education

Department of Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

SEPTEMBER 2019

TIME: 3 HOURS

MARKS: 150

This question paper consists of 14 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

The following table shows the test marks (in %) of Grade 11 learners in Frances Baard High School.

INTERVAL OF TEST MARKS	**NUMBER OF LEARNERS**
$0 \leq x < 20$	4
$20 \leq x < 40$	5
$40 \leq x < 60$	9
$60 \leq x < 80$	13
$80 \leq x < 100$	10
Totals	**41**

1.1 Write down the modal class. (1)

1.2 Calculate the estimated mean. (3)

1.3 Complete the cumulative frequency table provided in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency curve (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 Use the cumulative frequency curve (ogive) to determine the interquartile range for the data. (3)

[12]

QUESTION 2

Research is done to determine if the number of hours reading over a certain period of time has an effect on the results of a candidate's mark in a general knowledge test (out of 120).

Number of hours	15	20	22	25	32	40	44	50	55	58
Mark out of 120	40	30	55	80	70	75	100	105	98	79

2.1 Determine the equation of the least squares regression line. (3)

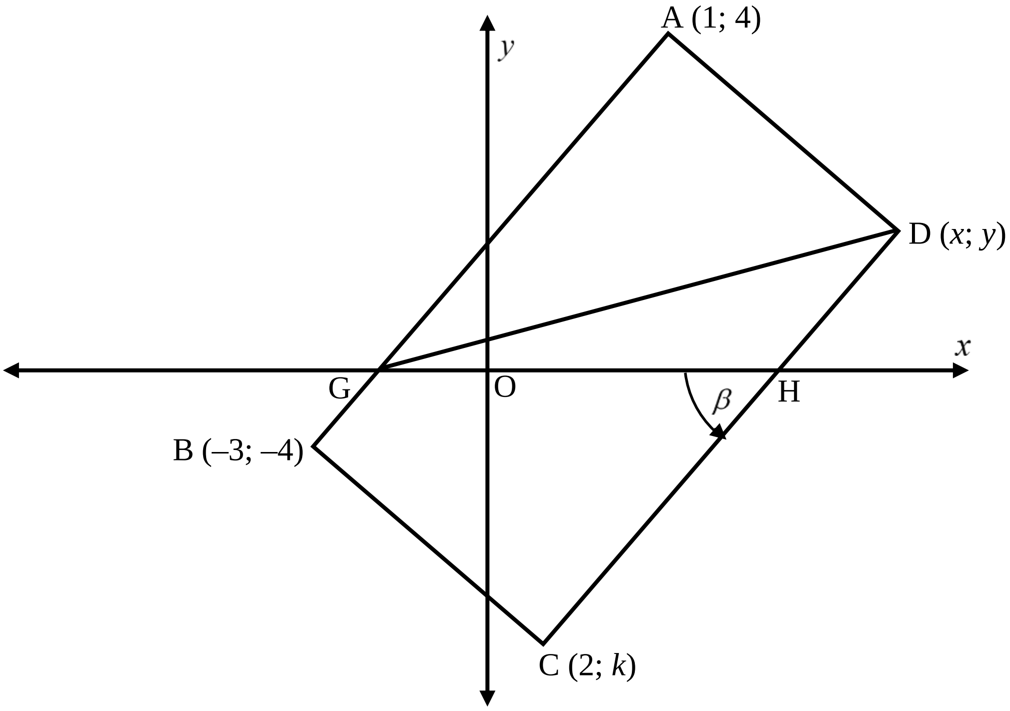
2.2 Estimate the mark that a person that reads 36 hours in that same period will obtain in the test. (2)

2.3 What is the correlation between the number of hours reading and the mark a person scores for the test? Motivate your answer. (3)

[8]

QUESTION 3

In the diagram below, A $(1\,;4)$, B $(-3\,;-4)$, C $(2\,;k)$ and D $(x\,;y)$ are the vertices of a rectangle. AB and DC cuts the $x-$axis at G and H respectively. GD is drawn. $G\hat{H}C = \beta$.



3.1 Calculate the gradient of BG. (2)

3.2 Determine the equation of AB in the form $y = mx + c$. (2)

3.3 Calculate the:

3.3.1 Value of k (y-coordinate of C). (4)

3.3.2 Coordinates of D. (3)

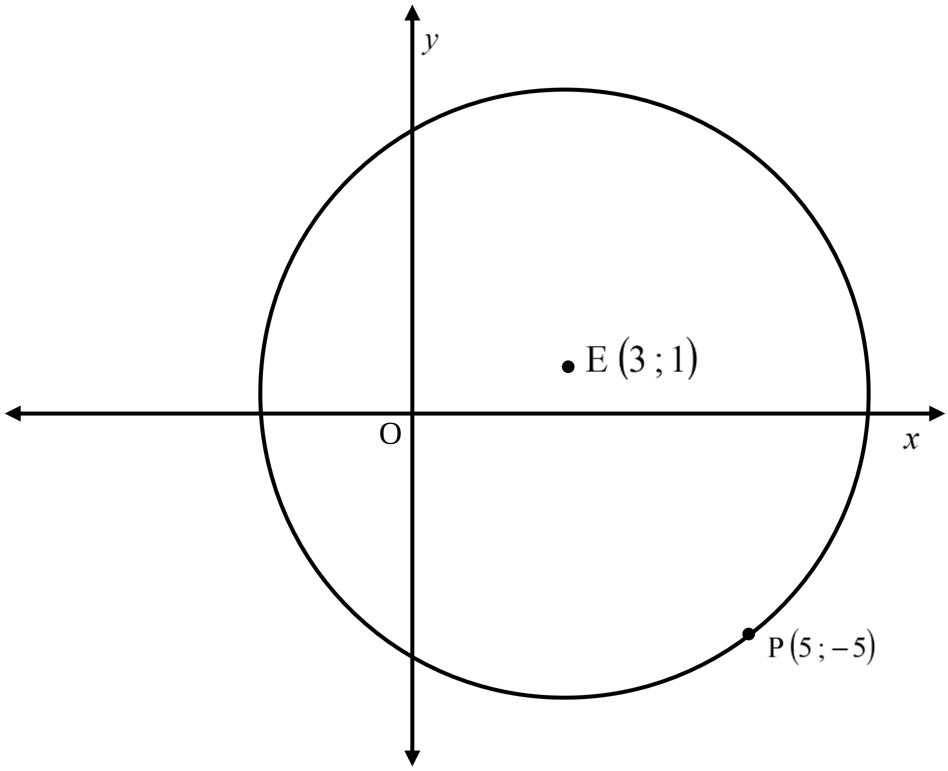
3.3.3 Size of β . (3)

3.3.4 Area of Δ DHG. (7)

[21]

QUESTION 4

In the diagram below, the circle centred at $E(3\,;1)$ passes through point $P(5\,;-5)$.



4.1 Determine the equation of:

4.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)

4.1.2 The tangent to the circle at $P(5\,;-5)$ in the form $y = mx + c$. (5)

4.2 A smaller circle is drawn inside the circle. Line EP is a diameter of the small circle. Determine the:

4.2.1 Coordinates of the centre of the smaller circle. (3)

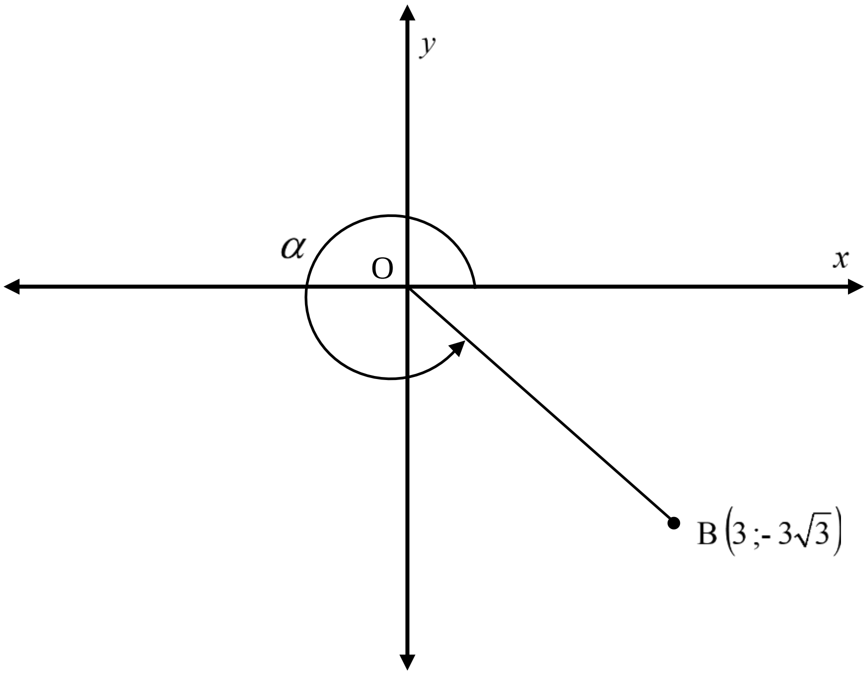
4.2.2 Length of the radius. (3)

4.3 Hence, or otherwise, determine whether point $C(9\,;3)$ lies inside or outside the circle centre at E. (3)

[18]

QUESTION 5

5.1 In the Cartesian plane below, the point $B\left(3\,;-3\sqrt{3}\right)$ and the reflex angle, α, are shown.



Determine **(without using a calculator)** the value of:

5.1.1 OB (2)

5.1.2 $\cos\left(\alpha + 30^{\circ}\right)$ (4)

5.2 Simplify:

$$\frac{\sin^{2}\left(90^{\circ} - x\right).\tan\left(360^{\circ} - x\right)}{\sin\left(-x\right)}$$ (4)

5.3 Prove that:

$$\cos\left(60^{\circ} + \theta\right) - \cos\left(60^{\circ} - \theta\right) = -\sqrt{3}\sin\theta$$ (3)

5.4 Consider the identity: $\dfrac{1 - \sin 2A}{\sin A - \cos A} = \sin A - \cos A$

5.4.1 Prove the identity. (4)

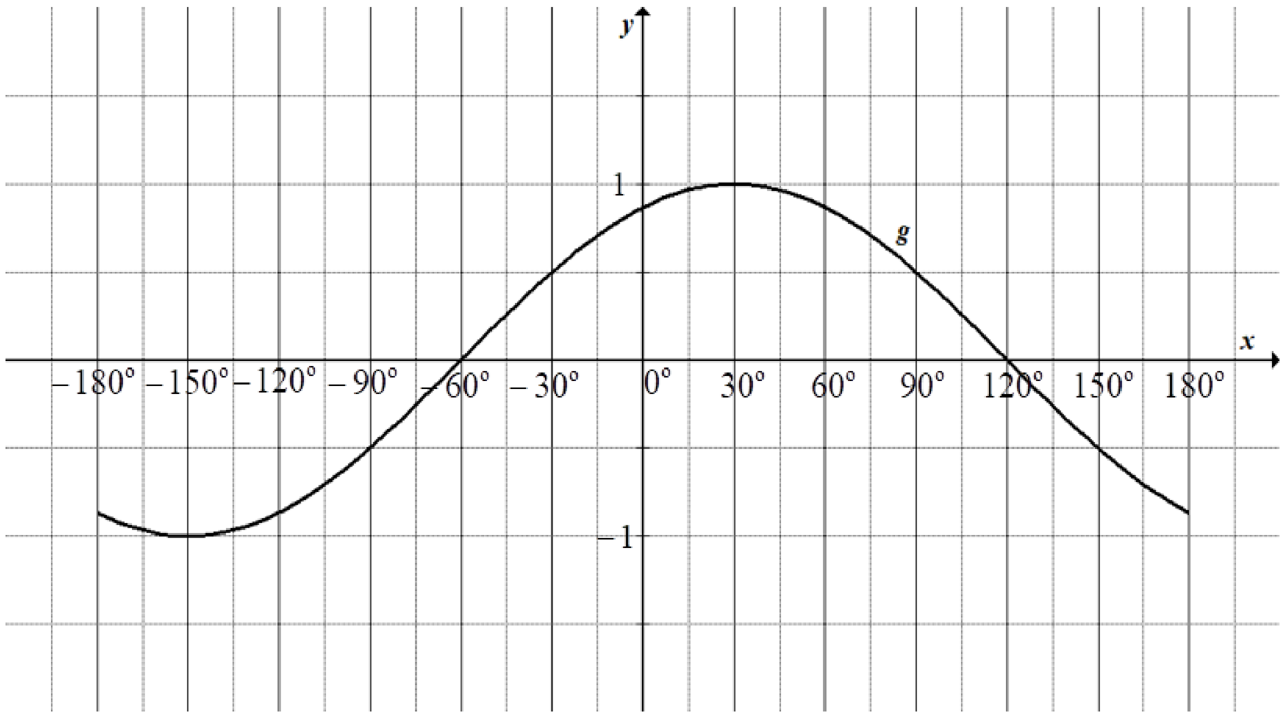
5.4.2 For which values of A in the interval $0^{\circ} < A < 180^{\circ}$ will the identity be undefined? (2)

[**19**]

QUESTION 6

6.1 Determine the general solution for $\sin 2x = \cos\left(x - 30^\circ\right)$. (5)

6.2 The diagram below shows the graph of $g(x) = \cos\left(x - 30^\circ\right)$ for the interval $x \in \left[-180^\circ; 180^\circ\right]$.



6.2.1 Write down the period of g. (1)

6.2.2 Determine the values of x for which the graph of g increasing. (2)

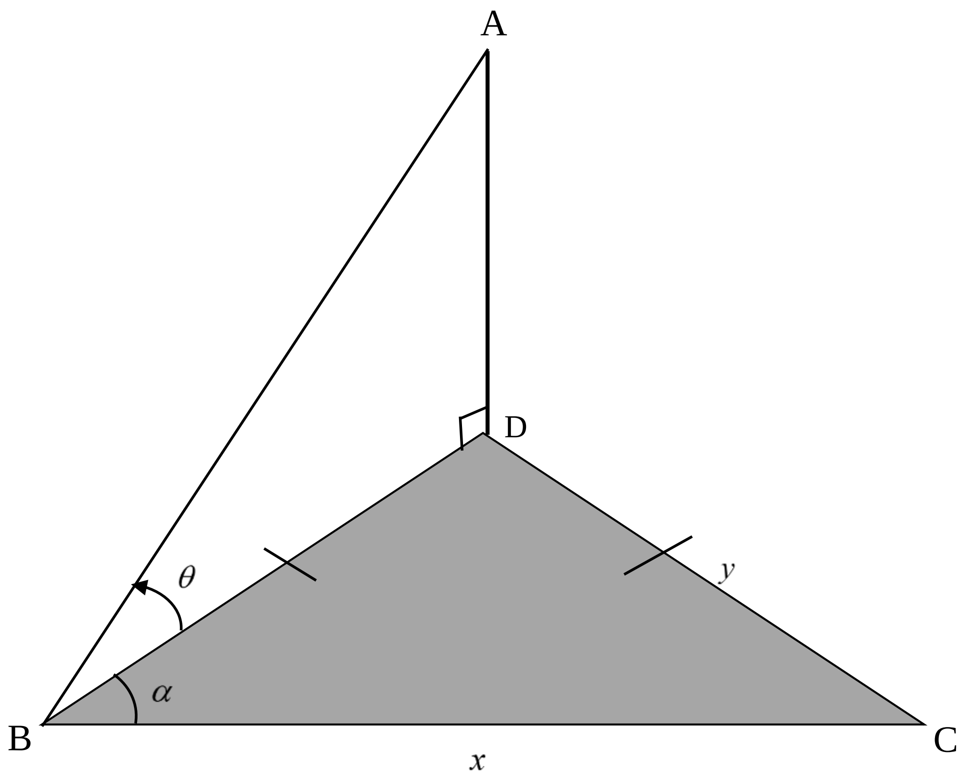
6.2.3 On the same system of axes draw the graph of $f(x) = \sin 2x$ for $x \in \left[-180^\circ; 180^\circ\right]$ in your ANSWER BOOK. (3)

6.2.4 Hence or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 180^\circ$ for which $f(x).g(x) < 0$. (3)

[14]

QUESTION 7

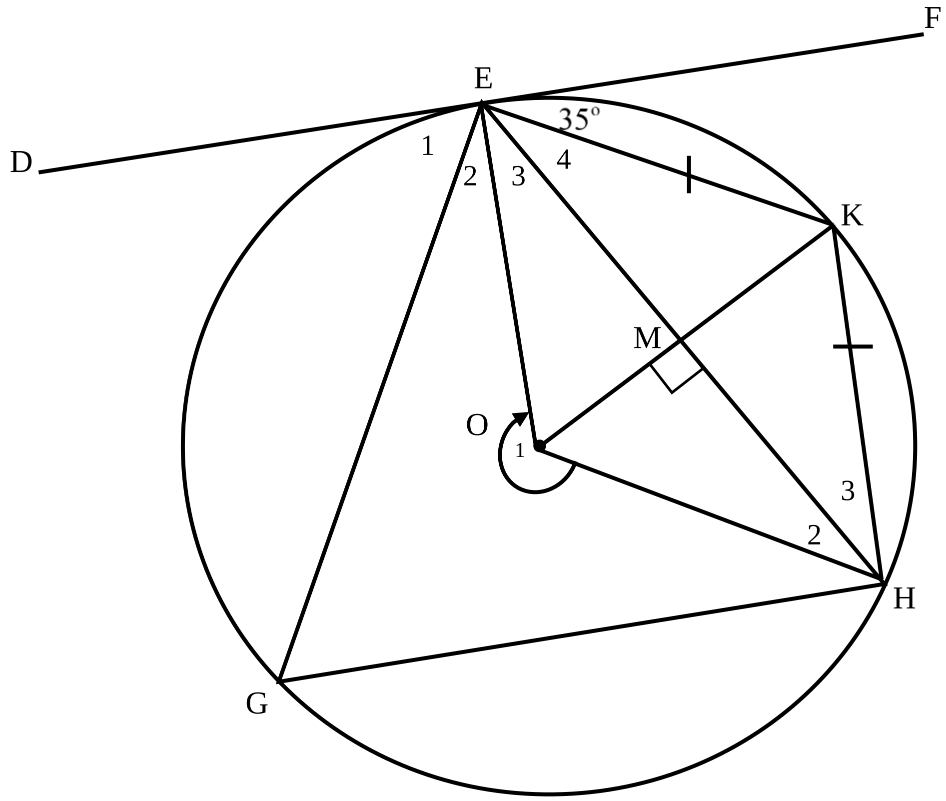
In the diagram below, B, C and D are three points on the same horizontal plane such that $BD = DC = y$. $C\hat{B}D = \alpha$ and $A\hat{B}D = \theta$. Line $BC = x$.



Prove that $AB = \dfrac{x}{2\cos\alpha\cos\theta}$ **[7]**

QUESTION 8

DF is a tangent to the circle at E. EKHG is a cyclic quadrilateral. $K\hat{E}F = 35^\circ$.
O is the centre of the circle. $OK \perp EH$ and $EK = HK$.



8.1 Determine, with reasons, the size of each of the following:

8.1.1 $\hat{E}_4$ (3)

8.1.2 $E\hat{K}H$ (2)

8.1.3 $\hat{G}$ (2)

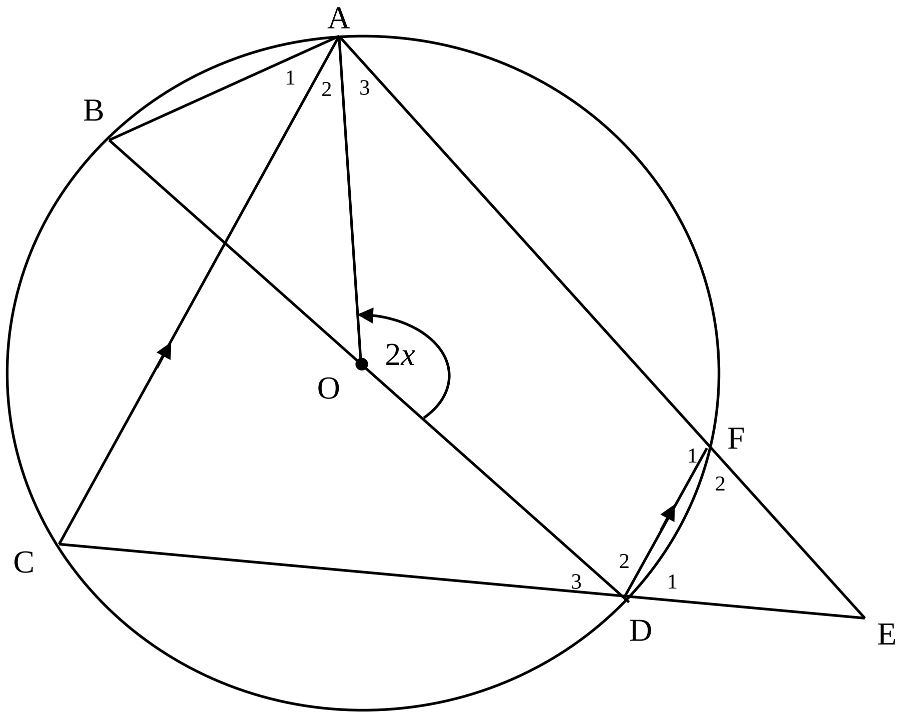
8.1.4 $\hat{O}_1$ (2)

8.2 It is further given that EH = 24 units. KM = 4 units and the radius of the circle EKHG is x. Determine the value of x. (4)

[13]

QUESTION 9

9.1 A circle with centre O is given below. Lines CD and AF are produced to E.
$\hat{AOD} = 2x$ and BD is the diameter. AC||FD.



9.1.1 Determine, with reasons, four other angles that are each equal to x. (6)

9.1.2 Express $\hat{E}$ in terms of x. (2)

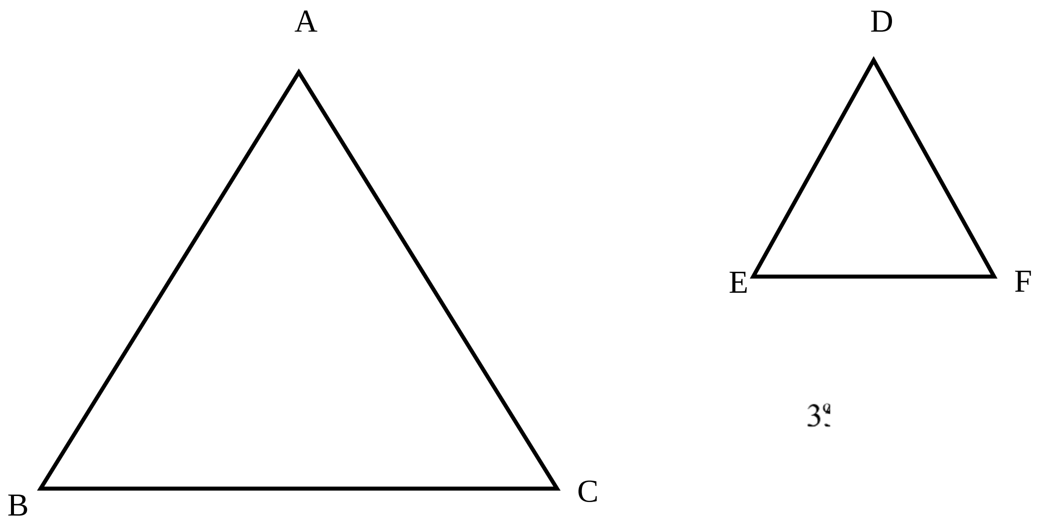
9.1.3 Prove that AODE is a cyclic quadrilateral. (2)

9.2 It is further given that ED : DC = 8 : 12 and FE = 10. Calculate the length of AF. (3)

[13]

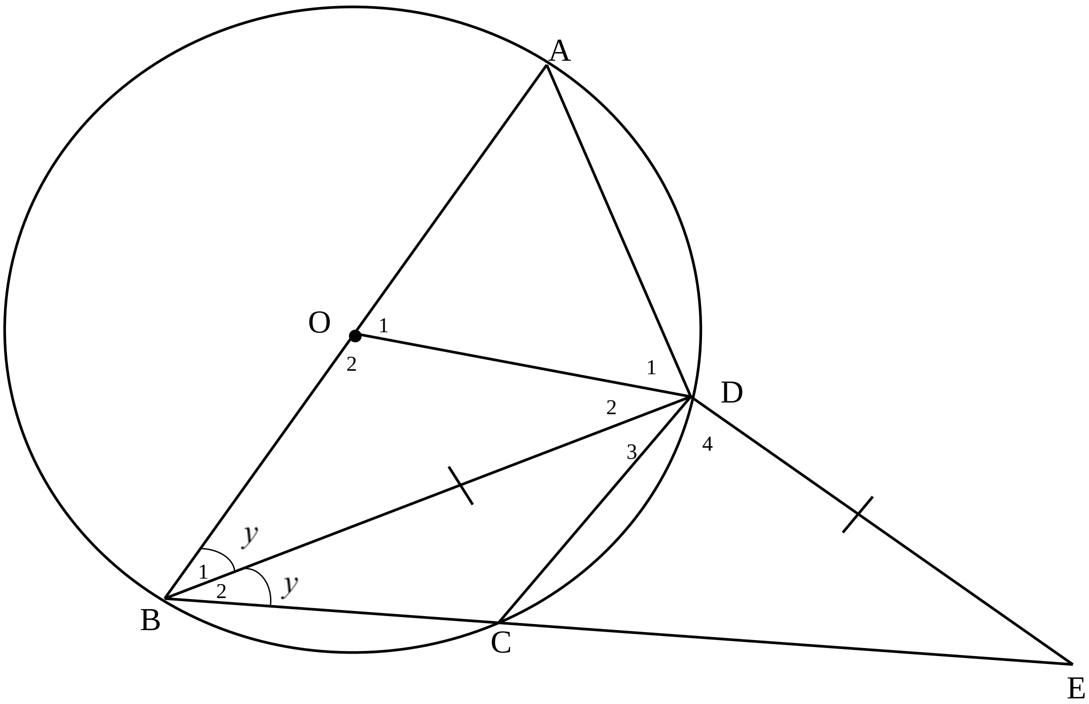
QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem that states that if two triangles are similar, then the sides are proportional, i.e. $\dfrac{DE}{AB} = \dfrac{EF}{BC} = \dfrac{DF}{AC}$. (5)

10.2 In the diagram below, AB is the diameter of a circle with centre O. BD and BC are chords. BD = DE. BCE is a line. $\hat{B}_1 = \hat{B}_2 = y$.



Prove that:

10.2.1 $\hat{D}_4 = 90^{\circ}$ (5)

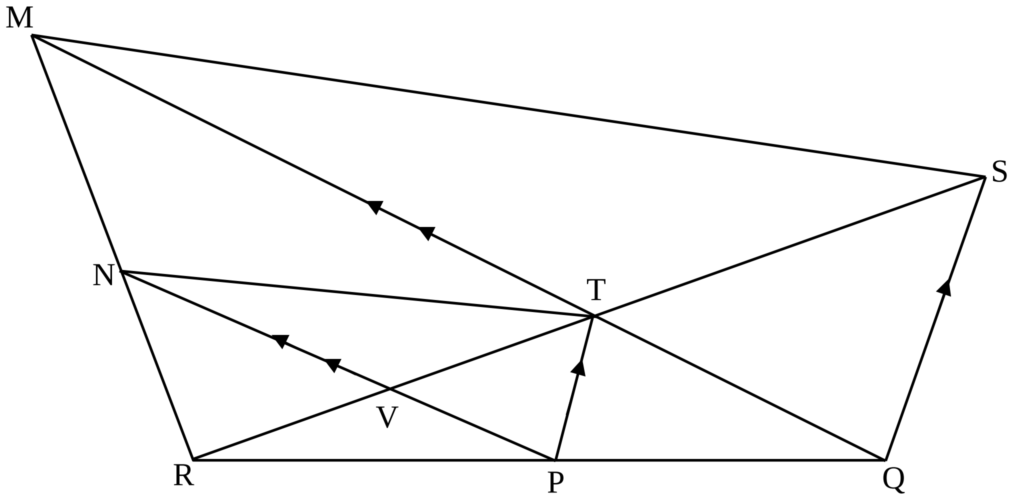
10.2.2 $\Delta BOD \,|||\, \Delta BDE$ (3)

10.2.3 $DE^2 = BE.OD$ (4)

[17]

QUESTION 11

In the diagram below, RQSM is a quadrilateral. N and P are points on MR and RQ respectively such that MQ || NP. The diagonals intersect at T. P is a point on RQ such that TP || SQ. TR and NP intersect at V.



11.1 Prove that NT || MS. (4)

11.2 If $RN = \frac{3}{5}NM$ and RS = 32, determine VT. (4)

[8]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1-r}\;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\,;\,y) \to (x\cos\theta + y\sin\theta\,;\, y\cos\theta - x\sin\theta)$$
$$(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$