LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

SEKHUKHUNE SOUTH DISTRICT

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

MEMORANDUM

PRE-TRIAL 2021

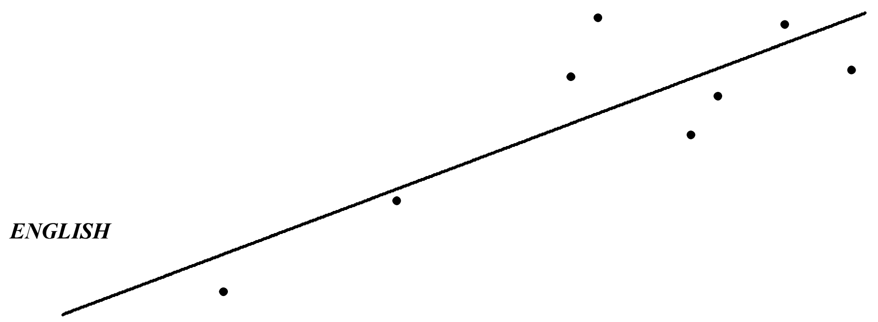
MARKS: 150

This memorandum consists of 12 pages.

QUESTION 1

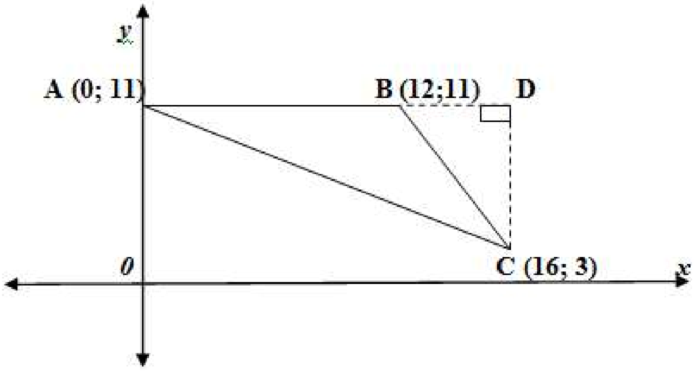
1.1

MATHS LIT VS ENGLISH



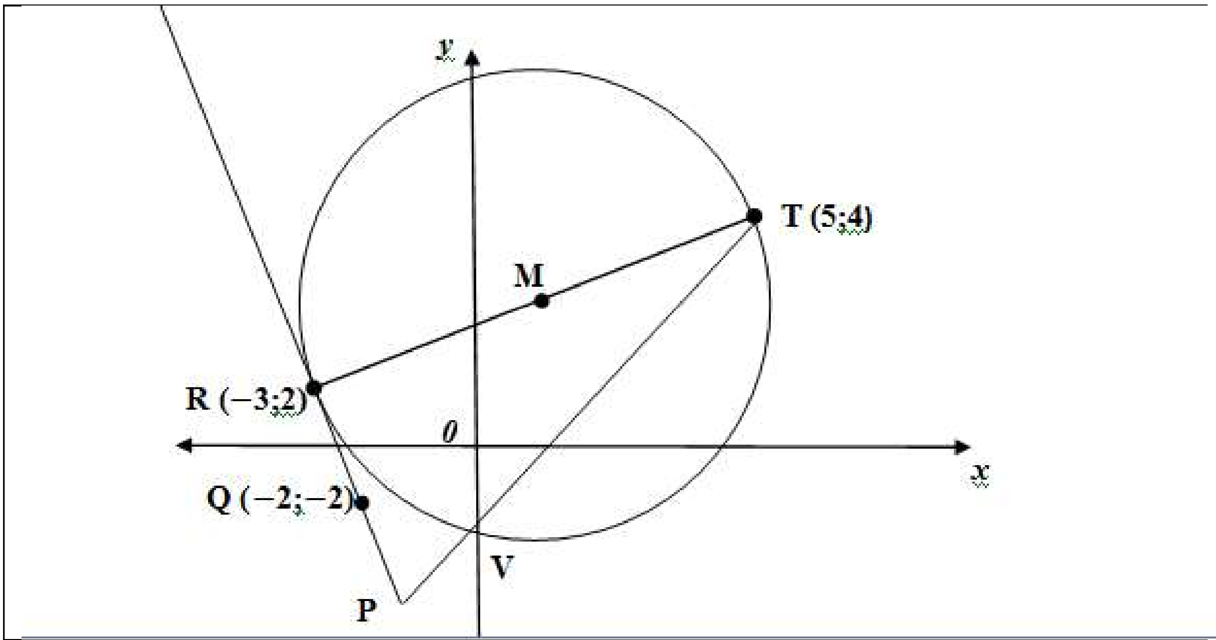
1.1.1	$\bar{x}=\frac{324}{8}$ $= 40{,}5$	✓$\frac{324}{8}$ ✓ 40,5 (2)
1.1.2	$\delta = 14{,}5688$ $= 14{,}57$	✓ ✓ accuracy (2)
1.2	$(40{,}5 - 14{,}57; 40{,}5 + 14{,}57)$ (25,93; 55,07) ∴ 5 learners.	✓method ✓(25,93; 55,07) ✓5 (3)
1.3	See scatter plot above	✓ 2-4 points ✓✓ 5-7pts correct ✓✓✓ all pts correct (3)
1.4	a =16,89 b = 0,75 y = 16,89 +0,75x	✓ a ✓ b ✓equation (3)
1.5	See above	✓ positive gradient ✓c-value betw 15 and 20 (2)
1.6	r = 0,82 It is a strong positive relationship	✓r = 0,82 ✓strong ✓positive (3)
1.7	54,81%	✓ ✓accuracy (2)
		[20]

QUESTION 2



2.1	$y = 11$ AB = 12	✓✓y = 11 ✓ AB = 12 (3)
2.2	D(16;11)	✓✓ (2)
2.3	M(8; 7)	✓✓ (2)
2.4	$m_{AC} = \frac{3-11}{16} = -\frac{8}{16} = -\frac{1}{2}$ $m_{line} = 2$ $y - 7 = 2(x - 8)$ $y = 2x - 9$	✓$-\frac{1}{2}$ ✓$m_{line} = 2$ ✓substitution ✓ equation (4)
2.5	$y = 2(12) - 9$ $= 15$ $\neq 11$ No, it does not pass through B	✓substitution ✓ $\neq 11$ No, it does not pass through B (2)
2.6	$\tan\theta = m_{BC} = \frac{11-3}{12-16}$ $\tan\theta = -2$ $\theta = 116{,}57°$	✓tan θ ✓-2 ✓116,57° (3)
2.7	$m_{new\ line} = -\frac{1}{2}$ $y - 11 = -\frac{1}{2}(x - 16)$ $y = -\frac{1}{2}x + 19$	✓ $-\frac{8}{13}$ ✓substitution ✓equation (3)
2.8	Area $\Delta ABC = \frac{1}{2} base\ height$ $= \frac{1}{2} \times 12 \times 8$ = 48 sq units	✓h=8 ✓ substitution ✓answer (3)
		[22]

QUESTION 3



3.1	M(1; 3) $r^2 = (5-1)^2 + (4-3)^2$ $r^2 = 16 + 1 = 17$ $(x-1)^2 + (y-3)^2 = 17$	✓✓M ✓substitution ✓$r^2 = 17$ ✓$(x-1)^2 + (y-3)^2 = 17$ (5)
3.2	$m_{PR} = \frac{-2-2}{-2+3} = -4$ $m_{RT} = \frac{4-2}{5+3} = \frac{1}{4}$ $m_{PR} \times m_{RT} - 1$ PR is a tangent	✓m_{PR} ✓m_{RT} ✓product = -1 (3)
3.3	Y int: $(0-1)^2 + (y-3)^2 = 17$ $1 + y^2 - 6y + 9 = 17$ $y^2 - 6y - 7 = 0$ $(y-7)(y+1) = 0$ $y = -1$ or $y = 7$ V(0; -1)	✓let $x = 0$ ✓standard form ✓$y = -1$ or $y = 7$ ✓V(0; -1) (4)
3.4	$m_{PT} = \frac{4+1}{5-0} = 1$ $\tan\alpha = 1$ $\alpha = 45°$ $\tan\beta = -4$ $\beta = 104°$ $\theta = 59°$	✓m_{PT} ✓$\tan\alpha = 1$ ✓$\alpha = 45°$ ✓$\tan\beta = -4$ ✓$\beta = 104°$✓ $\theta = 59°$ (6)
		[18]

QUESTION 4

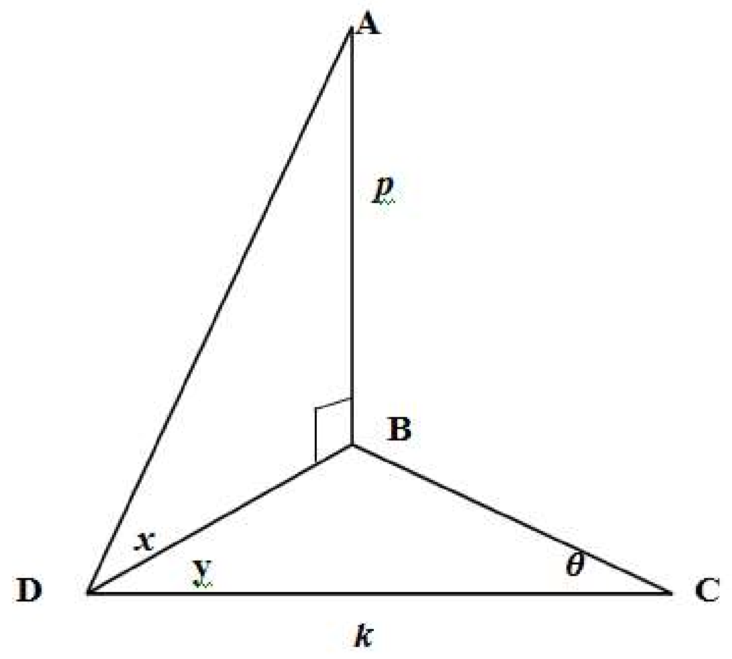
4.1.1	$\frac{2\sin(180°+x)\sin(90°+x)}{\cos^4x-\sin^4x}$ $=\frac{-2\sin x.\cos x}{(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)}$ $=\frac{-\sin 2x}{\cos 2x.(1)}$ $=-\tan 2x$	✓$-2\sin x$ ✓$\cos x$ ✓factorisation ✓$-\sin 2x$ ✓$\cos 2x$ (5)
4.1.2	At $\cos 2x = 0$ $2x = 90°$ or $2x = 270°$ $x = 45°$ or $x = 135°$	✓$\cos 2x = 0$ ✓$2x = 90°$ or $2x = 270°$ ✓$x = 45°$ $x = 135°$ (3)
4.2	$=\frac{(cos13°)(-sin13°)}{(-tan45°).(cos64°)}$ $=\frac{cos13°.-sin13°}{-1.cos64°}$ $=\frac{\mathbf{2}\times sin13°cos13°}{\mathbf{2}cos64°}$ $=\frac{sin26°}{2sin26°}$ $=\frac{1}{2}$	✓$cos13°$ ✓$-sin13°$ ✓$-tan45°$ ✓multiply by 2 in numerator **and** denominator ✓$\frac{sin26°}{2sin26°}$ (5)
4.3	LHS: $\frac{\cos(2x+x)}{cosx}$ $=\frac{cos2x.cosx-sin2x.sinx}{cosx}$ $=\frac{cos2x.cosx-2sinxcosx.sinx}{cosx}$ $=\frac{cosx(cos2x-2sin^2x)}{cosx}$ $= cos2x -1+1-2sin^2x$ $= cos2x -1+cos2x$ $= 2cos2x -1$ **OR**	✓$cos2x.cosx - sin2x.sinx$ ✓replacing $\sin 2x$ ✓factorise ✓ +1−1 ✓replacing $1-2sin^2x$ (5)

	$\frac{\cos(2x+x)}{cosx}$ $= \frac{cos2x.cosx - sin2x.sinx}{cosx}$ $= \frac{cos2x.cosx - 2sinxcosx.sinx}{cosx}$ $= \frac{cosx(cos2x - 2sin^2x)}{cosx}$ $= cos2x - 2sin^2x$ $= 2cos^2x - 1 - 2sin^2x$ $= 2(cos^2x - sin^2x) - 1$ $= 2cos2x - 1$	✓$cos2x.cosx - sin2x.sinx$ ✓replacing sin2x ✓factorise ✓ replacing cos2x ✓replacing $cos^2x - sin^2x$ (5)
		[18]

QUESTION 5

5.1	360°	✓ (1)
5.2	$sin(x+30°) = -2cosx$ $sinxcos30° + cosxsin30° = -2cosx$ $\sin x\left(\frac{\sqrt{3}}{2}\right) + \cos x\left(\frac{1}{2}\right) = -2\cos x$ $\sqrt{3}\sin x + \cos x = -4cosx$ $\sqrt{3}\sin x = -5cosx$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 180° - 70{,}89° + k.180°$ $x = 109.11° + k.180°$, $k \in Z$ $x = -70{,}89°$ or $x = 109{,}11°$	✓equating f and g ✓expanding $sin(x+30°)$ ✓special angle values ✓$\tan x = -\frac{5}{\sqrt{3}}$ ✓$x = -70{,}89°$ ✓$x = 109.11° + k.180°$ ✓$x = 109{,}11°$ (7)
5.3.1	$x \in [-90°; -70{,}89°] \cup [109{,}11°; 180°]$	✓✓boundaries ✓correct notation (3)
5.3.2	$x \in (-90°; -30°) \cup (90°; 150°)$	✓$(-90°; -30°)$ ✓$(90°; 150°)$ ✓correct notation (3)
		[14]

QUESTION 6

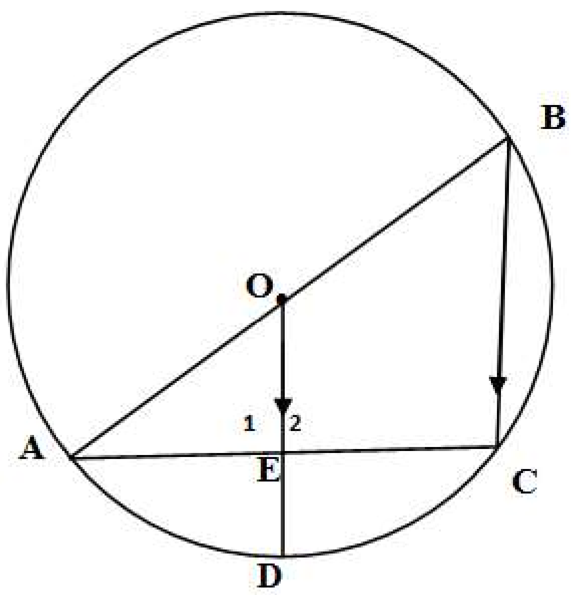


6.1.1	In $\Delta\, ABD$: $\tan x = \frac{p}{DB}$ $p = DB.\tan x$	✓$\tan x = \frac{p}{DB}$ ✓ $p = DB \tan x$ (2)
6.1.2	$\frac{DB}{sin\theta} = \frac{k}{\sin(180-(y+\theta)}$ $DB = \frac{k.sin\theta}{\sin(y+\theta)}$ $p = \frac{k.sin\theta}{\sin(y+\theta)} \times \tan x$ $= \frac{ksin\theta.tanx}{sinycos\theta + cosy.sin\theta}$	✓$B\hat{D}C = 180 - (y+\theta)$ ✓$\frac{DB}{sin\theta} = \frac{k}{\sin(180-(y+\theta)}$ ✓reduction formula ✓replacing DB ✓expanding sin(y+θ) (5)
6.2	$tan51{,}7° = \frac{80}{DB}$ $DB = \frac{80}{tan51{,}7°} = 63{,}18\ m$ $BC^2 = (63{,}18)^2 + 95^2 - 2(63{,}18)(95)cos62{,}5°$ $= 7473{,}789697 \ldots$ $\therefore BC = 86{,}45 \approx 86\ m$	✓$tan51{,}7° = \frac{80}{DB}$ ✓$DB = 63{,}18\ m$ ✓application of cosine formula. ✓$86m$ (4)
		[11]

QUESTION 7

7.1	is perpendicular to the chord	✓ (1)
7.2	The line from the centre of the circle perpendicular to the chord, bisects the chord	✓The line from the centre of the circle perpendicular to the chord ✓ bisects the chord (2)

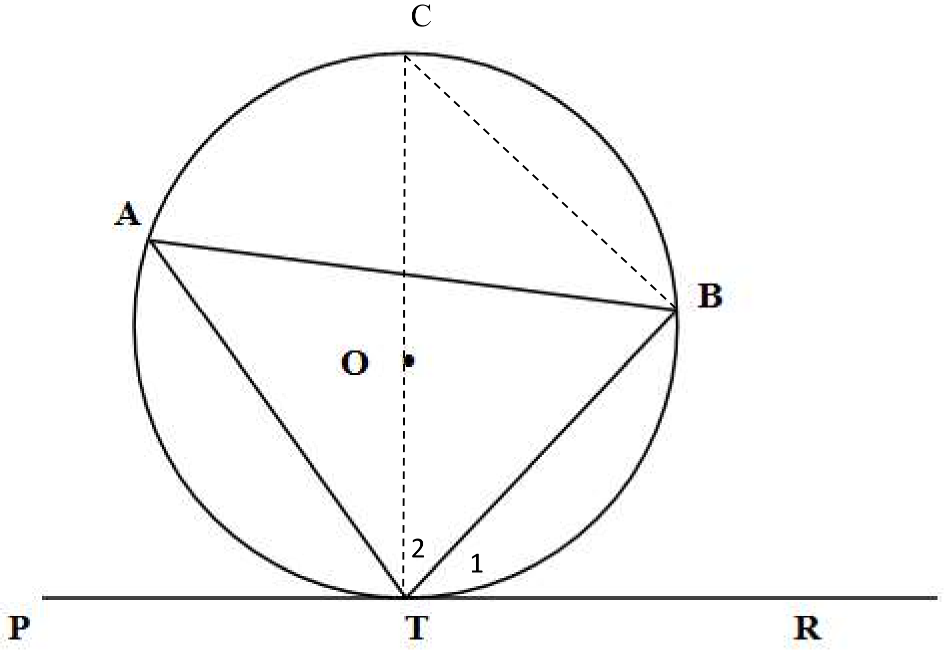
7.3



7.3.1	$\frac{AO}{OB} = \frac{AE}{EC} \cdots\cdots OE \parallel BC$ AO=OB.....Radii $\Rightarrow$ AE = EC	✓S✓R (2)
7.3.2	$\hat{C} = 90°$ (angle in semi⊙) $\hat{E}_1 = 90°$ (corr. angles; OD ∥ BC)	✓S/R ✓R (2)
7.3.3	$OE^2 = 10^2 - 8^2$ (theorem of Pyth) $OE^2 = 100 - 64 = 36$ OE = 6 cm ∴ED = 4 cm	✓S ✓OE = 6 cm ✓answer (3)
		[10]

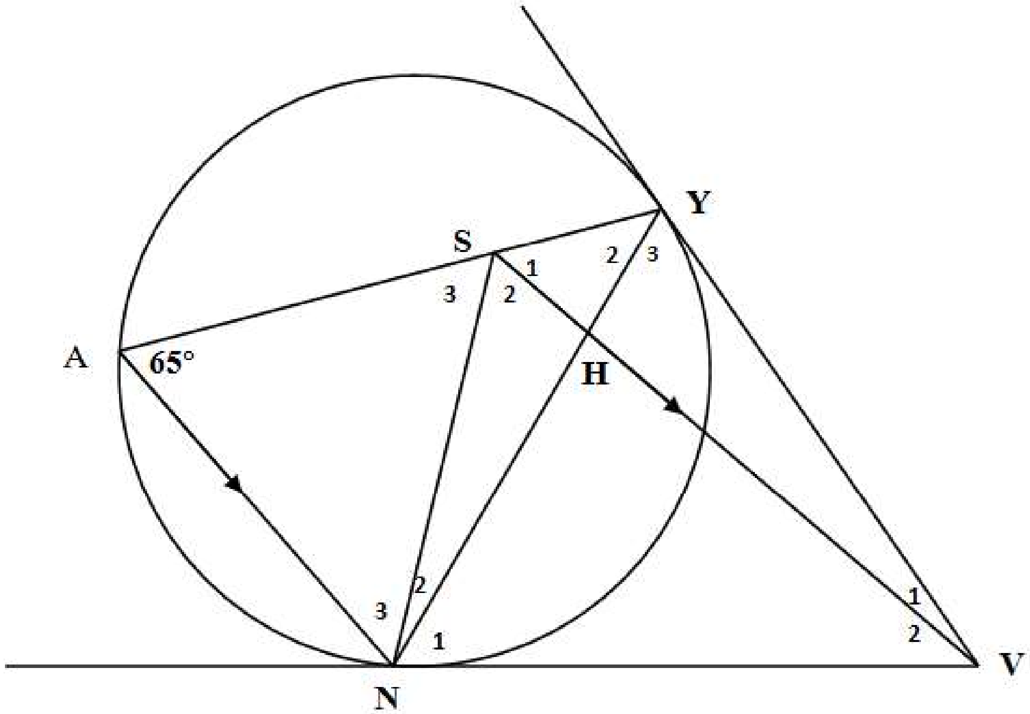
QUESTION 8

8.1



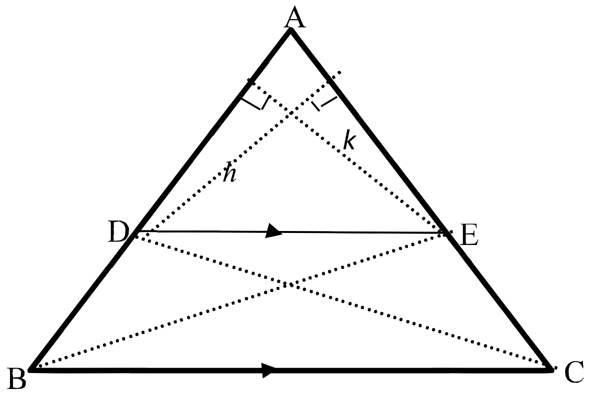
8.1	Construction: Draw diameter TC and join BC. $C\hat{B}T = 90°$ (∠ in semi ⊙) $\hat{C}+\hat{T}_2 = 90°$ (∠'s of Δ) $\hat{T}_1+\hat{T}_2 = 90°$ (tangent ⊥ r) $\therefore\ \hat{C}=\hat{T}_1$ But $\hat{C} = \hat{A}$ (∠'s in same segment) $\therefore \hat{T}_1 = \hat{A}$	✓construction ✓S / R ✓S ✓S/ R ✓S/ R ✓conclusion (6)

8.2

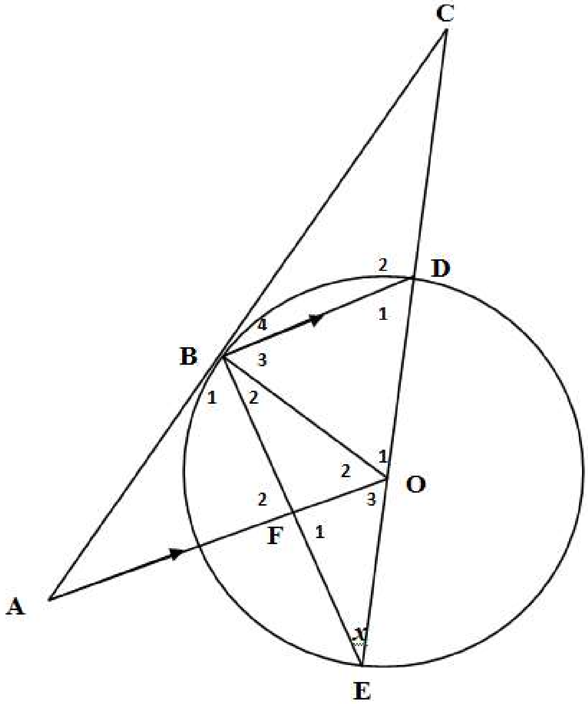


8.2.1	$\hat{S}_1 = 65°$ (corr ∠'s; AN ∥ SV) $\hat{Y}_3 = 65°$ (tan-chord th) $\hat{N}_1 = 65°$ (tan-chord th)	✓S R ✓S R ✓S R (3)
8.2.2	$\hat{S}_1 = \hat{N}_1$ VYSN is a cyclic quad (YV subtends equal angles)	✓$\hat{S}_1 = \hat{N}_1$ ✓YV subtends equal angles (2)
8.2.3	$\hat{S}_2 = 65°$ (∠'s in same segment) $\hat{N}_3 = 65°$ (alt. ∠'s; AN ∥ SV) $\therefore \hat{A} = \hat{N}_3$ AS = SN (sides opp equal angles)	✓S✓R ✓S✓R ✓R (5)
		[16]

QUESTION 9

	Use the diagram below to prove the theorem which states that if DE||BC then $\frac{BD}{AD} = \frac{EC}{AE}$.	
		✓Construction
	Construction: In ΔADE draw *altitudes h and k* $\frac{area\ \Delta BDE}{area\ \Delta ADE} = \frac{\frac{1}{2}BD \times k}{\frac{1}{2}AD \times k}$ $= \frac{BD}{AD}$ $\frac{area\ \Delta CED}{area\ \Delta ADE} = \frac{\frac{1}{2}EC \times h}{\frac{1}{2}AE \times h}$ $= \frac{EC}{AE}$ But $area\ \Delta BDE = area\ \Delta CED$ *Same base, same height* $\therefore \frac{area\ \Delta BDE}{area\ \Delta ADE} = \frac{area\ \Delta CED}{area\ \Delta ADE}$ $\therefore \frac{BD}{AD} = \frac{EC}{AE}$	✓ S ✓ S ✓ S ✓S & R ✓ S **[6]**

QUESTION 10



10.1	Subtended by a diameter / Angle in a semi-circle	✓ Answer (1)
10.2	$\hat{B}_2 = x$ (radii =) $\hat{B}_4 = x$ (tan-chord th $\hat{A} = x$ (corr ∠'s; BD ∥ AO)	✓ S ✓SR ✓S (3)
10.3	$\hat{A} = \hat{E} = x$ Converse $\angle's$ subtended by the same cord	✓ Answer (1)
10.4	$\hat{B}_2 + \hat{B}_3 = 90°$ (∠ in semi ⊙) $C\hat{B}E = 90° + x$	✓ R ✓$90° + x$ (2)
10.5.1	In Δ CBD and Δ CEB: $\hat{C} = \hat{C}$ $\hat{B}_4 = \hat{E} = x$ $\hat{D}_2 = C\hat{B}E$ ∴ Δ CBD ∣∣∣ Δ CEB (∠∠∠)	✓S ✓S (2)
10.5.2	$\frac{CB}{CE} = \frac{BD}{EB}$ (∣∣∣ triangles) EB.CB = CE. BD $\hat{F}_1 = 90°$ (corr ∠'s; BD ∥ AO) BF = FE (line from centre to mdpt of chord) ∴ BE = 2EF ∴ 2EF.CB = CE.BD	✓S✓R ✓SR ✓SR ✓replacing BE (5)
10.5.3	$\frac{2EF}{CE} = \frac{BD}{BC}$ out of 10.4 But Δ BCD ∣∣∣ Δ ACO (∠∠∠) $\therefore \frac{BD}{AO} = \frac{BC}{AC}$ $\frac{BD}{BC} = \frac{AO}{AC}$ $\frac{2EF}{CE} = \frac{AO}{AC}$	✓S ✓SR ✓S ✓S (4)