Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sportontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko

NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

SEPTEMBER 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 23 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

The distances (in cm) of the best attempts of 11 long jump athletes during an athletics event are given below:

287	328	374	486	492	501
522	583	601	619	685	

1.1 Calculate the mean distance of the athletes' best attempts. (2)

1.2 Calculate the standard deviation of the above data. (2)

1.3 Determine how many distances lie outside ONE standard deviation from the mean attempt. Show ALL your calculations. (3)

1.4 The official that measured the distances of the long jump athletes, mistakenly measured y cm short from the correct measuring mark. Hence all distances measured were y cm shorter than what it was supposed to be.

This scenario is shown in the diagram below.

- A = The correct mark from where the distance should have been measured.
- B = The incorrect mark from where the distances were measured.
- C = The mark up to where the distance was measured.

When the correction is made to the distances, the sum of the athletes' best jumps is now 5555 cm, i.e. $\sum_{n=1}^{11} k_n = 5555$.

1.4.1 Calculate the value of y. (2)

1.4.2 Write down the standard deviation of the new correct distances. (2)
[11]

QUESTION 2

The goal-scorers in a netball game practice scoring at training during the week. In the tournament during the weekend, the number of goals they score from the total number of attempts they made, is recorded as a percentage. This statistic is referred to as the successful goal-shoot average.

The table below shows the number of goal shots practiced during the week and the successful goal-shoot average during the tournament for 8 goal-scorers.

Number of goal shots practiced	280	400	540	595	375	430	500	650
Successful goal shoot average (%)	73	75	83	89	80	76	82	91

2.1 Determine the equation of the least squares regression line. (3)

2.2 Calculate the correlation coefficient for the data. (2)

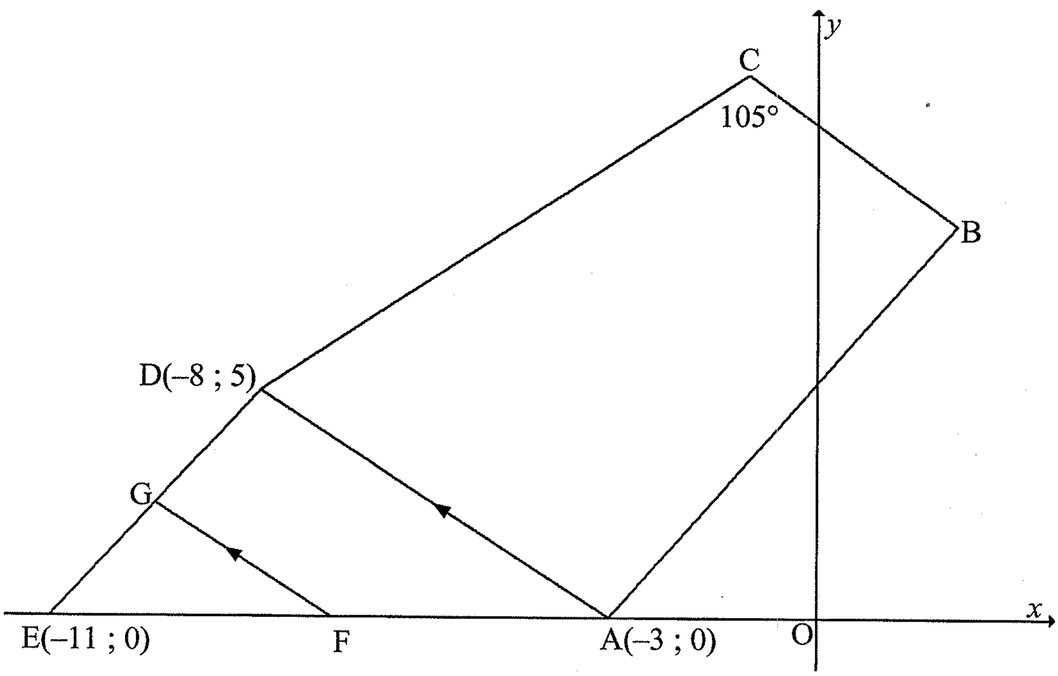
2.3 Comment on the correlation between the number of goal shots practiced and the successful goal-shoot average. (2)

2.4 A player practiced 465 goal shots. What is her expected successful goal-shoot average for the next tournament? (2)

[9]

QUESTION 3

In the diagram below, A(–3 ; 0), B, C and D(–8 ; 5) are the vertices of a quadrilateral with $B\hat{C}D = 105°$. E(–11 ; 0) and F are points on the x-axis. ED is a straight line.
FG || AD with G on ED.



3.1 Prove that the perimeter of Δ ADE, rounded off to TWO decimal places, is 20,90 units. (5)

3.2 It is given that F is the midpoint of AE.

3.2.1 Give a reason why G is the midpoint of DE. (1)

3.2.2 Hence, determine the coordinates of G. (3)

3.2.3 Write down the length of FG. (1)

3.2.4 Determine the equation of FG. (4)

3.3 Calculate the size of $D\hat{A}O$. (2)

3.4 It is given that ABCD is a cyclic quadrilateral and that the equation of CB is $y = \frac{-12+5\sqrt{3}}{3}x + \frac{24+5\sqrt{3}}{3}$.

Determine the coordinates of B. (7)

[23]

QUESTION 4

4.1 Determine the equation of a circle with its centre at the origin in the Cartesian plane and with a radius of $2\sqrt{3}$ units. (2)

4.2 A circle is defined by the equation $x^2 - 6x + y^2 - 2y = 6$.

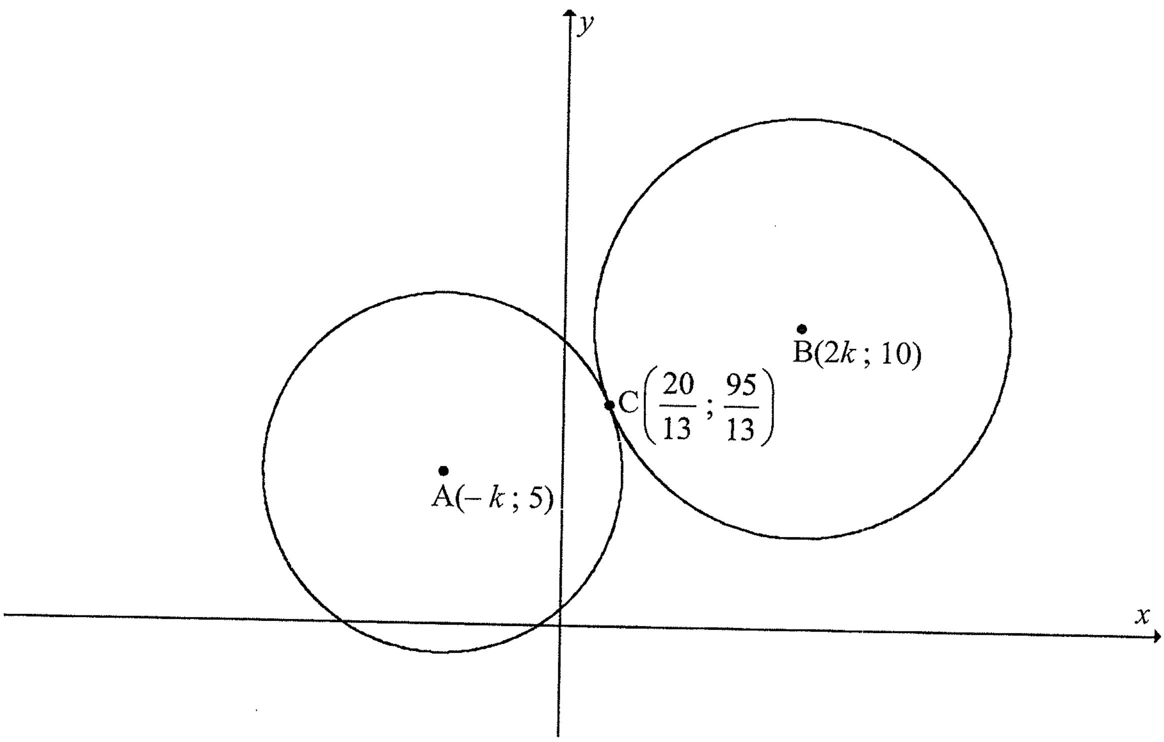
4.2.1 Rewrite the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.2.2 Write down the coordinates of M, the centre of the circle. (2)

4.2.3 Write down the length of the radius of the circle. (1)

4.2.4 A tangent PQ is drawn from the point P(6 ; –2) to the circle, with Q the point of contact. Calculate the length of the tangent PQ. (4)

4.3 In the diagram below are two circles that touch each other externally in the point $C(\frac{20}{13};\frac{95}{13})$.
The radius of circle A with centre $A(-k\,;5)$ is 6 units.
The radius of circle B with centre $B(2k\,;10)$ is 7 units.
ACB is a straight line.



4.3.1 If $k > 0$, prove that the gradient of AB is $\frac{5}{12}$ (5)

4.3.2 Hence, or otherwise, determine the equation of the common tangent to the circles A and B. (3)

[19]

QUESTION 5

5.1 Given: $\cos 16° = \dfrac{\sqrt{k}}{t}$.

Without using a calculator, determine in the simplest form, the value of each of the following in terms of k and t:

5.1.1 $\sin 106°$ (2)

5.1.2 $\sin 16°$ (3)

5.1.3 $\cos 8°$ (4)

5.2 Prove that:

$$\frac{\sqrt{4(1-\cos\theta)(1+\cos\theta)}}{\sin 2\theta} = \frac{1}{\cos\theta}$$ (4)

5.3 It is given that $\sin p + \sqrt{3}\cos p = 1$.

5.3.1 Show that the equation can be written as $\sin(60° + p) = \dfrac{1}{2}$ (3)

5.3.2 Hence, determine the general solution of $\sin p + \sqrt{3}\cos p = 1$. (4)

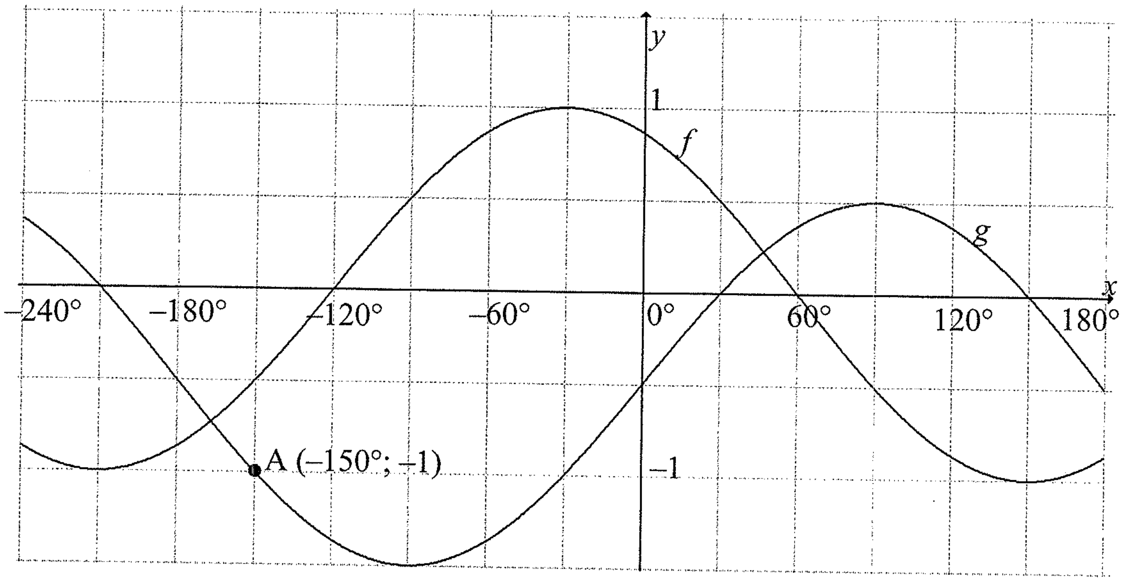
5.4 **Without using a calculator**, determine the value of:

$\cos 0° + \cos 1° + \cos 2° + ... + \cos 178° + \cos 179° + \cos 180° + 2$ (3)

[23]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = \sin x + q$ are drawn on the same set of axes for $-240° \leq x \leq 180°$. A(–150° ; –1) is a point on g.



6.1 Determine the values of p and q. (4)

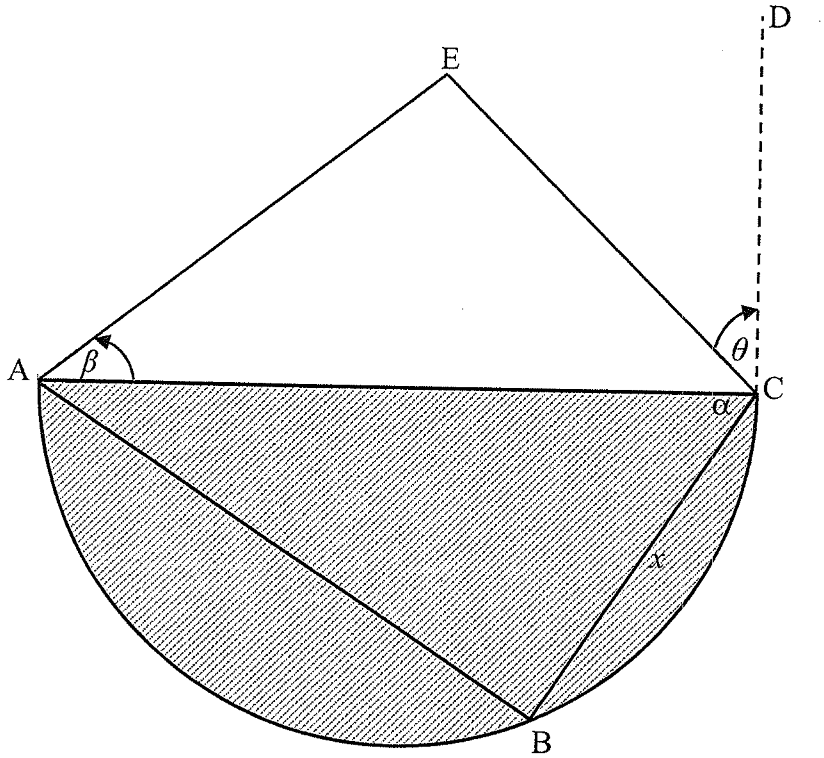
6.2 Determine graphically the values of x, for $-240° \leq x \leq 180°$, where $f(x) = g(x) + \frac{1}{2}$ (2)

6.3 Describe the transformation that the graph of f has to undergo to form the graph of h, where $h(x) = -\sin x$. (2)

[8]

QUESTION 7

A, B and C are three points on the circumference of a semi-circle in a horizontal plane. AC is the diameter of the semi-circle. EC is a slanted pole in the vertical plane through C, forming an angle of θ with the vertical. The angle of elevation of E at A is β.
$BC = x$ and $A\hat{C}B = \alpha$.



7.1 Show that $AC = \dfrac{x}{\cos\alpha}$ (2)

7.2 Determine an expression for $A\hat{E}C$ in terms of θ and β. (3)

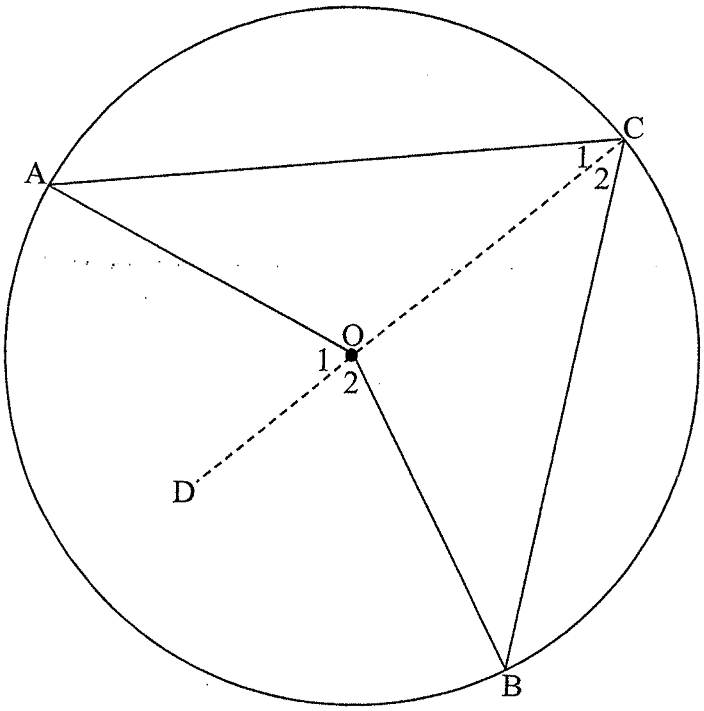
7.3 Prove that $EC = \dfrac{x.\sin\beta}{\cos\alpha.\cos(\beta-\theta)}$ (3)

[8]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

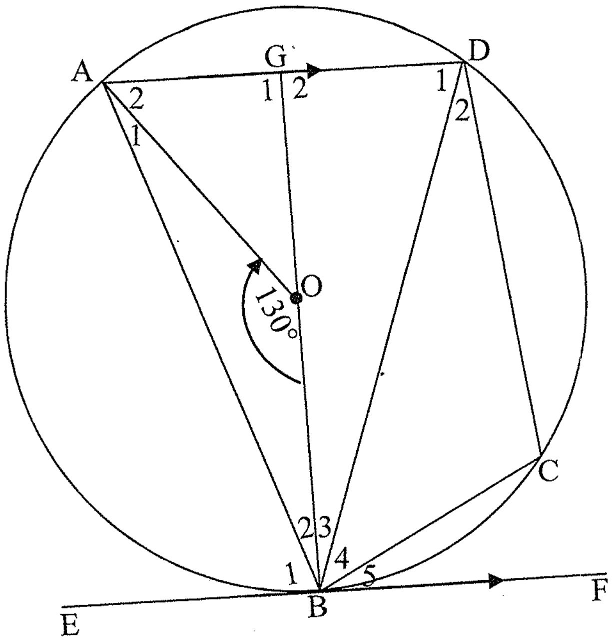
QUESTION 8

8.1 In the diagram below, A, B and C are points on a circle having centre O. DC is a construction line drawn through the centre O to the point C.



Use the above diagram to prove the theorem which states that $A\hat{O}B = 2\hat{C}$. (5)

8.2 In the diagram below, the circle having centre O, passes through A, B, C and D, with $A\hat{O}B = 130°$. EBF is a tangent to the circle at B with $EF \parallel AD$. BOG is a straight line.



Calculate, with reasons, the size of:

8.2.1 $\hat{D}_1$ (2)

8.2.2 $\hat{B}_1$ (2)

8.2.3 $B\hat{A}D$ (1)

8.2.4 $\hat{C}$ (2)

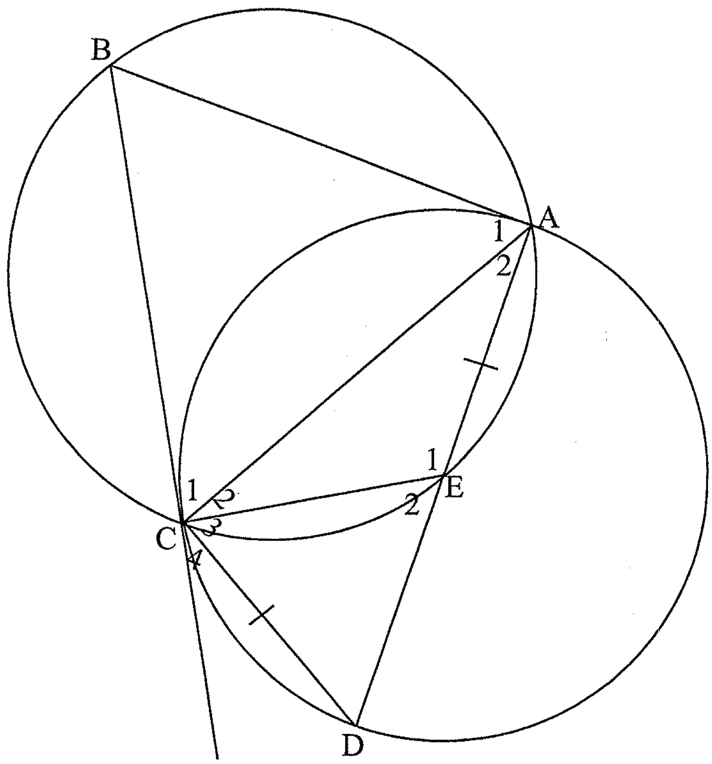
8.2.5 $\hat{B}_3$ (3)

8.2.6 Calculate the length of GD, if $AD = \frac{\sqrt{7}}{2}$ units. (3)

[18]

QUESTION 9

Two equal circles cut each other in A and C. BA and BC are tangents to one circle at A and C respectively and they are chords of the other circle. E is a point on the circumference of one circle and AE produced cuts the other circle in D. Chords AE and CD are equal.



Prove that:

9.1 $\hat{C}_2 = \hat{C}_4$ (4)

9.2 $\hat{C}_3 = \hat{A}_1$ (3)

9.3 E is the centre of the circle that passes through A, C and D. (4)

9.4 ΔECD is equilateral. (2)

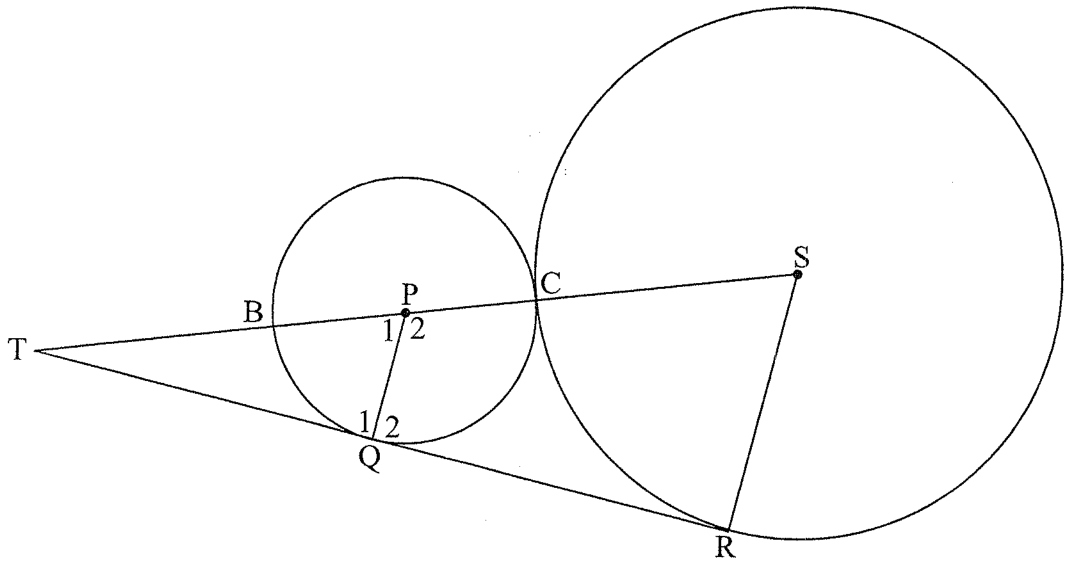
[13]

QUESTION 10

10.1 Complete the following theorem to make the statement TRUE:

If a line divides two sides of a triangle in the same proportion, then the line is ... (1)

10.2 Two circles with centres P and S touch each other externally at C. SP produced intersects circle P at B. A common tangent at R and Q meets SB produced at T.



Prove that:

10.2.1 $PQ \parallel SR$ (4)

10.2.2 $TP = \dfrac{TQ(BP+SR)}{QR}$ (4)

10.2.3 $\Delta TQP \,|||\, \Delta TRS$ (3)

10.2.4 $\sqrt{TS^2 - CS^2} = \dfrac{\sqrt{(TP^2 + BP^2 - 2TP.BP\cos S)}\,.\,CS}{BP}$ (6)

[18]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $A=P(1-ni)$ $A=P(1-i)^n$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $S_n=\frac{a(r^n-1)}{r-1}$; $r\neq 1$ $S_\infty=\frac{a}{1-r}$; $-1<r<1$;

$F=\frac{x[(1+i)^n-1]}{i}$ $P=\frac{x[1-(1+i)^{-n}]}{i}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $a^2=b^2+c^2-2bc.\cos A$

$area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$ $\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum x}{n}$ $\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y}=a+bx$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$