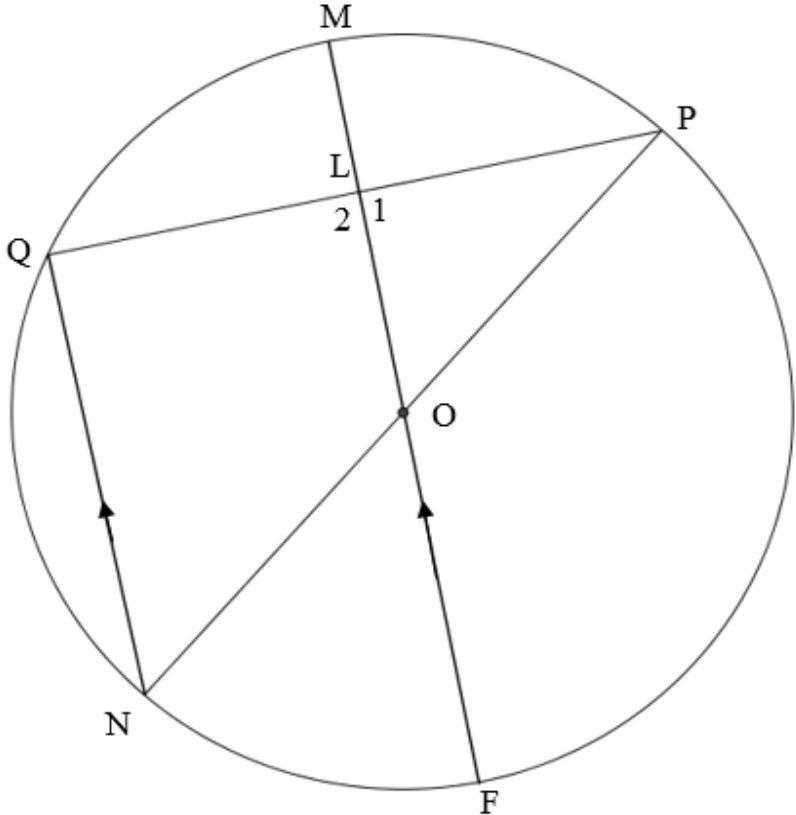
QUESTION 6 / VRAAG 6			
6.1	$\sin(x + 60^\circ) = \sin(90^\circ - 2x)$ $\therefore x + 60^\circ = 90^\circ - 2x + 360^\circ.k$ or/of $x + 60^\circ = 180^\circ - (90^\circ - 2x) + 360^\circ.k$ $3x = 30^\circ + 360^\circ.k$ or/of $x = -30^\circ + 360^\circ.k$ $x = 10^\circ + 120^\circ.k$; $k \in \mathbb{Z}$ $\therefore x = -30^\circ ; 10^\circ ; 130^\circ$ OR/OF $\cos 2x = \cos(30^\circ - x)$ $2x = 30^\circ - x + 360^\circ.k$ or $2x = -(30^\circ - x) + 360^\circ.k$; $k \in \mathbb{Z}$ $3x = 30^\circ + 360^\circ.k$ or $x = -30^\circ + 360^\circ.k$ $\therefore x = -30^\circ ; 10^\circ ; 130^\circ$	✓ co-ratio / ko-funksie ✓ both gen. solns <i>beide algemene oplossings</i> ✓ -30° ✓ 10° ✓ 130° ✓ co-ratio / ko-funksie ✓ both gen. solns <i>beide algemene oplossings</i> ✓ -30° ✓ 10° ✓ 130°	(5)
6.2		g: ✓ x-intercept. <i>x-afsnit</i> ✓ y-intercept. <i>y-afsnit</i> ✓ shape / vorm f: ✓ x-intercept. <i>x-afsnit</i> ✓ y-intercept. <i>y-afsnit</i> ✓ shape / vorm	(6)
6.3	240°	✓ answer / antwoord	(1)
6.4	$h(x) = \cos(2x - 90^\circ) - 1 = \sin 2x - 1$	✓ substitution / vervanging ✓ $\sin 2x - 1$	(2)
			[14]

QUESTION 7 / VRAAG 7

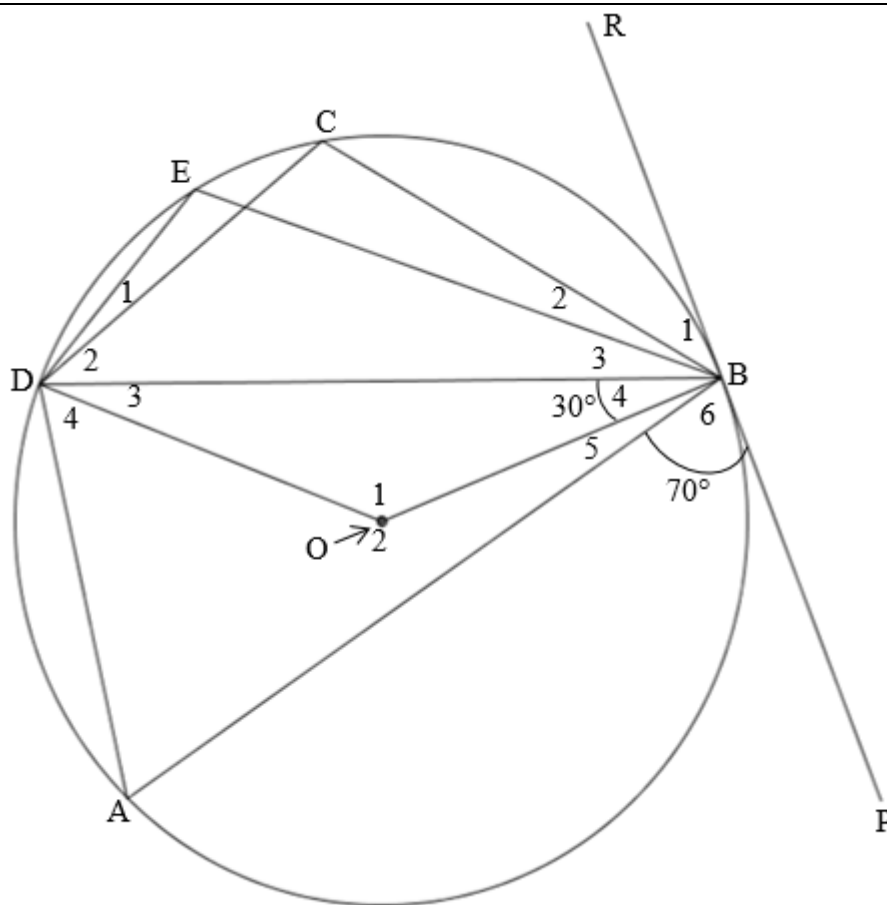


7.1	$NP^2 = ON^2 + OP^2$ $= (103)^2 + (145)^2$ $\therefore NP = 177,86$	✓ using Pyth theorem correctly <i>korrekte gebruik van stelling van Pythagoras</i> ✓ answer / <i>antwoord</i>	(2)
7.2	$PQ^2 = (103)^2 + (103)^2 - 2(103)(103)\cos(120^\circ)$ $PQ = 178,40$	✓ subst. into cosine rule <i>verv. in cosinus formule</i> ✓ answer / <i>antwoord</i>	(2)
7.3	$\cos \hat{N} = \frac{(177,86)^2 + (177,86)^2 - (178,40)^2}{2(177,86)(177,86)}$ $\therefore \hat{N} = 60,20^\circ$	✓ substitution / <i>vervangings</i> ✓ $\hat{N}$	(2)
[6]			

QUESTION 8 / VRAAG 8

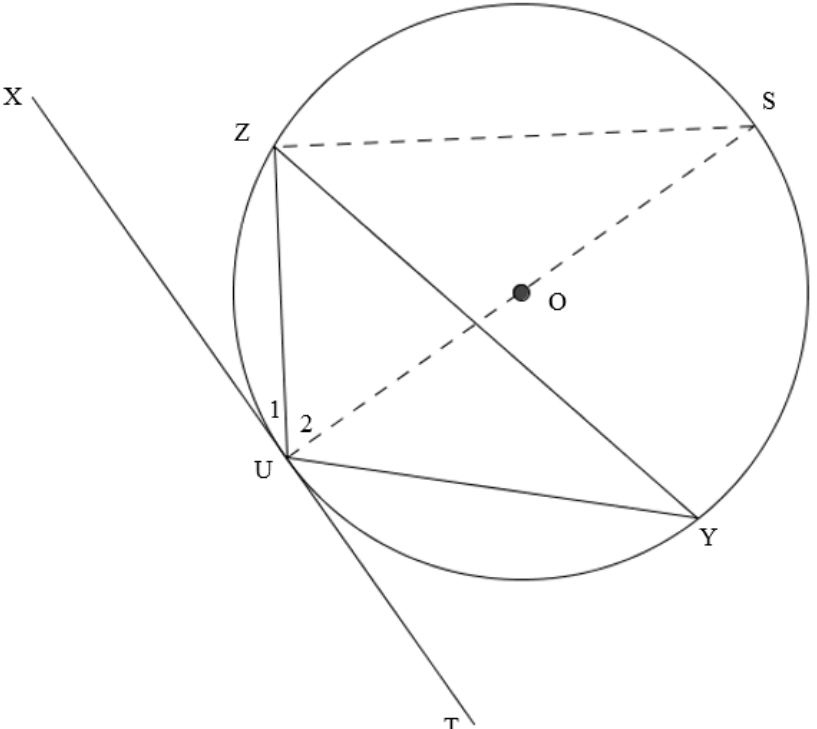
8.1	Bisects the chord / <i>Halveer die koord</i>	✓ answer / <i>antwoord</i>	(1)
8.2			
8.2.1	$\hat{Q} = 90^\circ$ [$\angle$ in semi circle.] / [$\angle$ in halwe sirkel] $\hat{L}_1 = \hat{Q} = 90^\circ$ [corresp. $\angle^s$, $QN \parallel LO$] <i>[ooreenk. $\angle^e$, $QN \parallel LO$]</i> $QL = LP$ [line from centre perp. to chord] <i>[lynstuk vanaf middelpnt is loodreg op koord]</i>	✓ S ✓ R ✓ S/R ✓ R	(4)
8.2.2	$MF = 8ML$	✓	(1)
8.2.3	$OP^2 = OL^2 + LP^2$ $(4ML)^2 = (3ML)^2 + 7^2$ $16ML^2 = 9ML^2 + 49$ $7ML^2 = 49$ $\therefore ML = \sqrt{7}$	✓ using Pyth correctly <i>korrekte gebruik van Pythagoras</i> ✓ simplification / <i>vereenvoudiging</i> ✓ ML	(3)
			[9]

QUESTION 9 / VRAAG 9

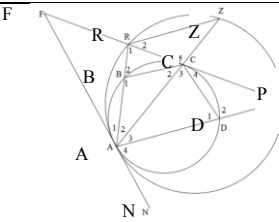


9.1	$\hat{D}_3 = 30^\circ$ [$\angle$ s opp equal sides] / [$\angle$ teenoor gelyke sye] $\therefore \hat{O}_1 = 120^\circ$ [sum of $\angle$ s of a Δ .] / [som v/d $\angle$ van 'n Δ]	✓ S ✓R ✓ S/R	(3)
9.2	$\hat{A} = 60^\circ$ [$\angle$ at centre = 2 $\angle$ at circumf.] / [middelpunts $\angle = 2 \times$ omtrekshoek]	✓ S ✓R	(2)
9.3	$\hat{C} = 120^\circ$ [opp. $\angle$ s of cyclic quad.] / [teenoorst. $\angle$ van k.v]	✓ S ✓R	(2)
9.4	$\hat{A}DB = 70^\circ$ [tan chord theorem.] / [raaklyn koord stelling]	✓ S ✓R	(2)
			[9]

QUESTION 10 / VRAAG 10

10.1		✓ constr. konstr.	
	$\hat{U}_2 = 90^\circ - \hat{U}_1$ [tan $\perp$ radius.] / [<i>raaklyn $\perp$ radius</i>] $\hat{Z} = 90^\circ$ [$\angle$ in semi circle] / [$\angle$ in halwe sirkel] $\hat{S} = 180^\circ - (90^\circ - \hat{U}_1) - 90^\circ$ [sum of $\angle$ s of a Δ] [som van die $\angle$e van 'n Δ] $\therefore \hat{S} = \hat{U}_1$ $\hat{S} = \hat{Y}$ [$\angle$ s in same segment] / [$\angle$ e in dieselfde segment] $\therefore \hat{U}_1 = \hat{Y}$	✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)

10.2			
10.2.1	$\hat{A}_2 = x$ [∠s opp. = sides] / [<i>teenoor gelyke sye</i>] $\hat{C}_2 = \hat{A}_2 = x$ [tan chord theo.] / [<i>raaklyn koord stelling</i>] $\hat{A}_1 = \hat{C}_1$ [tan chord theo.] / [<i>raaklyn koord stelling</i>] $\hat{C}_2 = \hat{S}_1$ [alt. ∠s, CB ∥ TS] / [<i>verw. ∠, CB ∥ TS</i>] $\hat{C}_1 = \hat{T}$ [corresp ∠s, CB ∥ TS] / [<i>ooreenk. ∠, CB ∥ TS</i>]	✓ S/R ✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)
10.2.2	$\hat{S}_1 = \hat{T} = x$ [proven in 10.2.1] / [<i>bewys in 10.2.1</i>] $\therefore CS = CT$ [sides opp. = ∠s] / [<i>sye teenoor gelyke hoeke</i>]	✓ S ✓ R	(2)
10.2.3	$\frac{AR}{BR} = \frac{AT}{CT}$ [line ∥ to one side of a Δ] [lynstuk ∥ aan een sy van 'n Δ] $CS = CT$ [proved in 10.2.2] / [<i>bewys in 10.2.2</i>] $AT = \frac{3}{2} \times 4$ $AT = 6\text{cm}$	✓ S/R ✓ CS = CT ✓ substitution <i>vervanging</i> ✓ AT	(4)
			[16]

QUESTION 11 / VRAAG 11			
			
11.1	$\hat{A}_1 = \hat{C}_2$ [tan chord theo.] / [raaklyn koord stelling] $= \hat{Z}$ [tan chord theo.] / [raaklyn koord stelling] $\therefore \hat{C}_2 = \hat{Z}$ [both = x] / [beide = x] $\therefore BC \parallel RZ$ [corresp. $\angle$ s =] / [ooreenk. $\angle$ e =]	$\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ R	(4)
11.2	$\hat{Z} = \hat{P}$ [$\angle$ s in same segment] / [$\angle$ e in dieselfde segment] $= \hat{C}_2$ [corresp. $\angle$ s ; $BC \parallel RZ$] / [ooreenk. $\angle$ e ; $BC \parallel RZ$] $\therefore BC$ is a tangent to circle ACP [conv. of tan chord theorem] BC is 'n raaklyn aan sirkel ACP [omgekeerde van raaklyn koord stelling]	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R	(3)
11.3	$\hat{B}_1 = \hat{D}_2$ [ext. $\angle$ of a cyclic quad.] [buite $\angle$ van koordevierhoek] $\hat{R} = \hat{B}_1$ [corresp. $\angle$ s , $BC \parallel RZ$] [ooreenk. $\angle$ e , $BC \parallel RZ$] $\therefore \hat{R} = \hat{D}_2$ $\hat{Z} = \hat{P}$ [$\angle$ s in same segment] [$\angle$ e in dieselfde segment] $\hat{A}_2 = \hat{C}_4$ [3^{rd} $\angle$] / [3^{de} $\angle$] $\therefore \triangle RZA \parallel \triangle DPC$ [equiangular or $\angle\angle\angle$] [gelykhoekig of $\angle\angle\angle$]	$\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S/R $\checkmark$ R	(5)
11.4	$\frac{ZA}{PC} = \frac{RA}{DC}$ [similar Δ^s] / [gelykvormige Δ^e] $\therefore RA = \frac{ZA \times DC}{PC} \dots (1)$ $\frac{AR}{AB} = \frac{AZ}{AC}$ [line $\parallel$ to one side of a Δ] [lynstuk $\parallel$ aan een sy van 'n Δ] $\therefore AR = \frac{AZ \times AB}{AC} \dots (2)$ $\frac{ZA \times DC}{PC} = \frac{AZ \times AB}{AC}$ $\therefore \frac{DC}{PC} \times \frac{AC}{AB} = 1$	$\checkmark$ S/R $\checkmark$ $RA = \frac{ZA \times DC}{PC}$ $\checkmark$ S/R $\checkmark$ $RA = \frac{AZ \times AB}{AC}$ $\checkmark$ simplification	(5)
			[17]
		TOTAL/TOTAAL:	150