education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

September 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and an information sheet.

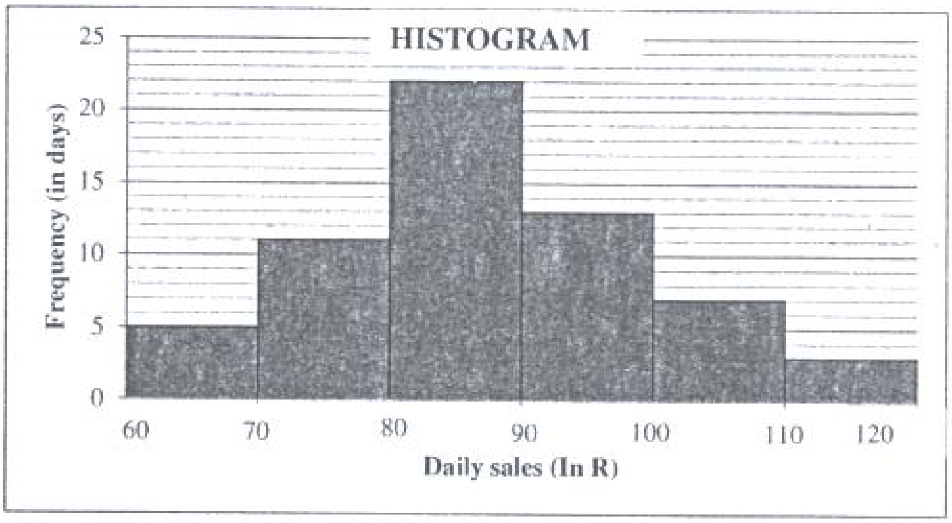
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A street vendor has kept record of his daily sales for November and December 2017. The daily sales(in Rand) is shown in the histogram below.



1.1 A table summarising the daily sales for November and December 2017 is drawn in the answer book. Complete the cumulative frequency column in the table. (3)

1.2 Draw an ogive (cumulative frequency graph) of the daily sales for November and December 2017 on the grid provided IN THE ANSWER BOOK. (4)

1.3 On how many days did the daily sales exceed 100? (2)

1.4 Use the ogive IN THE ANSWER BOOK to estimate the median value of the daily sales. (2)

[11]

QUESTION 2

2.1 A learner conducted an experiment to investigate the relationship between the age and resting heart rate (in beats per minute). He sought the assistance of the local clinic. The information for 12 people is shown in the table below:

Age	59	32	42	50	22	39	21	20	27	40	29	47
Resting heart rate (beats per minute)	88	74	74	93	85	71	78	82	70	75	95	75

2.1.1 Determine the equation of the least squares regression line. (3)

2.1.2 Predict the resting heart rate of a person aged 57 years. (2)

2.1.3 Write down the correlation coefficient for the data. (1)

2.1.4 Use the correlation coefficient to comment on the prediction done in Question 2.1.2. (1)

2.2 A group of students wrote a statistics test. The following information was obtained from the marks obtained in this test:

$$\sum_{n=1}^{22} x_n = 1320$$

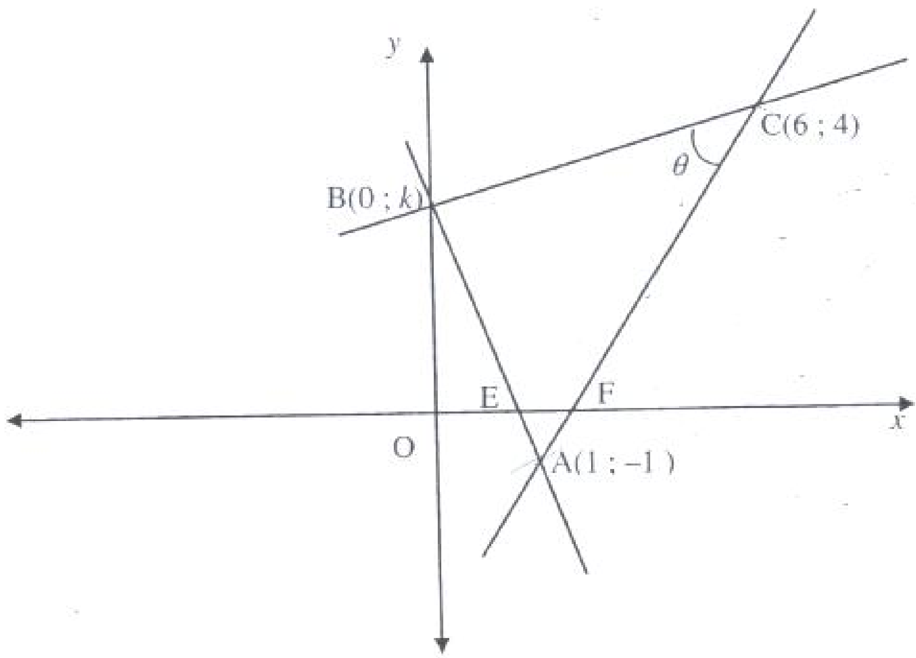
2.2.1 How many students wrote the test? (1)

2.2.2 Calculate the mean mark for the test. (2)

[10]

QUESTION 3

In the diagram, $A(1;-1)$, $B(0;k)$ and $C(6;4)$ are the vertices of ΔABC. The equations of the sides AB and AC are $y+3x-2=0$ and $y=x-2$ respectively. AB cuts the x-axis at E and AC cuts the x-axis at F.



3.1 Write down the value of k. (2)

3.2 Calculate the length of AC and leave your answer in simplest surd form. (2)

3.3 Prove that $A\hat{B}C = 90°$. (3)

3.4 Calculate the size of θ. (5)

3.5 Determine the equation of the circle passing through A, B and C in the form $(x-a)^2+(y-b)^2=r^2$. (4)

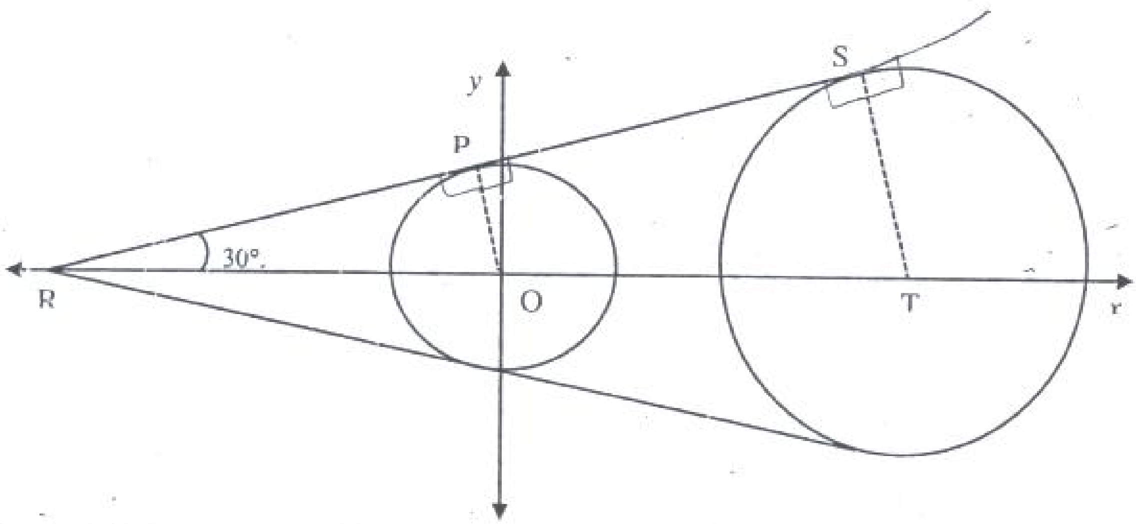
3.6 If D is a point in the first quadrant, calculate the coordinates of D such that ABCD in that order, forms a parallelogram. (4)

[20]

QUESTION 4

The diagram below shows a representation of the chain of a bicycle attached to two circular cogs as represented in the Cartesian plane. The equations of the circles are given by $x^2 + y^2 = 1$ and $(x-6)^2 + y^2 = 9$.

RPS is a common tangent to the smaller and larger circles at P and S respectively, with R a point on the negative x-axis. T is the centre of the larger circle. The angle of inclination between the tangent RPS and the horizontal axis is 30°.



4.1 Write down the coordinates of T. (1)

4.2 Write down the length of the radius of the larger circle. (1)

4.3 If R(−3 ;0), determine the equation of tangent PRS in the form $y = mx + c$. Leave answer in surd form. (4)

4.4 Determine the equation of the radius ST in the form $y = mx + c$. Leave answer in surd form. (3)

4.5 Determine the coordinates of S. (5)

4.6 Determine the distance between the two circular cogs. (4)

[18]

QUESTION 5

5.1 Simplify to one trigonometric ratio:

$$\frac{\sin(A-180^\circ).\tan(180^\circ-A).\cos A}{\cos(90^\circ+A)}$$ (5)

5.2 If $\sin 28^\circ = a$ and $\cos 32^\circ = b$, determine the following in terms a and b.

5.2.1 $\cos 28^\circ$ (2)

5.2.2 $\cos 64^\circ$ (2)

5.2.3 $\sin 4^\circ$ (4)

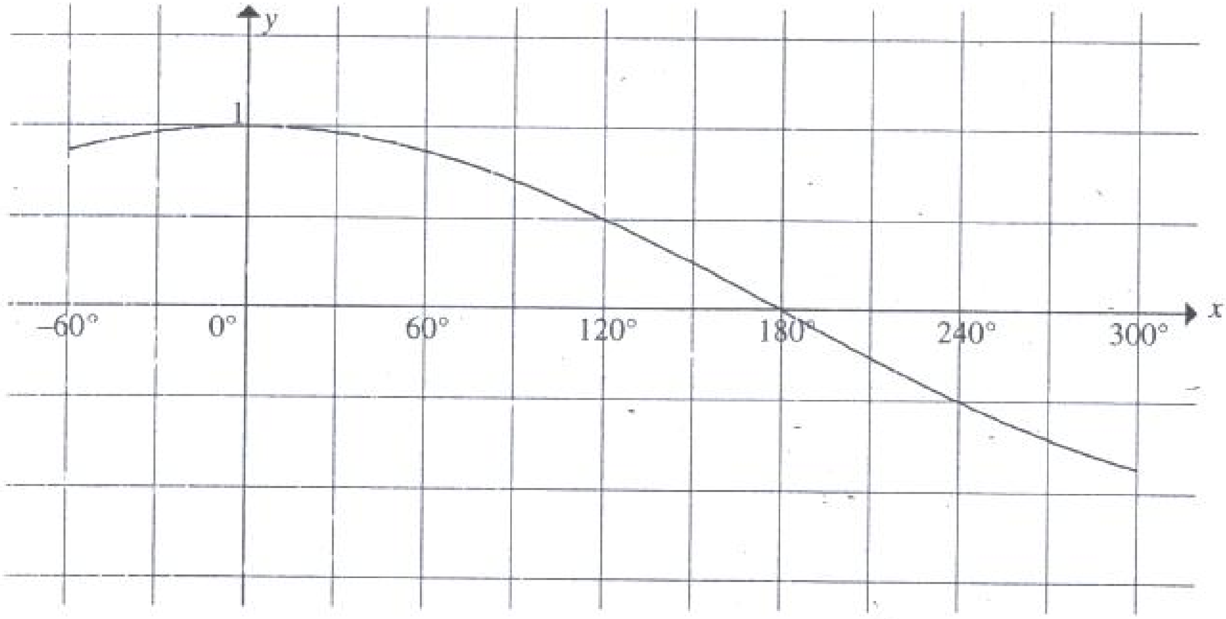
5.3 Prove the identity: $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=\frac{2\tan x}{\cos x}$ (5)

[18]

QUESTION 6

6.1 Solve for x if $\sin(x+60^\circ)=\cos\frac{1}{2}x$ and $x\in[-60^\circ;300^\circ]$. (7)

6.2 In the diagram below, the graph of $f(x)=\cos\frac{1}{2}x$ is drawn for $x\in[-60^\circ;300^\circ]$.



6.2.1 On the grid provided IN THE ANSWER BOOK, draw the graph of $g(x)=\sin(x+60^\circ)$ for $x\in[-60^\circ;300^\circ]$. Clearly show all maximum and minimum points, the intercepts with the axes and the endpoints. (3)

6.2.2 Use the solutions obtained in 6.1 as well as the graphs drawn in 6.2.1 to determine the value(s) of $x\in[-60^\circ;300^\circ]$ for which:

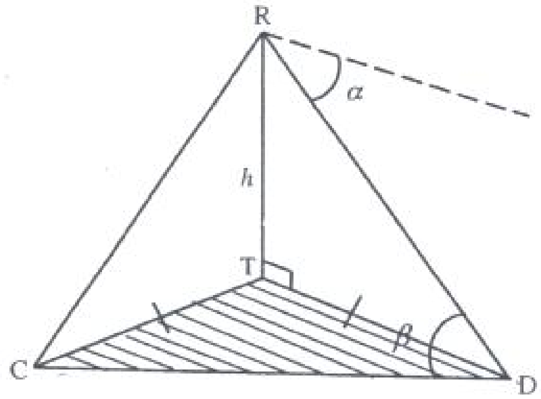
6.3.1 $f(x)<g(x)$ (3)

6.3.2 $f(x).g(x)\le 0$ (3)

[16]

QUESTION 7

In the diagram, RT represents the height of a vertical tower. C and D represent two points equidistant from T and which lie on the same horizontal plane as T. The height of the tower is h. The angle of depression of D from R is α. $R\hat{D}C = \beta$.



7.1 Determine the size of $C\hat{R}D$ in terms of β. (2)

7.2 Prove that $CD = \dfrac{2h.\cos\beta}{\sin\alpha}$. (5)

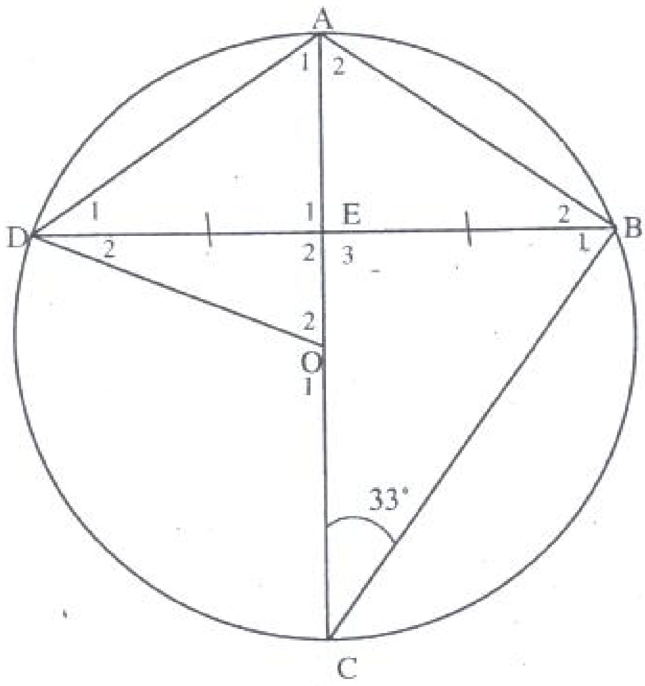
7.3 Calculate the height of the tower, rounded off to nearest unit, if

$CD = 5{,}4$ units, $\alpha = 51°$ and $\beta = 65°$. (2)

[9]

QUESTION 8

In the diagram, AC is the diameter of the circle with centre O. AC and chord DB intersect at E such that DE = EB. Chords AB, BC and AD and radius OD are drawn. $A\hat{C}B = 33°$.



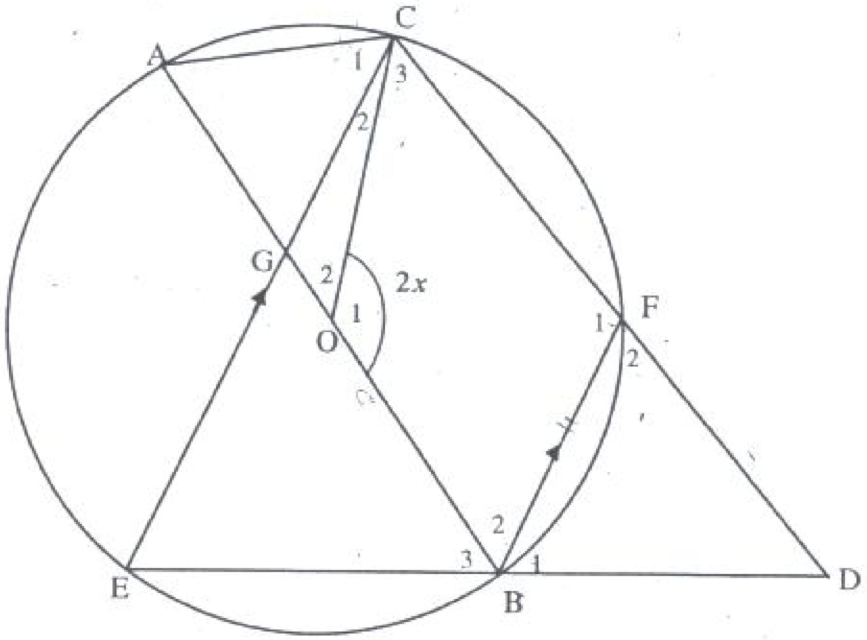
Determine the size of:

8.1	$\hat{D}_1$	(2)
8.2	$\hat{A}_1$	(3)
8.3	$\hat{O}_1$	(2)
8.4	$\hat{D}_2$	(2)
8.5	$\hat{A}_2$	(3)

[12]

QUESTION 9

In the diagram below, AB passes through the centre O of the circle. Chords CF and EB are produced to meet at D. EC ∥ BF. Chord AC and radius OC are drawn. Let $\hat{O}_1 = 2x$.



9.1 Determine, in terms of x, the size of $\hat{F}_2$. (3)

9.2 Prove that DF = BD. (4)

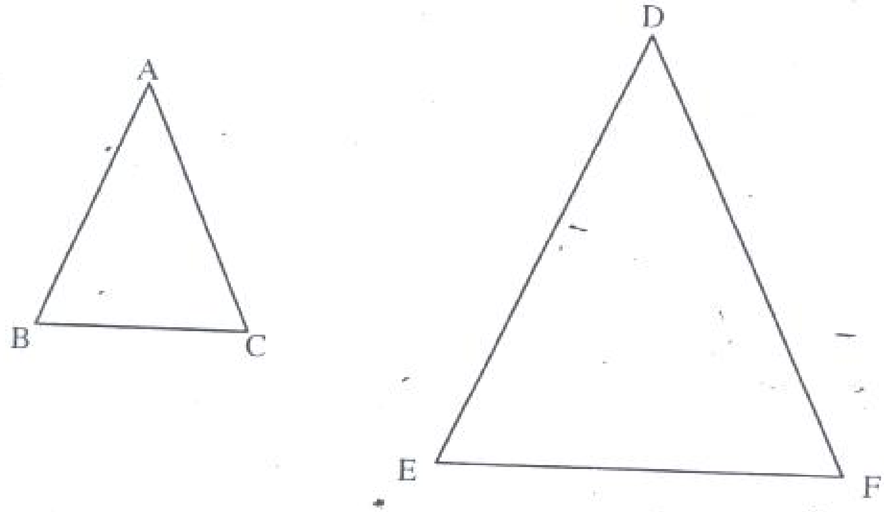
9.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)

9.4 If it is further given that CG = FB, FD = 1 and BG = 2, determine the value of $\frac{BD}{ED}$. (5)

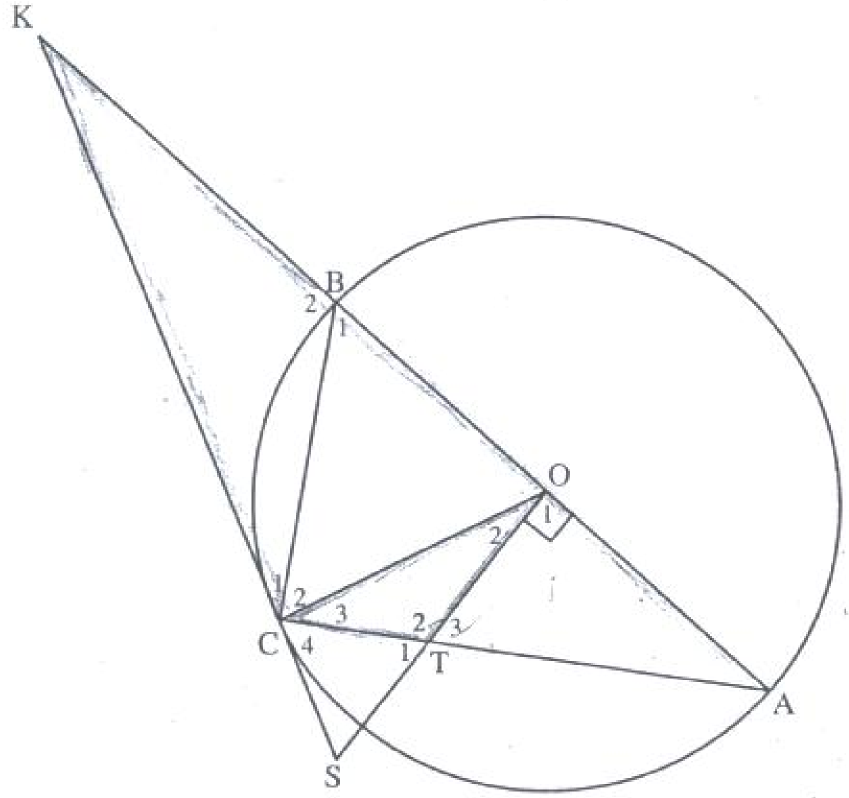
[16]

QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Proof the theorem that states that $\frac{AB}{DE} = \frac{AC}{DF}$. (6)



10.2 In the diagram, AB is the diameter of the circle with centre O. AB is produced to K such that SK is a tangent to the circle at C. $SO \perp AB$. CA and SO intersect at T.



Prove that:

10.2.1 $\Delta CKB \parallel\!\mid \Delta AKC$ (3)

10.2.2 $K\hat{C}T = \hat{T}_2$ (4)

10.2.3 $\Delta COT \parallel\!\mid \Delta AKC$ (3)

10.2.4 $BK.AK = \frac{OT^2.CA^2}{CT^2}$ (4)

[20]

TOTAL: 150 MARKS