Metro North Education District

Grade 12

Mathematics P2
September 2019

MARKS: 150
TIME: 3 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included at the end of the question paper.

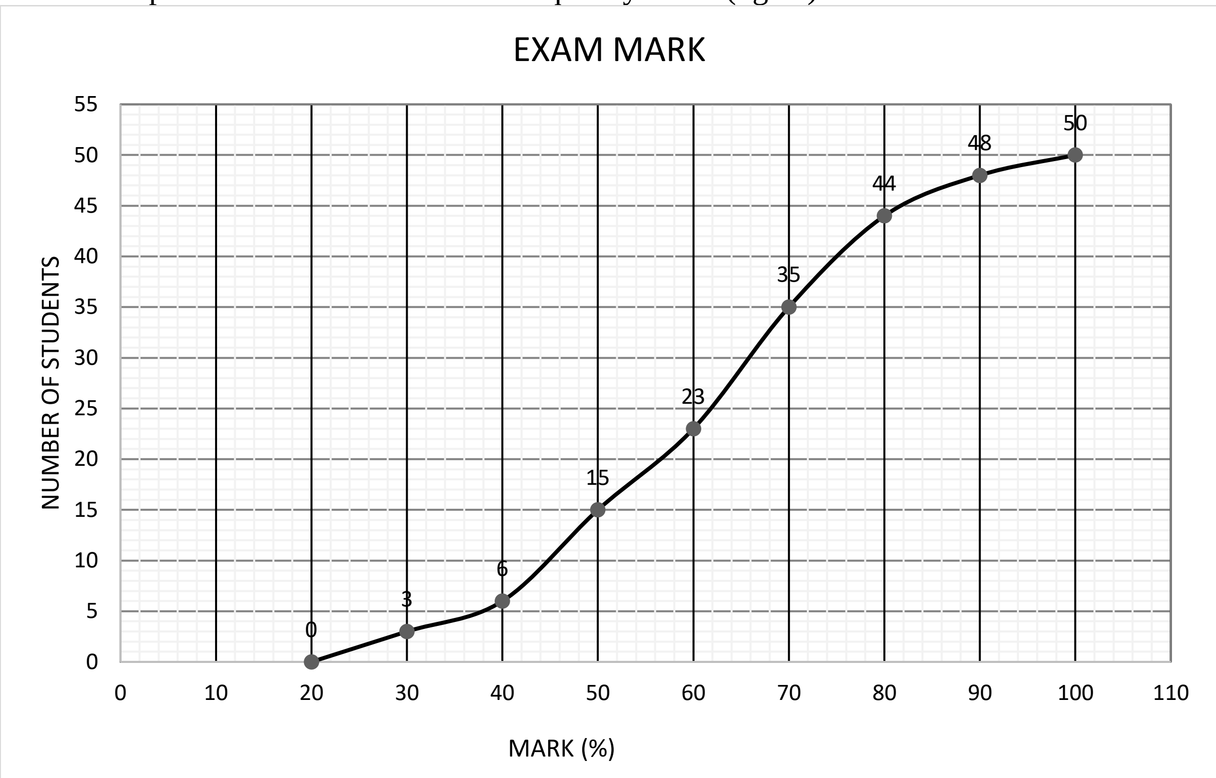
   Number the answers correctly according to the numbering system used in this question paper.
9. Write legibly and present your work neatly.

This question paper consists of 13 pages and 1 information sheet and

an answer book of 22 pages.

QUESTION 1

1.1 A number of learners' marks for acertain exam were requested in a survey. Marks from the exam are represented in the cumulative frequency curve (ogive) below:



1.1.1 How many learners participated in this survey? (1)

1.1.2 Complete the frequency column in the table given in the ANSWER BOOK.

Interval	Frequency	Cumulative frequency
$20 \leq x < 30$		3
$30 \leq x < 40$		6
$40 \leq x < 50$		15
$50 \leq x < 60$		23
$60 \leq x < 70$		35
$70 \leq x < 80$		44
$80 \leq x < 90$		48
$90 \leq x < 100$		50

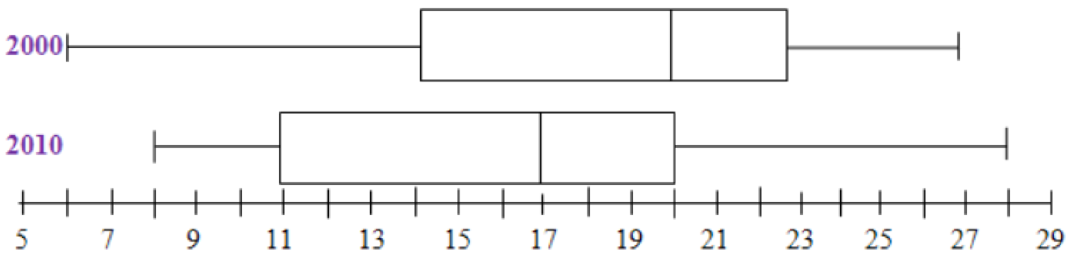
(2)

1.1.3 Use the graph to determine the median mark. (2)

1.1.4 Write down the modal class. (1)

1.1.5 If a learner needs 70% to qualify for a reward, how many learners qualified? (2)

1.2 The results from a test for students for the year 2000 and the year 2010 are illustrated in the box and whisker plot. The total mark for the test was 30.



1.2.1 Determine the interquartile range for 2010. (2)

1.2.2 In which year did students perform better in the test? Motivate your answer. (2)

[12]

QUESTION 2

A group of 12 learners have been asked to measure their resting heart rate (beats per minute) and the time (in minutes) that they exercise in a week. The data below was gathered.

Minutes of exercise per week	Resting heart rate (BPM)
30	82
40	77
60	75
90	70
140	68
180	67
270	60
350	58
360	52
420	50
440	48
500	45

2.1 Represent the data as a scatter plot on the grid provided (3)

2.2 Calculate the correlation coefficient for the given data. (2)

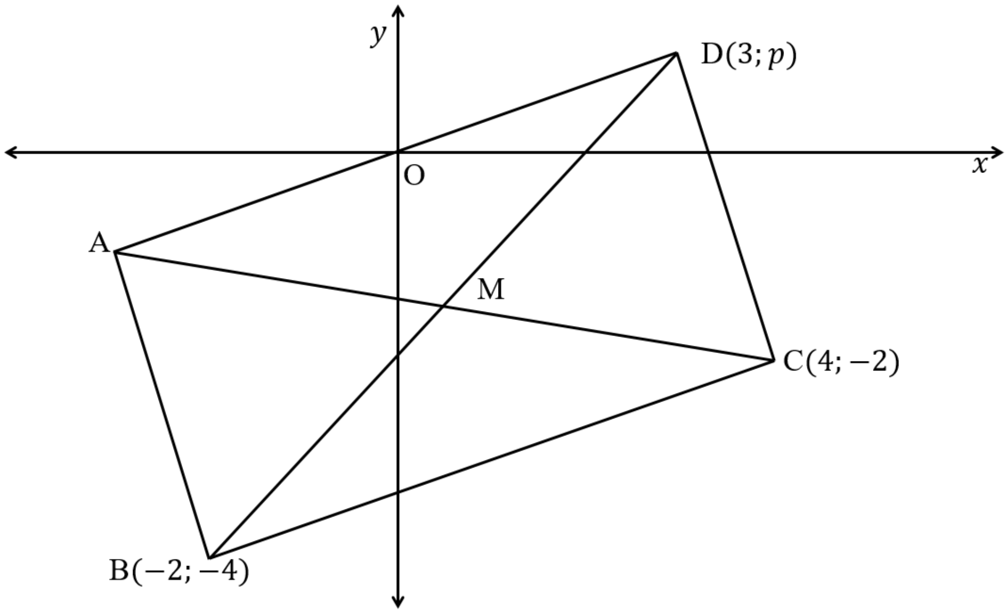
2.3 Use the data to calculate the equation of the least squares regression line. (3)

2.4 If a learner has a resting heart rate of 65 beats per minute, how many hours would you expect him to exercise per week? (3)

[11]

QUESTION 3

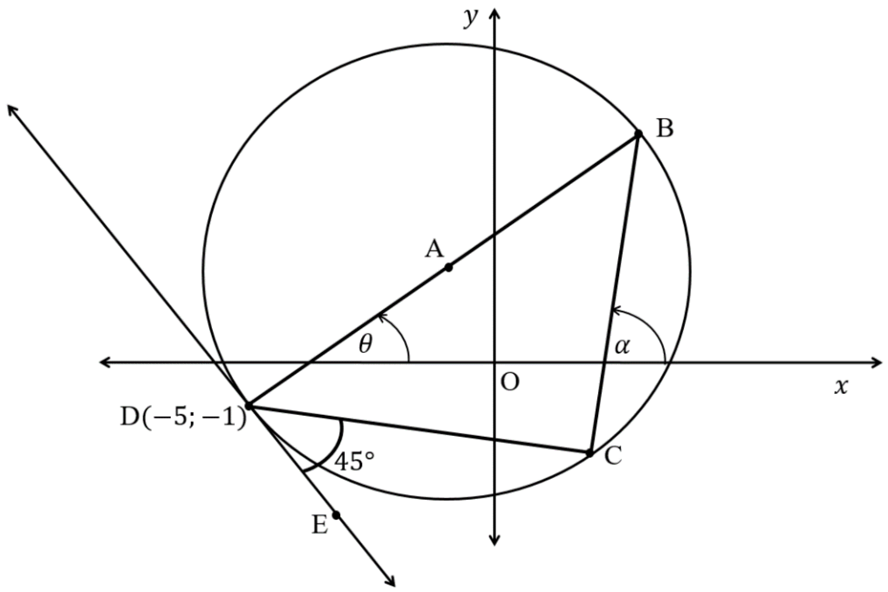
In the diagram A, B$(-2;-4)$, C$(4;-2)$ and D$(3;\ p\ )$ are the vertices of a rectangle. The diagonals AC and BD intersect at M.



3.1	Given that the length of AC is $\sqrt{50}$ units, show that $p = 1$.	(4)
3.2	Determine the coordinates of M.	(3)
3.3	Calculate the gradient of DC	(2)
3.4	Determine the equation of line AB in the form $y = mx + c$.	(2)
		[11]

QUESTION 4

In the diagram is the circle with equation $(x+1)^2 + (y-2)^2 = 25$.
DB is the diameter of the circle and A the centre of the circle. DE is a tangent to the circle at D$(-5;-1)$. The angle E$\widehat{D}$C = 45°. The inclination angles of AD and BC is θ and α respectively. B and C are points on the circumference of the circle.



4.1 Determine:

4.1.1 The coordinates of A, the centre of the circle. (2)

4.1.2 The coordinates of B (3)

4.1.3 The gradient of AD. (2)

4.1.4 The value of θ, the inclination angle of AD. (2)

4.1.5 The equation of the tangent DE. (3)

4.2 Calculate the gradient of BC. (4)

4.3 Another circle with equation $x^2 + y^2 - 6x + 2y = 8$ is given.

Show that:

4.3.1 The coordinates of the centre of the circle is M$(3;-1)$. (4)

4.3.2 The two circles will intersect each other. Show all calculations. (4)

[24]

QUESTION 5

5.1 If $\sin 40°.\cos 22° + \cos 40°.\ \sin 22° = k,$ determine without the use of a calculator, the value of the following in terms of k.

5.1.1 $\sin 62°$ (2)

5.1.2 $\tan 118°$ (4)

5.1.3 $\sin 14°.\ \cos 14°$ (3)

5.2 Prove the following identity:

$$\frac{1 - \cos 2\theta}{\sin 2\theta \times \tan\theta} = 1$$ (4)

5.3 For which value(s) of A will the following expression be real?

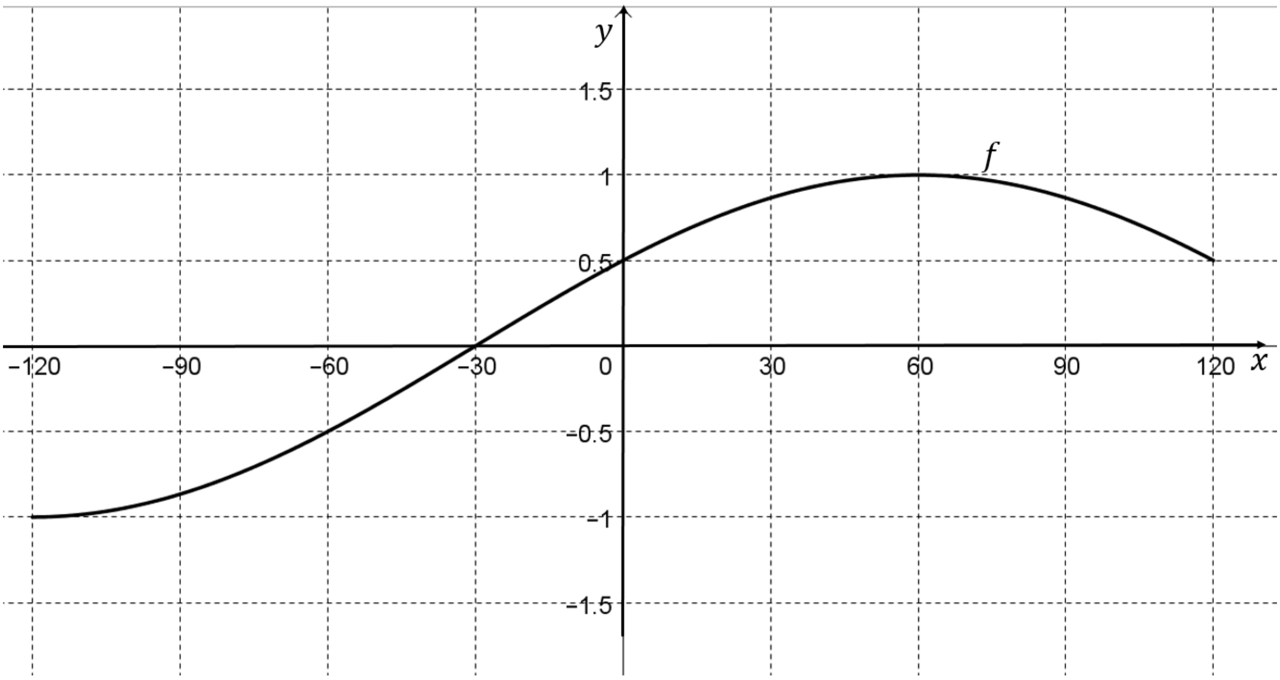
$$\sqrt{\sin(180° + \mathrm{A}).\cos(90° + \mathrm{A}) - \tan 45°}$$

(6)

[19]

QUESTION 6

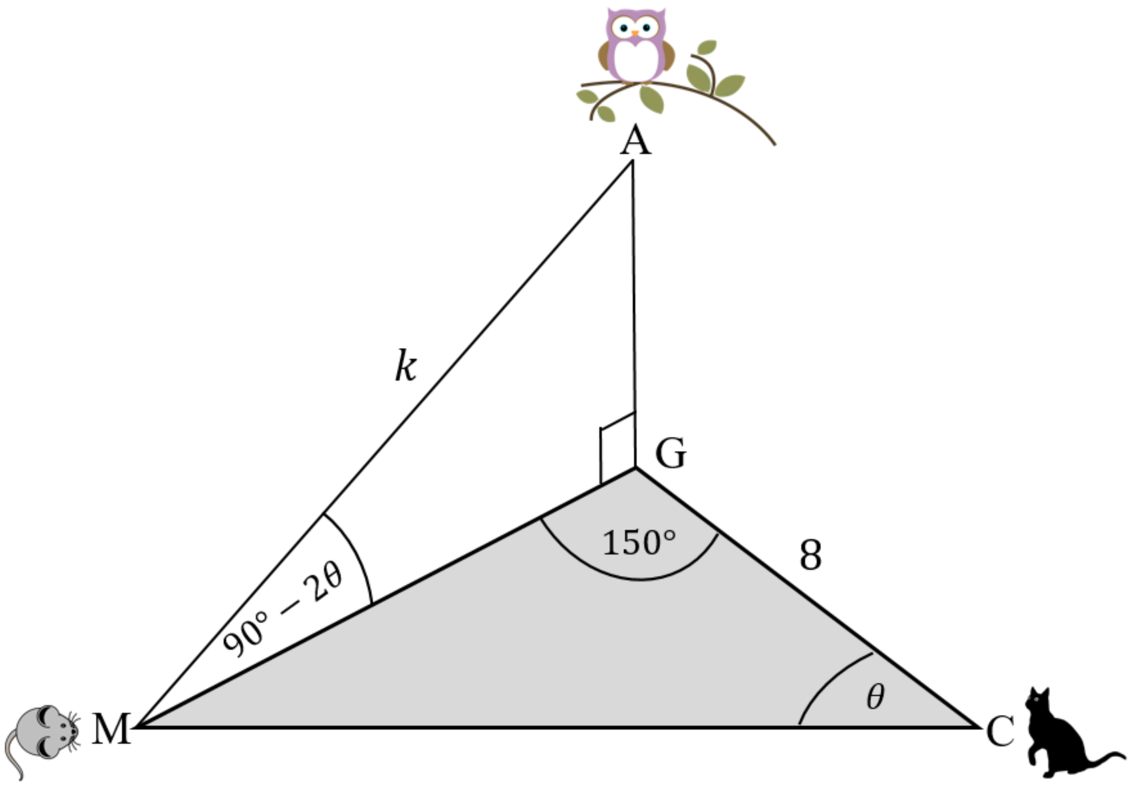
In the diagram is the graph of $f(x) = \sin(x + a)$ for the interval [-120°;120°]



6.1	Determine the numerical value of a.	(1)
6.2	On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(3x)$ for the interval $x \in [-120°; 120°]$. Clearly show ALL intercepts with the axes, the turning point(s) and endpoint(s) of the graph.	(4)
6.3	Determine the general solution for the following: $f(x) = g(x)$	(5)
6.4	Determine the values of x in the interval $x \in [0°; 120°]$, for which $f(x) > g(x)$.	(2)
6.5	Describe the transformation from graph g to the graph of $k(x) = \cos(60° - 3x)$.	(2)
		[14]

QUESTION 7

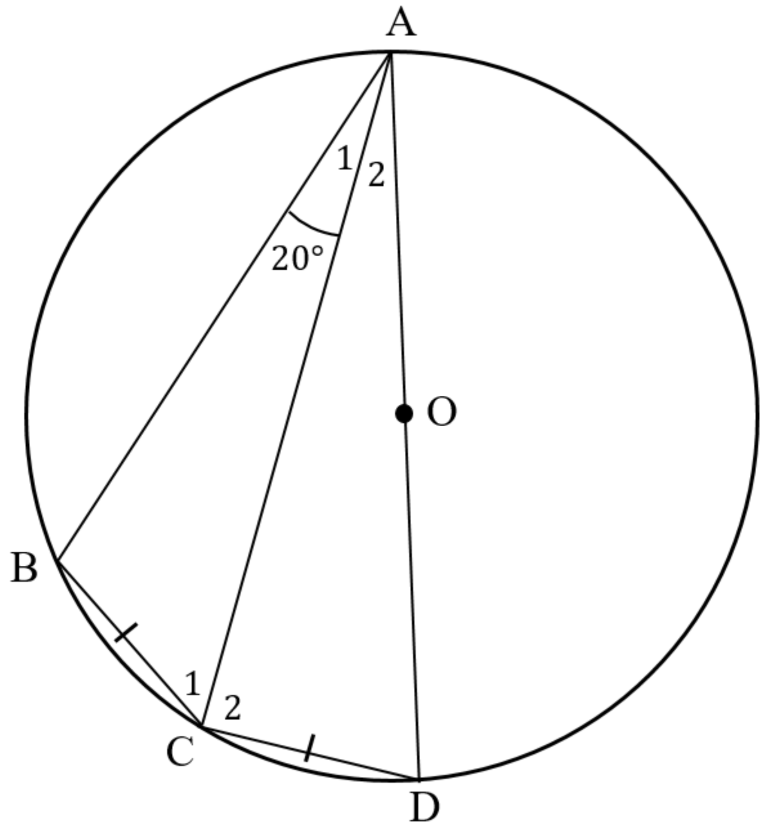
A mouse on the ground is looking up to an owl in a tree and a cat to his right on the ground. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$.
$\text{AM} = k\ units$, $\text{GC} = 8\ units$, $\text{M}\hat{\text{G}}\text{C} = 150°$ and $\text{M}\hat{\text{C}}\text{G} = \theta$



7.1	Give the size of $\text{M}\hat{\text{A}}\text{G}$ in terms of θ.	(1)
7.2	Show that $\text{MG} = k \sin 2\theta$	(2)
7.3	Show that $\text{MC} = k \cos \theta$	(4)
7.4	Show that the area of $\Delta\text{MGC} = 2k \sin 2\theta$	(2) **[9]**

QUESTION 8

8.1 In the diagram, AOD is a diameter of the circle centred at O.
BC = CD and $\hat{A}_1 = 20°$.



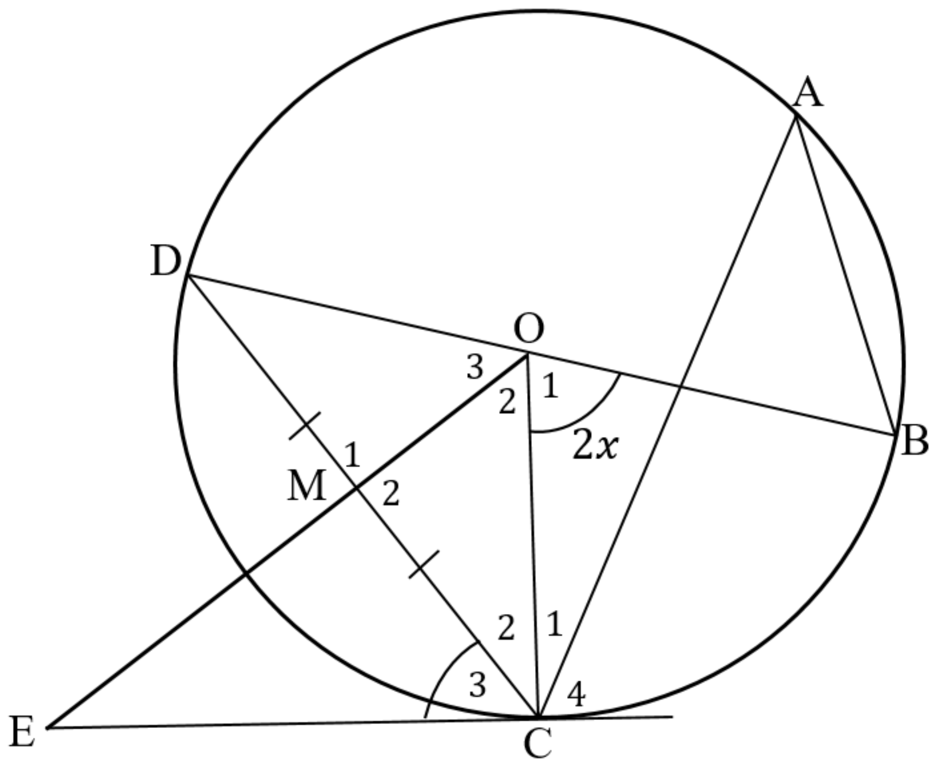
Determine, with reasons, the size of each of the following angles:

8.1.1 $\hat{A}_2$ (2)

8.1.2 $\hat{C}_2$ (2)

8.1.3 $A\hat{B}C$ (3)

8.2 In the diagram, O is the centre of the circle and EC is a tangent to the circle at C.
$DM = MC$ and OME is a straight line.
Let $\hat{O}_1 = 2x$.



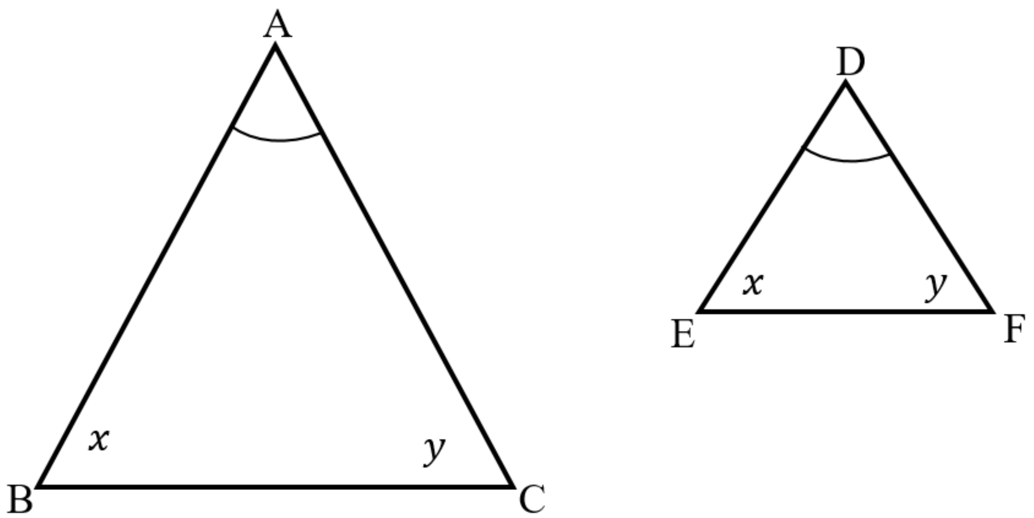
8.2.1 Give, with reasons, THREE angles equal to x. (6)

8.2.2 Prove that $\hat{O}_2 = 90° - x$ (3)

8.2.3 Prove that DOCE is a cyclic quadrilateral. (4)
[20]

QUESTION 9

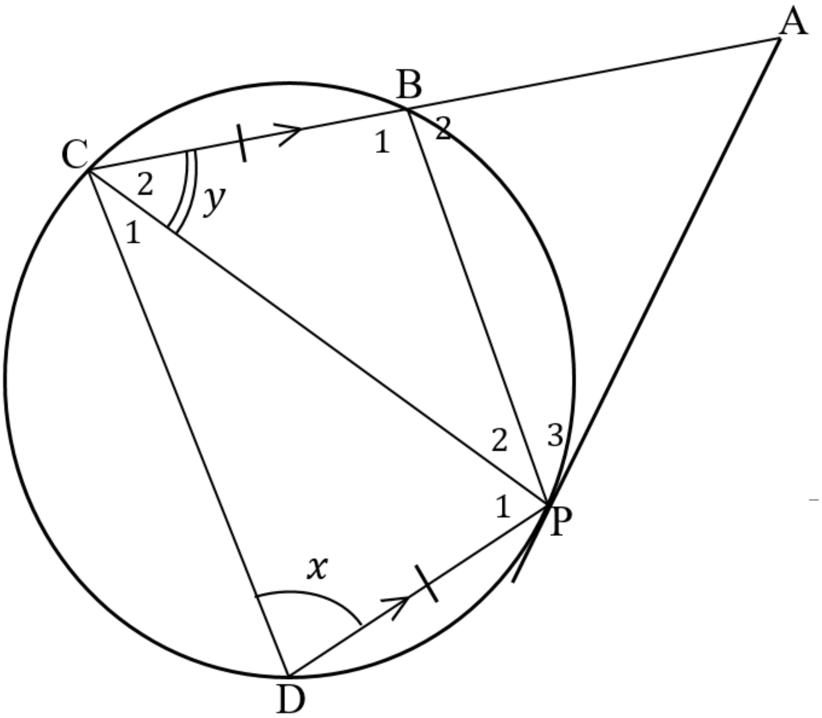
9.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{DE}{AB} = \frac{DF}{AC}$

(6)

9.2 AP is a tangent to the circle at P. CB ∥ DP and CB=DP. CBA is a straight line.
Let $\widehat{D} = x$ and $\hat{C}_2 = y$.



Prove, with reasons that:

9.2.1 $\Delta APC \,|||\, \Delta ABP$ (4)

9.2.2 $AP^2 = AB \cdot AC$ (1)

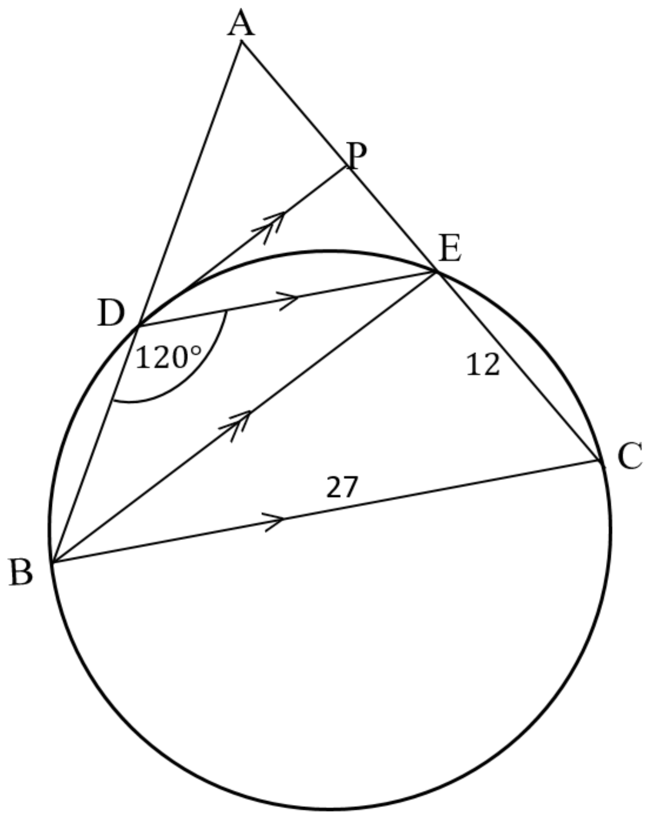
9.2.3 $\Delta APC \,|||\, \Delta CDP$ (4)

9.2.4 $AP^2 + PC^2 = AC^2$ (4)

[19]

QUESTION 10

In Δ ABC in the diagram, D is a point on AB such that AD : DB = 5 : 4.
P and E are points on AC such that DE ∥ BC and DP ∥ BE.
BC is NOT a diameter of the circle.
Given: $\text{B}\hat{\text{D}}\text{E} = 120°$, EC = 12 units and BC = 27units.



10.1 Determine, with reasons:

10.1.1 The length of AE (3)

10.1.2 $\dfrac{\text{area of } \Delta \text{ AEB}}{\text{area of } \Delta \text{ ECB}}$ (2)

10.2 Hence, determine the length of DP if ΔADP ||| ΔABE (6)

[11]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$