education

Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

SEPTEMBER 2018

MARKS: 150

TIME: 3 HOURS

This question paper consists of 14 pages an information sheet and an answer book of 22 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. Number the answers correctly according to the numbering system used in this question paper.
8. A special ANSWER BOOK is provided in which to answer ALL the questions.
9. Answers only will NOT necessarily be awarded full marks.
10. Write neatly and legibly.

QUESTION 1

The tuck shop sells cans of soft drinks. The Environmental Club decided to have a can-collection project for three weeks to make learners aware of the effects of litter on the environment. The data below shows the number of cans collected on each school day of the three week project.

58	83	85	89	94
97	98	100	105	109
112	113	114	120	145

1.1 Determine the lower and upper quartiles of the data. (2)

1.2 Use the scaled line in the ANSWER BOOK to draw a box and whisker diagram for this set of data. (3)

1.3 Comment on the skewness in the distribution of the data. (1)

1.4 Calculate the mean number of cans collected over the three week period. (2)

1.5 Calculate the standard deviation of the number of cans collected. (1)

1.6 On how many days did the number of cans collected lie outside one standard deviation of the mean? Show all calculations. (3)

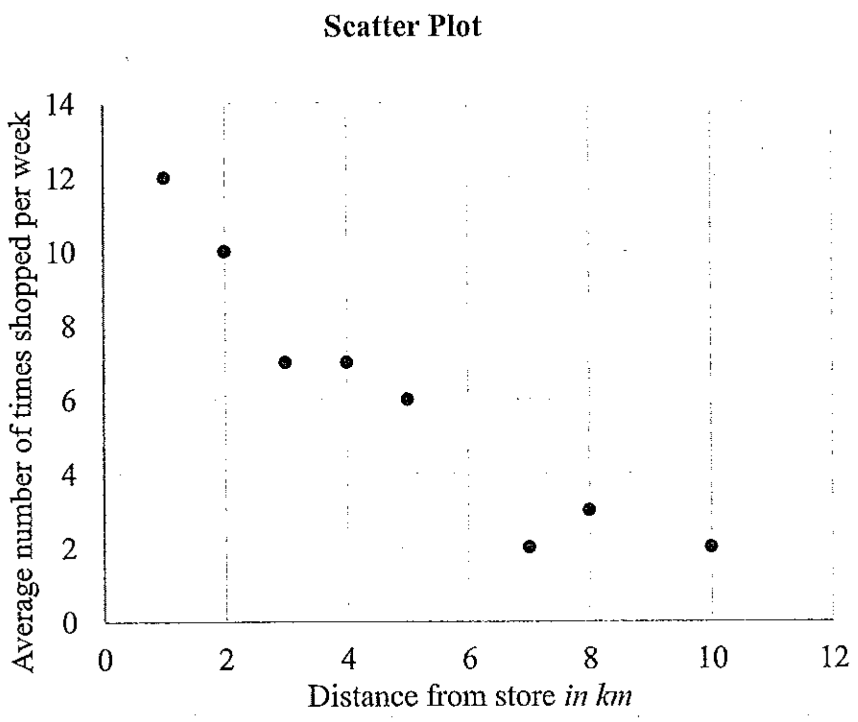
[12]

QUESTION 2

A survey was conducted at a local supermarket to establish the relationship between the distance (in kilometres) that shoppers stay from the store and the average number of times that they shop at the store in a week. The results are shown in a table below.

Distance from store *in km*	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2

The above data is represented in a scatter plot below.



2.1 Calculate the correlation coefficient of the data. (2)

2.2 Comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)

2.3 Calculate the equation of the least squares regression line of the data. (3)

2.4 Use your answer in QUESTION 2.3 to predict the average number of times that a shopper living 6 *km* from the supermarket will visit the store in a week. (2)

[8]

QUESTION 3

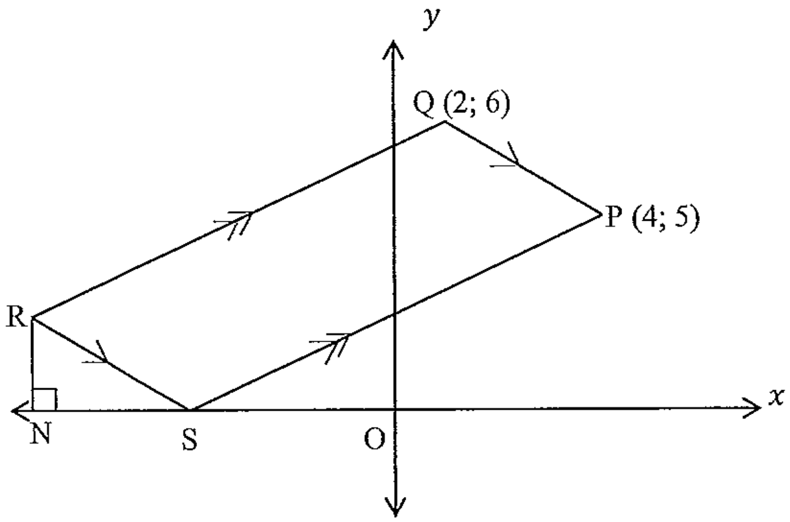
3.1 Given: $E(4;3), F(0;-1)$ and $G(t;1)$

Determine the value of t for which:

3.1.1 E, F and G lie on the same straight line. (4)

3.1.2 ΔFEG is right angled at F. (2)

3.2 In the diagram below $P(4;5)$, $Q(2;6)$, R and S are vertices of a parallelogram. N is on the x-axis such that RN is perpendicular to the x-axis. The equation of line QR is $2y = x + 16$.



3.2.1 Determine the equation of PS. (4)

3.2.2 Calculate the co-ordinates of S. (2)

3.2.3 Show that the mid-point of QS is $(-2;3)$. (2)

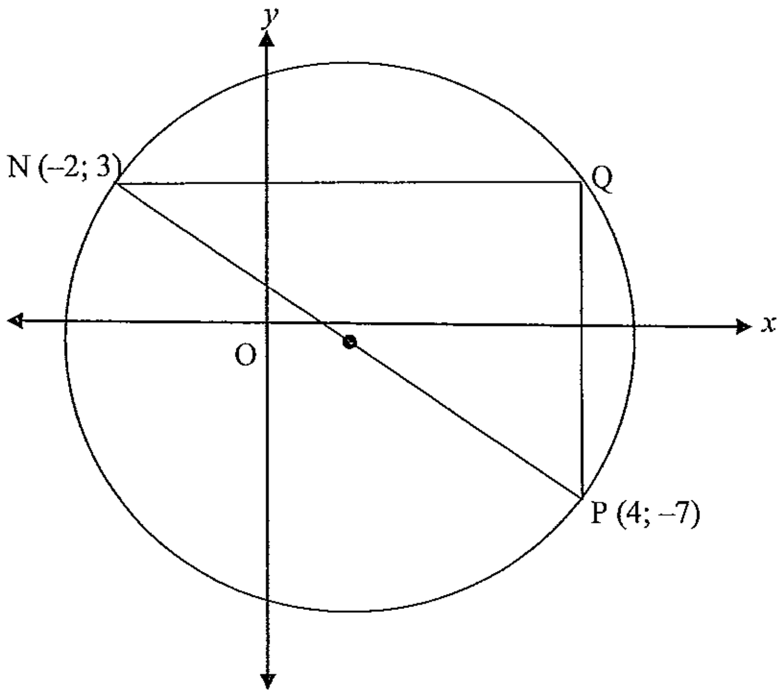
3.2.4 Determine the co-ordinates of R. (3)

3.2.5 Calculate the size of $R\hat{S}P$. (5)

[22]

QUESTION 4

In the diagram below, a circle with diameter NP is given with N$(-2;3)$ and P$(4;-7)$.



Determine:

4.1 The equation of the circle in the form: $(x-a)^2+(y-b)^2=r^2$. (5)

4.2 The equation of the tangent to the circle at N in the form $y = mx + c$ (4)

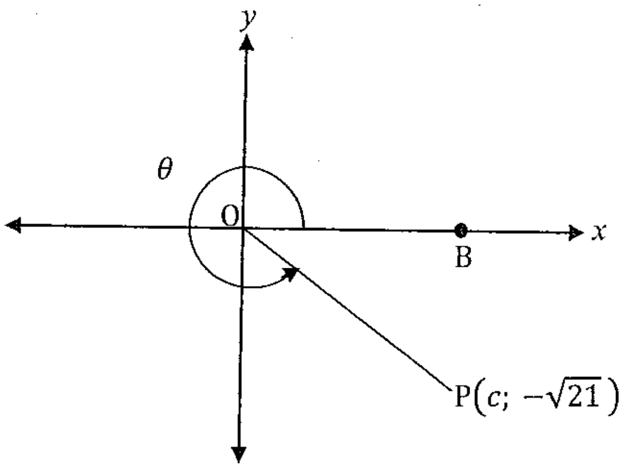
4.3 The size of $N\hat{P}Q$ if Q$(4;3)$ is a point on the circle. (4)

4.4 Calculate the value of: $\frac{Area\ of\ \Delta\ PQN}{Area\ of\ the\ circle}$ (4)

[17]

QUESTION 5

5.1 In the diagram below, P is the point $(c; -\sqrt{21})$ such that OP = 5 units.
Reflex $\hat{BOP} = \theta$ as indicated.



5.1.1 Calculate the numerical value of c. (2)

5.1.2 Determine **without the use of calculator**, the numerical value of the following:

a) $\cos\,\theta$ (1)

b) $\tan\theta + sin^2\theta$ (2)

c) $\sin 2\theta$ (2)

5.2 Simplify, **without the calculator**, to a single trigonometric ratio of x:

$$\frac{\sin(x-180^\circ).\tan x.\cos 690^\circ}{\cos^2(x-90^\circ)}$$ (5)

[12]

QUESTION 6

6.1 Prove the identity: $\frac{2\sin^2 x}{2\tan x - \sin 2x} = \frac{\cos x}{\sin x}$ (5)

6.2 Determine the general solution of $2 + 2\cos 2x = 0$ (4)

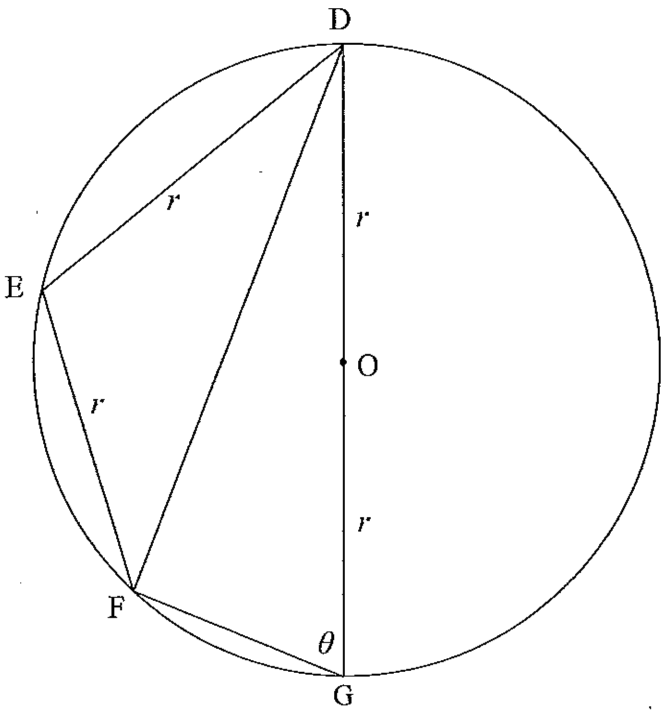
[9]

QUESTION 7

In the given diagram, DG is a diameter of the circle having radius r.

DE and EF are chords so that DE= EF = r.

FD and FG are joined.



7.1 Write down the size of $\widehat{E}$ in terms of θ. (1)

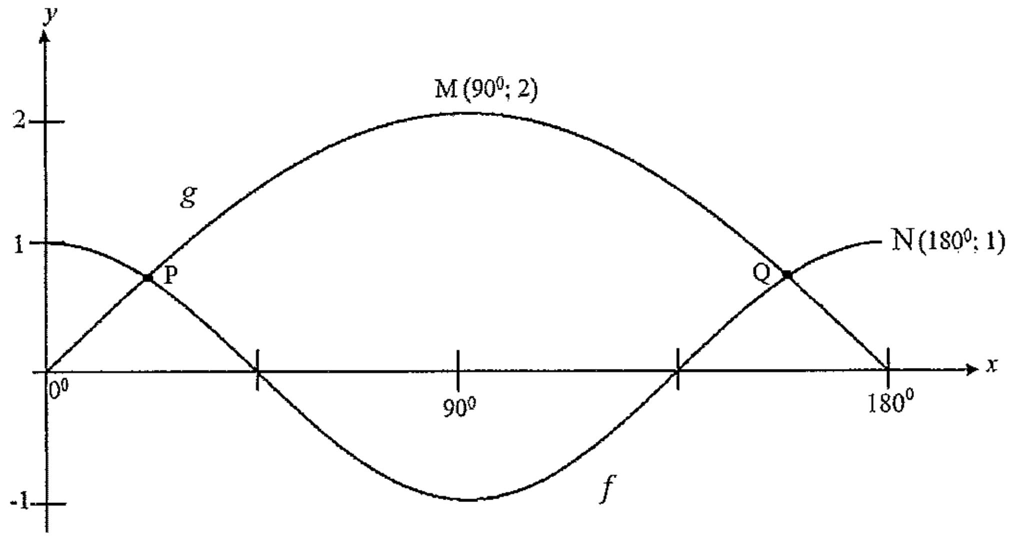
7.2 Prove that $DF = r\sqrt{2 + 2\cos\theta}$ (2)

7.3 Prove that $2\sin^2\theta = 1 + \cos\theta$ (3)

7.4 Hence, calculate the size of θ. (5)
[11]

QUESTION 8

The diagram below shows the graphs of $f(x) = a\cos bx$ and $g(x) = c\sin dx$ in the interval $x\epsilon[0°; 180°]$. The graphs f and g intersect at points P and Q. M$(90°; 2)$ is the turning point of g and N$(180°; 1)$ is an end point of f.



8.1 Write down the numerical value of a, b, c and d. (4)

8.2 If $(158{,}56°; 0{,}73)$ are the coordinates of Q, write down the co-ordinates of P. (2)

8.3 If $x\epsilon(0°; 180°)$, determine the value(s) of x for which:

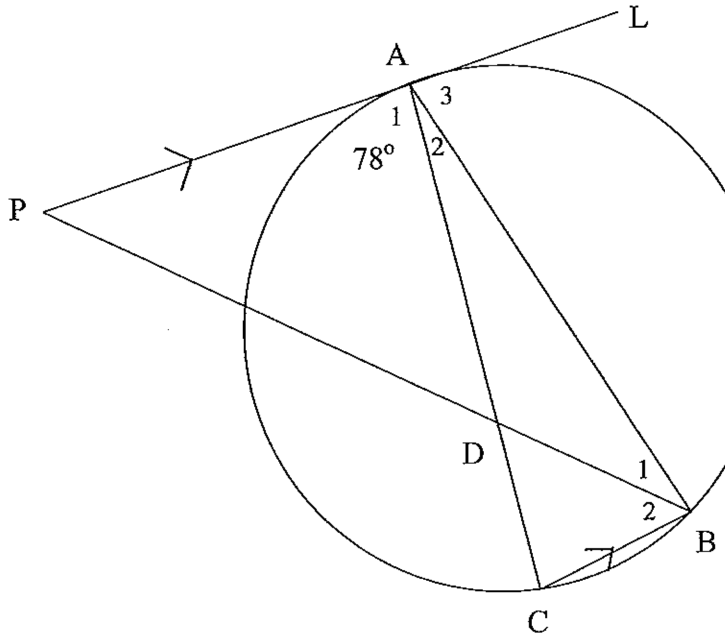
8.3.1 $g(x) - f(x) = 3$ (1)

8.3.2 $f(x).g(x) \leq 0$ (2)

[9]

QUESTION 9

9.1 In the diagram, PAL is a tangent to the circle at A. C and B are points on the circle such that CB||PAL. AB, AC and PB are drawn. PB intersects AC at D. $\hat{A}_1 = 78°$.



Determine the sizes of each of the following angels, with reasons:

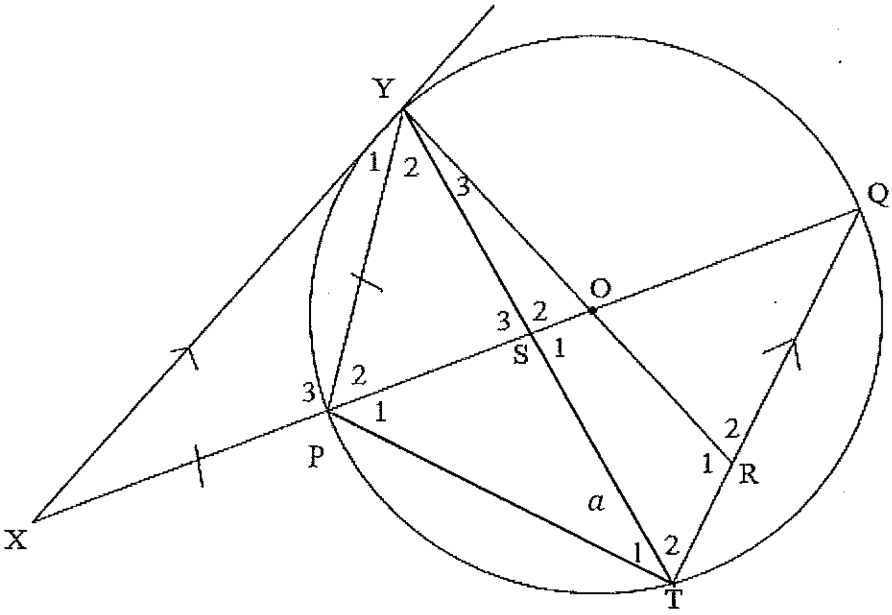
9.1.1 $\hat{C}$ (1)

9.1.2 $A\hat{B}C$ (2)

9.1.3 $\hat{A}_3$ (1)

9.1.4 $\hat{A}_2$ (1)

9.2 XY is a tangent to the circle with centre O. XPQ, YOR, YST are straight lines.
PX = PY, XY||TQ and $\hat{T}_1 = a$



9.2.1 Write down, with reasons, FOUR other angles each equal to a. (6)

9.2.2 Prove that $\hat{T}_2 = 2\hat{T}_1$ (2)

9.2.3 Prove that $\hat{T}_2 = 90° - a$. (2)

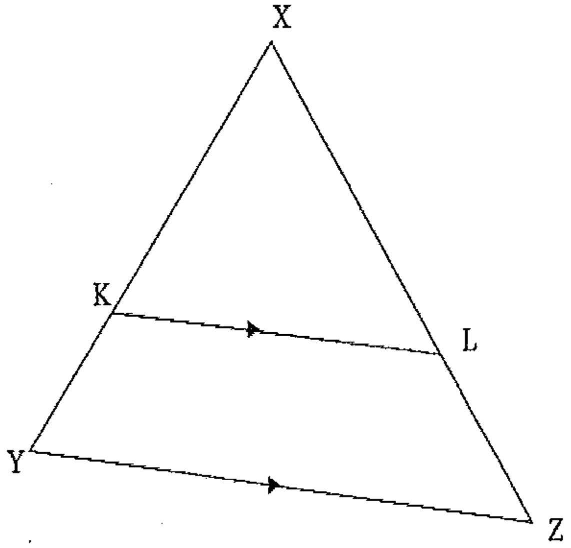
9.2.4 Prove that SORT is a cyclic quadrilateral. (2)

9.2.5 Determine the value of a. (2)

9.2.6 Show that TR = RQ. (3)
[22]

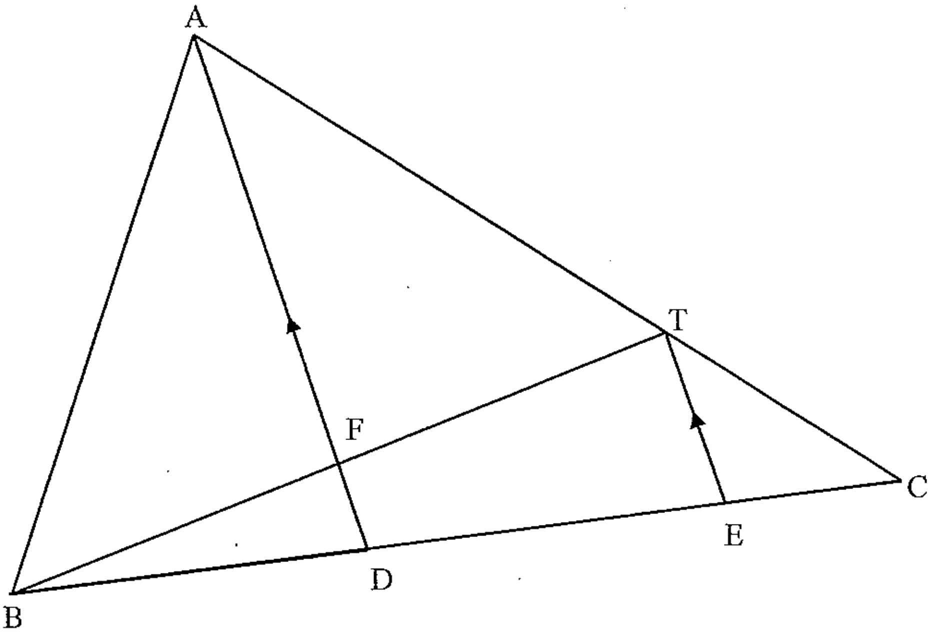
QUESTION 10

10.1 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY}=\frac{XL}{LZ}$.



(5)

10.2 In the figure below, D is a point on side BC of ΔABC such that BD = 6 *cm* and DC = 9 *cm*. T and E are points on AC and DC respectively and TE||AD and AT:TC = 2:1



10.2.1 Show that D is the midpoint of BE. (3)

10.2.2 If FD = 2 cm, calculate the length of TE. (3)

10.2.3 Calculate the numerical value of:

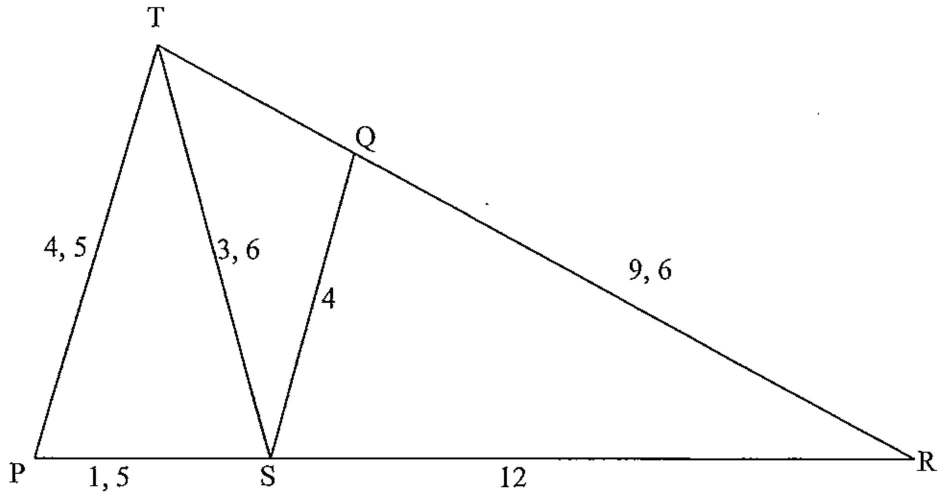
10.2.3(a) $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (2)

10.2.3(b) $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ (3)

[16]

QUESTION 11

In the diagram, TPR is a triangle with TP = 4,5 units. Q and S are points on TR and PR respectively. QR = 9,6 units, QS = 4 units, TS = 3,6 units, PS = 1,5 units and SR = 12 units.



11.1 Prove that PT is a tangent to the circle which passes through the points T, S and R. (7)

11.2 Calculate the length of TQ. (5)
[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$