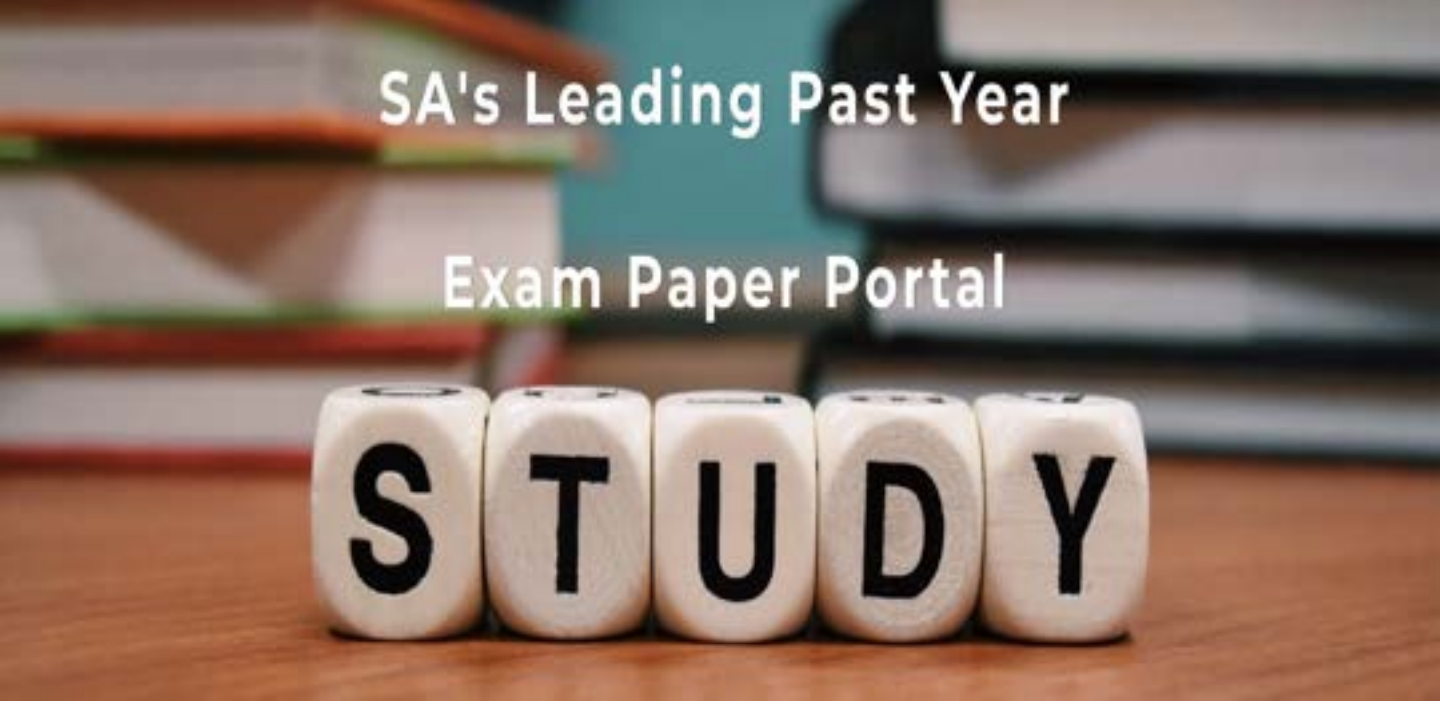


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS PAPER 2/
WISKUNDE VRAESTEL 2

MEMORANDUM

MARKS/PUNTE: 150

TIME/TYD: 3 HOURS/URE

This memorandum consists of 14 pages.

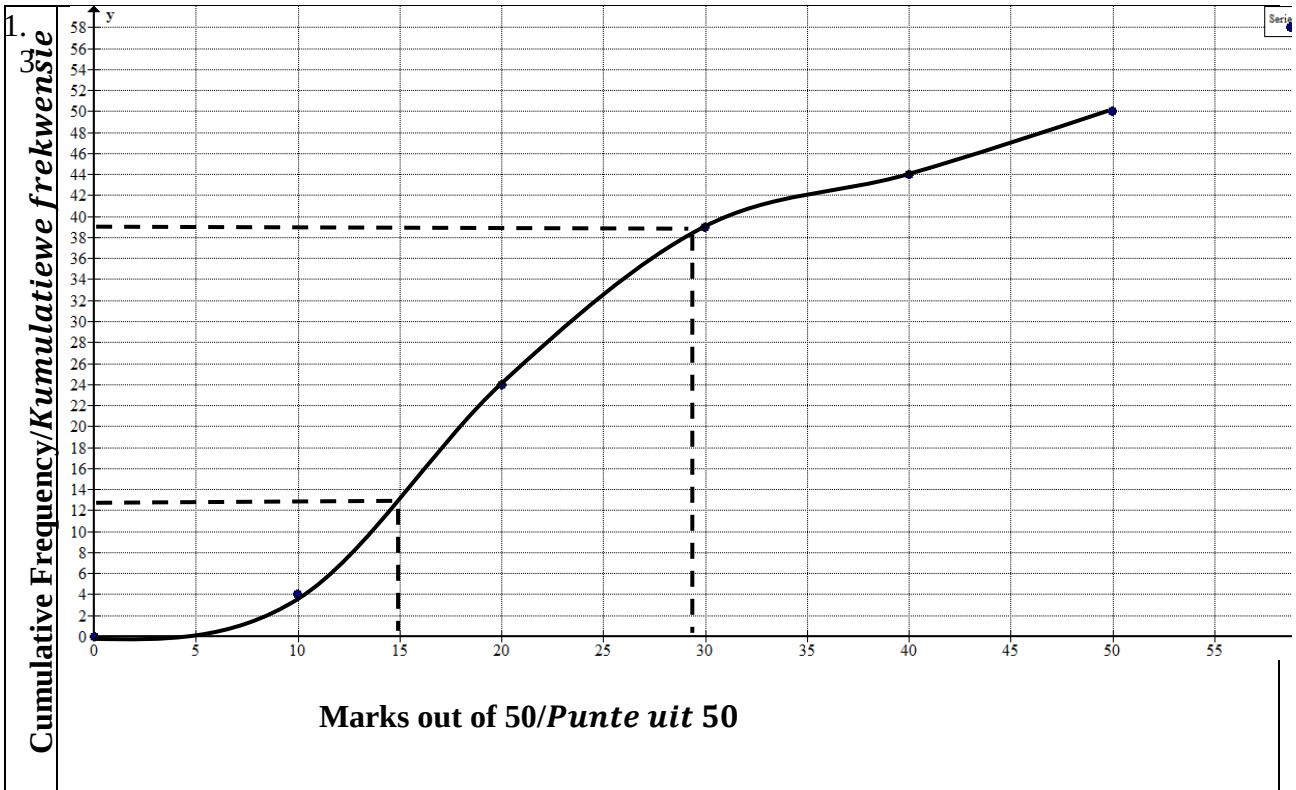
Hierdie memorandum bestaan uit 14 bladsye

QUESTION/*VRAAG* 1

1.1	$\bar{x} = \frac{1140}{50} = 22{,}8$	✓ $\frac{1140}{50}$ ✓ answer/ *antwoord* (2)
1.2		✓4;24;39 ✓44;50 (2)

Marks	Frequency	Cumulative Frequency
$0 \leq x < 10$	4	4
$10 \leq x < 20$	20	24
$20 \leq x < 30$	15	39
$30 \leq x < 40$	5	44
$40 \leq x < 50$	6	50

1.3



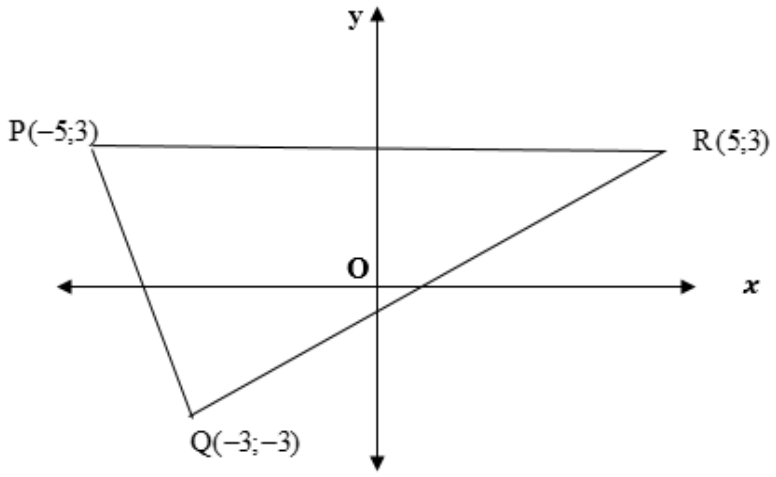
✓(0;0) ✓upper limits/*boonste grense*
✓all points correct /*al die punte korrek* (3)

1.4	$Q_1 = 15$ (accept 14 to16)/ (aanvaar van 14 tot 16) $Q_3 = 29$ (accept 28 to 30)/ (aanvaar van 28 tot 30) IQR/IKO = $29 - 15 = 14$	✓ $Q_1 = 15$ ✓ $Q_3 = 29$ ✓answer /*antwoord*(3)
		[10]

QUESTION/*VRAAG* 2

2.1	$\bar{x} = \frac{143}{10} = 14{,}3$ seconds/*sekondes*. δx = 2,87 seconds/*sekondes*.	✓ $\bar{x} = \frac{143}{10}$ ✓ 14,3 ✓ δx = 2,87 ✓ accuracy /*akkuraat* (4)
2.2	a = 17,93 b = − 0,07 $\hat{y} = -0{,}07x + 17{,}93$	✓ a ✓ b ✓ equation/ *vergelyking* (3)
2.3	$r = -0{,}69$	✓ $r = -0{,}69$ (1)
2.4	$y = -0{,}07(80) + 17{,}93$ y = 2,33 seconds.	✓ substitute/*vervang* ✓ answer/*antwoord* (2)
		[10]

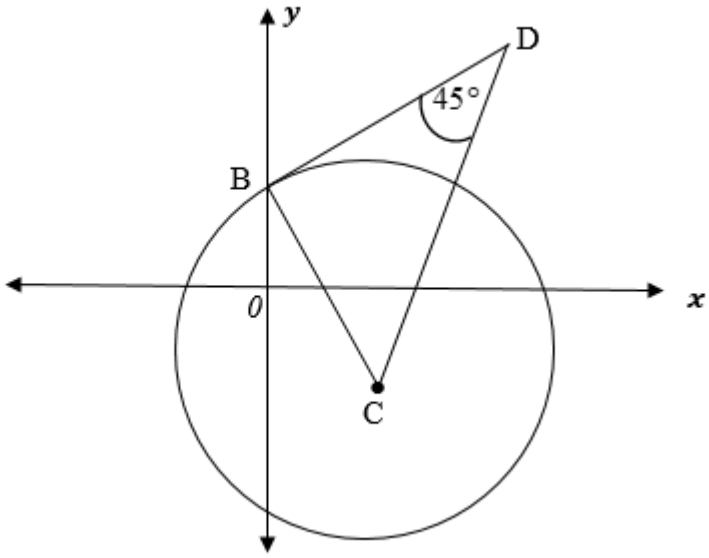
QUESTION/*VRAAG* 3



3.1	QR = $\sqrt{(5+3)^2 + (3+3)^2}$ $= \sqrt{64+36}$ $= \sqrt{100}$ =10	✓ substitution/ *vervang* ✓ 10 (2)

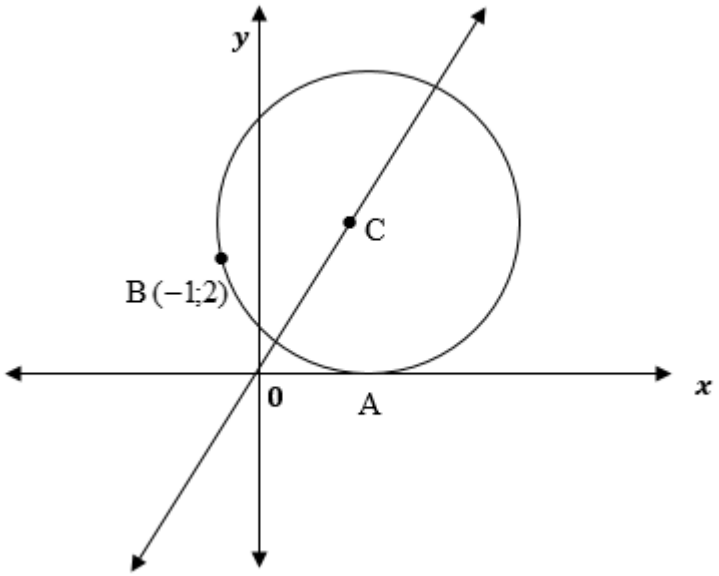
3.2	$M(\frac{5-3}{2};\frac{3-3}{2})$ $=M(1;0)$	✓ x-value/$waarde$ ✓ y-value /$waarde$ (2)
3.3	$P(-5;3)$ and $M(1;0)$: $m=\frac{0-3}{1+5}$ $=\frac{-1}{2}$ $y-0=-\frac{1}{2}(x-1)$ $y=-\frac{1}{2}x+\frac{1}{2}$	✓ m ✓subst of m and point/ $vervang\ m\ en\ punt$ ✓equation/$vergelyking$ (3)
3.4	$r=5$; centre $(1;0)$: $(x-1)^2+y^2=25$	✓ r = 5 and/en (1;0) ✓ LHS/LK ✓RHS/RK (3)
3.5	PM = $\sqrt{(1+5)^2+(-3)^2}$ $=\sqrt{45}$ $>\sqrt{25}$ $\therefore$ P lies OUTSIDE the circle.	✓ PM = $\sqrt{45}$ ✓$>\sqrt{25}$ ✓conclusion/$gevolgtrekking$ (3)
3.6	S(3;9)	✓ x-value/$waarde$ ✓ y −value/$waarde$ (2)
3.7	$m_{PQ}=\frac{3+3}{-5+3}=-3$ $\tan\theta=-3$ $\theta=180°-71{,}57°$ $=108{,}43°$ $\beta=71{,}57°$ co-interior angles, PR $\parallel$ x-axis.	✓ $m_{PQ}=-3$ ✓ $\tan\theta=-3$ ✓ $\theta=108{,}43°$ ✓ $\beta=71{,}57°$ (4)
		[19]

QUESTION/*VRAAG* 4



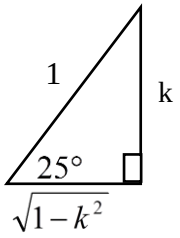
4.1.1	C(3; – 2)	✓ x-value/$waarde$ ✓ y-value/$waarde$ (2)
4.1.2	$-4y+8=0$ $y=2$ B(0;2)	✓ x-value/$waarde$ ✓ y-value/$waarde$(2)
4.1.3	$r^2=(3-0)^2+(-2-2)^2=25$ $r=5$ $x=3-5=-2$ $x=3+5=8$	✓ $r^2=25$ ✓ r = 5 ✓ $x=-2$ ✓ $x=8$ (4)
4.1.4	Let $D(x;y)$: $(x-0)^2+(y-2)^2=25$ and $y=\frac{3}{4}x+2$ $x^2+(\frac{3}{4}x+2-2)^2=25$ $x^2+\frac{9}{16}x^2=25$ $\frac{25x^2}{16}=25$ $x^2=16$ $x=-4$ *or* $x=4$ n/a $y=\frac{3}{4}(4)+2=5$ $D(4;5)$	✓ subst in distance formula/ $vervang\ in\ afstandformule$ ✓ $y=\frac{3}{4}x+2$ ✓ replace y/$Vervang\ y$ ✓ $x^2=16$ ✓ x-value/ $waarde$ ✓ y-value/$waarde$ (6)

4.2



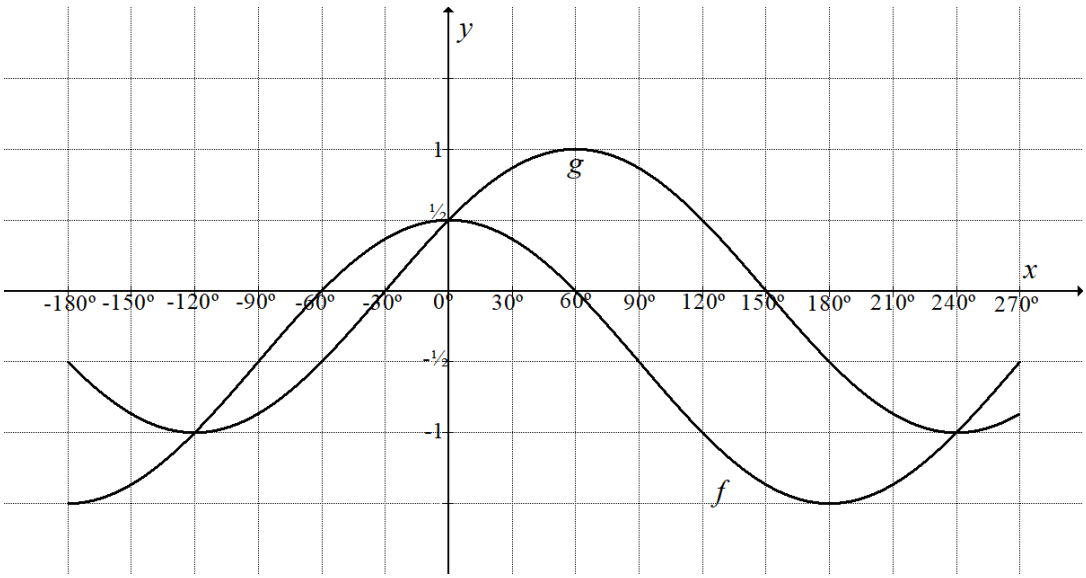
4.2.1	Let $C(x;2x)$ and $A(x;0)$ $(x+1)^2+(2x-2)^2=(x-x)^2+(2x-0)^2$ $x^2+2x+1+4x^2-8x+4=4x^2$ $x^2-6x+5=0$ $(x-1)(x-5)=0$ $x=1$ or $x=5$ $C(1;2)$ or $C(5;10)$	✓coordinates of C and A/ *koördinate A en C* ✓equating two r^2 / *Stel twee r^2 gelyk* ✓standard form/ *standaardvorm* ✓factors/*faktore* ✓ $C(1;2)$ ✓ $C(5;10)$ (6)
4.2.2	$r=2$ or $r=10$	✓ $r=2$ ✓ $r=10$ (2)
		[22]

QUESTION/*VRAAG* 5

5.1.1	$y^2 = 1^2 - (\sqrt{1-k^2})^2$ $= 1 - 1 + k^2$ $= k^2$ $y = k$  $\sin 25° = k$	✓diagram and/*en* Pythagoras ✓ $\sin 25° = k$ (2)
5.1.2	$\sin 50° = 2\sin 25° \cos 25°$ $= 2k\sqrt{1-k^2}$	✓double angle expansion/ *Brei dubbelhoek uit* ✓substitution/*vervang* (2)
5.2	$\sqrt{\dfrac{\tan(-207°)}{\tan 333°} - \dfrac{\sin^2(x-360°)}{\sin(x-90°)\cos x}}$ $= \sqrt{\dfrac{-\tan 27°}{-\tan 27°} - \dfrac{\sin^2 x}{-\cos x.\cos x}}$ $= \sqrt{1 + \dfrac{\sin^2 x}{\cos^2 x}}$ $= \sqrt{\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}}$ $= \sqrt{\dfrac{1}{\cos^2 x}}$ $= \dfrac{1}{\cos x}$	✓ $-\tan 27°$ ✓ $-\tan 27°$ ✓ $\sin^2 x$ ✓ $-\cos x$ ✓ $= \sqrt{\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}}$ ✓ $\sqrt{\dfrac{1}{\cos^2 x}}$ ✓answer /*antwoord* (7)
5.3	RHS: $\dfrac{\sin 2A}{\cos 2A} \times \dfrac{\cos A}{\sin A}$ $= \dfrac{2\sin A\cos A.\ \cos A}{\sin A.\cos 2A}$ $= \dfrac{2\cos^2 A}{\cos 2A}$ LHS: $\dfrac{1 + 2\cos^2 A - 1}{\cos 2A}$ $= \dfrac{2\cos^2 A}{\cos 2A}$ ∴ LHS = RHS	✓replace/*vervang* $\tan 2A$ ✓expansion of $\sin 2A$ /*uitbrei van* $\sin 2A$ ✓ $\dfrac{2\cos^2 A}{\cos 2A}$ ✓ replacing $\cos 2A$ in numerator/ *Vervang* $\cos 2A$ *in teller* ✓ $\dfrac{2\cos^2 A}{\cos 2A}$ (5)

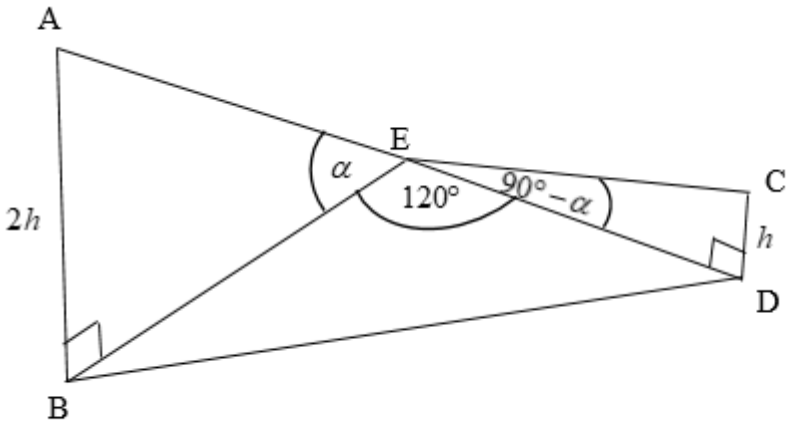
5.4	$2\sin 2x = -\cos 2x$ $\frac{2\sin 2x}{\cos 2x} = \frac{-\cos 2x}{\cos 2x}$ $2\tan 2x = -1$ $\tan 2x = -\frac{1}{2}$ $2x = 153{,}43° + k180°$ $x = 76{,}72° + k.90°, k \in Z$	✓ dividing by $\cos 2x$ / *Deel deur* $\cos 2x$ ✓ $\tan 2x = -\frac{1}{2}$ ✓ $2x = 153{,}43° + k180°$ ✓ $x = 76{,}72° + k.90°$ ✓ $k \in Z$ (5)
		[22]

QUESTION/*VRAAG* 6



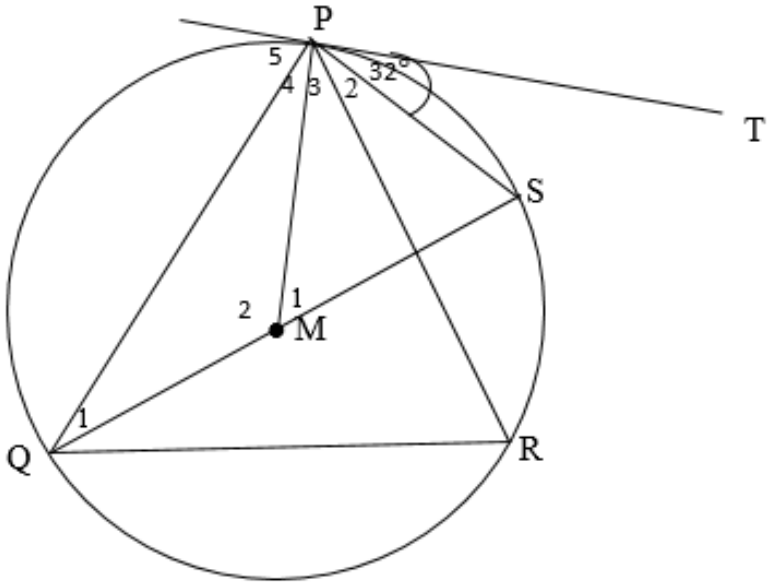
6.1	Graph done/*Grafiek geskets*	g: ✓ intercepts/*as – afsnitte* ✓turning points/*draaipunte* ✓shape/*vorm* (3)
6.2.1	$x \in (-120°; 0°) \cup (240°; 270°]$	✓ $(-120°; 0°)$ ✓ $(240°; 270°]$ ✓ notation /*notasie* (3)
6.2.2	$x \in [-180°; -60°] \cup [-30°; 60°] \cup [150°; 270°]$	✓ $x \in [-180°; -60°]$ ✓ $[-30°; 60°]]$ ✓ $[150°; 270°]$ (3)
6.3	$y = \sin(x + 30° - 120°)$ $= \sin(x - 90°)$ $= -\cos x$ $= -\cos x$	✔ $y = \sin(x + 30° - 120°)$ ✔ answer/*antwoord* (2)
		[11]

QUESTION/*VRAAG* 7



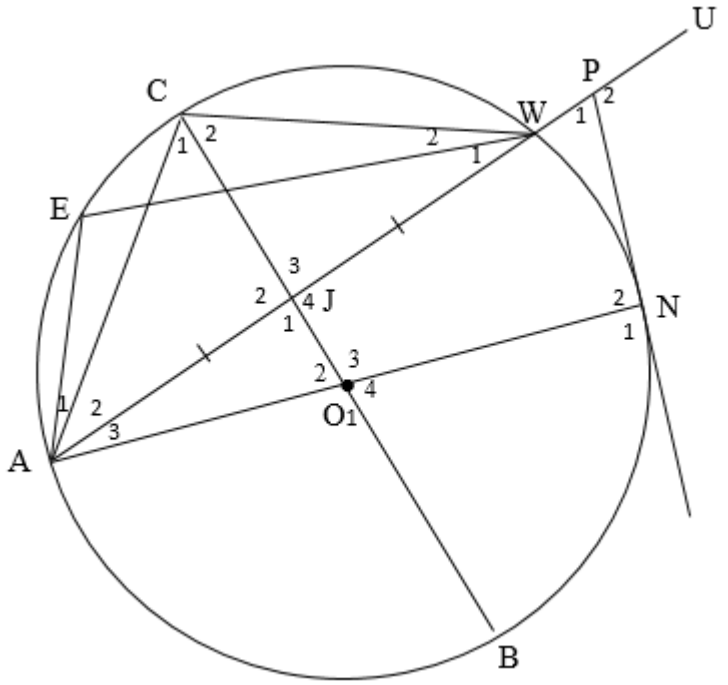
7.1	$\tan\alpha = \dfrac{2h}{BE}$ BE = $\dfrac{2h}{\tan\alpha}$	✓ ratio/*trig verhouding* ✓ BE = $\dfrac{2h}{\tan\alpha}$ (2)
7.2	$\tan\alpha = \dfrac{ED}{h}$ ∴ED = $h\tan\alpha$	✓ $\tan\alpha$ ✓ correct ratio /*korrekte verhouding* (2)
7.3	$BD^2 = BE^2 + ED^2 - 2BE.DE.\cos B\hat{E}D$ $= \left(\dfrac{2h}{\tan\alpha}\right)^2 + (h\tan\alpha)^2 - 2\left(\dfrac{2h}{\tan\alpha}\right)(h\tan\alpha)(\cos 120°)$ $= \dfrac{4h^2}{\tan^2\alpha} + h^2\tan^2\alpha - 4h^2(-\cos 60°)$ $= \dfrac{4h^2}{\tan^2\alpha} + h^2\tan^2\alpha - 4h^2\left(-\dfrac{1}{2}\right)$ $= \dfrac{4h^2}{\tan^2\alpha} + h^2\tan^2\alpha + 2h^2$ $= \dfrac{4h^2 + h^2\tan^4\alpha + 2h^2\tan^2\alpha}{\tan^2\alpha}$ $= \dfrac{h^2(4 + \tan^4\alpha + 2\tan^2\alpha)}{\tan^2\alpha}$ $= \dfrac{h\sqrt{\tan^4\alpha + 2\tan^2\alpha + 4}}{\tan^2\alpha}$	✓ Correct use of cos rule/*korrekte vervanging in kos − reël* ✓ substitution/*vervanging* ✓ simplification/*vereenvoudig* ✓ simplify /*vereenvoudig* ✓ h^2 taken out as common factor/ *haal h^2 gemeenskaplik uit* (5)
		[9]

QUESTION/*VRAAG* 8



8.1	$\hat{Q}_1 = 32°$ (tan-chord theorem/ $\angle$ *tuss rkl en krd*)	✓S✓R	(2)
8.2	$\hat{P}_4 = \hat{Q}_1 = 32°$ ($\angle$'s opp equal sides/ $\angle$'*e teenoor gelyke sye*)	✓S✓R	(2)
8.3	$\hat{M}_1 = 64°$ ($\angle$ at centre is 2× $\angle$ at circumference/ *middelpunts* $\angle$ = 2× *omtreks* $\angle$)	✓S✓R	(2)
8.4	$\hat{M}_2 = 116°$ ($\angle$'*s* on a str line / $\angle$*e op 'n rt lyn*) $\hat{R} = 58°$ ($\angle$ at centre is 2× $\angle$ at circumference/ *middelpunts* $\angle$ = 2× *omtreks* $\angle$)	✓S✓R ✓S✓R	(4)
			[10]

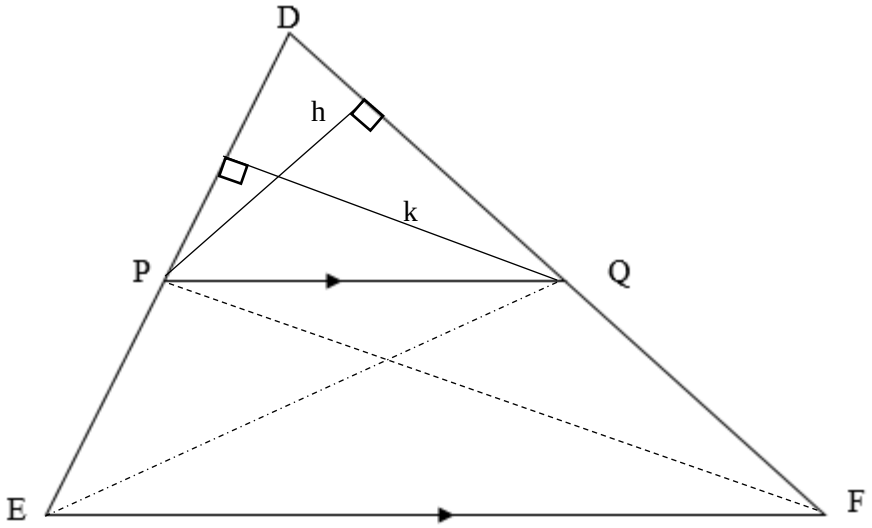
QUESTION/*VRAAG* 9



9.1	$\hat{J}_3 = 90°$ (line from centre to midpoint/ (*lyn van mdpt van sirkel na mdpt van koord*) $N_2 = 90°$ (tangent ⊥to radius/ *rkl* ⊥ *r*) $\hat{J}_3 = \hat{N}_2$ ONPJ is a cyclic quad /koordevierhoek (CONVERSE of ext.∠of cyclic quad/ *OMG van buite∠ van koordevierhoek*)	✓S ✓R ✓S ✓R ✓S ✓ R (6)
9.2	$\hat{O}_1 = 2\hat{C} = 2x$ (∠ at centre =2× ∠ at circumference *middelpunts ∠ = 2× omtreks∠*) $\hat{O}_4 = 180° - 2x$ (∠'s on a str line/∠ *e op 'n rt lyn*) $\hat{P}_1 = 180° - 2x$ (ext. ∠ of a cyclic quad/ *buite ∠ van koordevierhoek*)	✓S ✓R ✓S✓R ✓S✓R (6)
		[12]

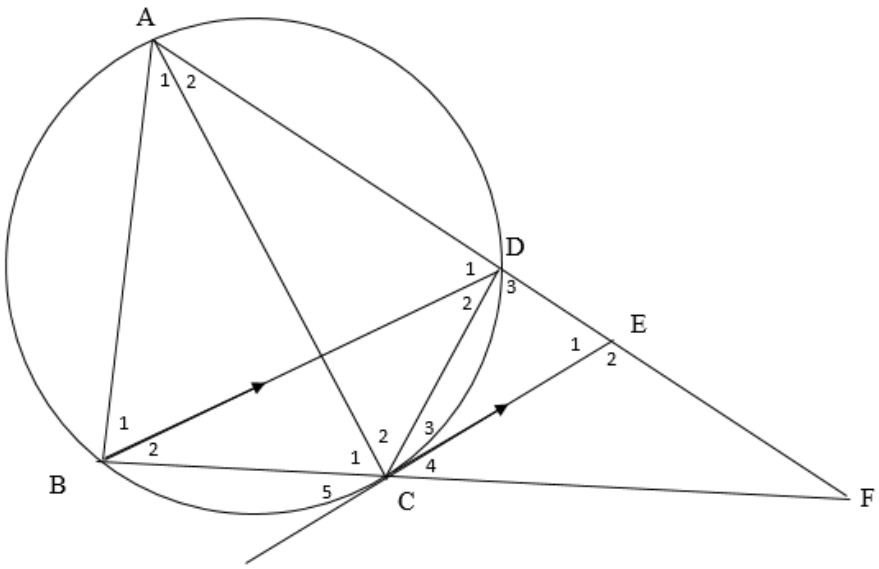
QUESTION/*VRAAG* 10

10.1



10.1	Construction: Join PQ and PF and draw altitudes h and k/ *Konstruksie: Verbind PQ en PF en trek hoogtelyne h en k*	✓construction/ *konstruksie*
	$\frac{area\Delta DPQ}{area\Delta PEQ} = \frac{\frac{1}{2}DP \times h}{\frac{1}{2}PE \times h} = \frac{DP}{PE}$	✓S
	$\frac{area\Delta DPQ}{area\Delta PQF} = \frac{\frac{1}{2}DQ \times k}{\frac{1}{2}QF \times k} = \frac{DQ}{QF}$	✓S
	But the $area\Delta PQE = area\ \Delta PQF$ (same base and same height/ *dieselfde basis en hoogte*)	✓S✓R
	$\therefore \frac{area\Delta DPQ}{area\Delta PEQ} = \frac{area\Delta DPQ}{area\Delta PQF}$	✓S
	$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$	
		(6)

10.2



10.2.1	$\hat{C}_3 = \hat{B}_2$ (tan-chord theorem/$\angle$ *tuss rkl en krd*) $\hat{C}_3 = \hat{D}_2$ (alternate/ *verwisselende* $\angle$'s; BD∥CE) $\hat{B}_2 = \hat{D}_2$ BC = DC (sides opposite equal $\angle$'s/ *sye teenoor* $= \angle$e)	✓S✓R ✓S ✓R (4)
10.2.2	In ΔBAF and/ en ΔDCF: $\hat{B}_1 + \hat{B}_2 = \hat{D}_3$ (ext. $\angle$ of a cyclic quad/ *buite* $\angle$ *van kvh*) $\hat{A}_1 + \hat{A}_2 = \hat{C}_3 + \hat{C}_4$ (ext. $\angle$ of a cyclic quad/ *buite* $\angle$ *van kvh*) $\hat{F}$ is common/ *gemeenskaplik* ∴ ΔBAF ||| ΔDCF ($\angle\angle\angle$)	✓S✓R ✓S ✓R (4)
10.2.3	$\frac{BA}{DC} = \frac{AF}{CF}$ (similar/ *gelykvormige* Δ's) $\frac{BA}{AF} = \frac{DC}{CF}$ $\frac{BA}{AF} = \frac{BC}{CF}$ (BC = DC) But $\frac{BC}{CF} = \frac{DE}{EF}$ (BD ∥ CE; prop theorem/*eweredigh st*) ∴ $\frac{BA}{AF} = \frac{DE}{EF}$	✓S ✓S ✓Replacing/*vervang* DC ✓S✓R (5)

10.2.4	In ΔECD and ΔEAC: $\hat{E}_1$ s common/*gemeenskaplik* $\hat{C}_3 = \hat{A}_2$ (tan-chord theorem/∠ *tuss rkl en krd*) $\hat{D}_3 = \hat{C}_2 + \hat{C}_3$ (∠'s of a Δ/ ∠'e van 'n Δ) ∴ ΔECD ||| ΔEAC (∠∠∠)	✓S✓R ✓S ✓R (4)
10.2.5	$\frac{EC}{EA} = \frac{ED}{EC}$ (||| Δ's) $EC^2 = EA.ED$ But $ED = \frac{BA.EF}{AF}$ (from/ *uit* 10.2.3) $EC^2 = \frac{EA.BA.EF}{AF}$	✓S ✓S ✓S (3)
		[26]

TOTAL/*TOTAAL*: 150