GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2022

10612

MATHEMATICS

PAPER 2

TIME: 3 hours

MARKS: 150

14 pages + 1 information sheet and an answer book of 24 pages

MATHEMATICS: Paper 2

10612E

X10

P.T.O.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before you answer the paper:

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

Mrs Molefe decided to research the effectiveness of her online classes. She divided the Grade 12 pupils fairly into Grade 12A and Grade 12B. Grade 12A attended face-to-face classes and Grade 12B attended online classes. Both classes were taught by Mrs Molefe for the same duration.

The table below shows the time spent teaching (in hours) and the average results achieved by the learners in their weekly tests as percentages (%).

TIME SPENT TEACHING (in hours)	2	8	4	6	12	10	11
AVERAGE RESULT OF 12A (as %)	42	62	48	52	64	63	67
AVERAGE RESULT OF 12B (as %)	9	63	45	47	61	64	62

1.1 Determine the equation of the least squares regression line of Grade 12A. (3)

1.2 Write down the correlation coefficient of the Grade 12A results in respect of the time spent teaching. (1)

1.3 Comment on the correlation between the time spent teaching and the average result of Grade 12A. (1)

1.4 The equation of the least squares regression line of Grade 12B is $y = 15{,}74 + 4{,}54x$. Calculate the difference in the result achieved by each class had Mrs Molefe spent the average time to complete a particular section. (3)

1.5 Identify an outlier in Grade 12B. (1)

1.6 Indicate a valid reason for this outlier. (1)
[10]

QUESTION 2

Formula 1 (F1) race car drivers have to endure high G-forces at extremely high temperatures. They tend to lose close to 4 kg of weight after every race.

2.1 The table below shows the total weight lost by 40 different race car drivers after the duration of one race.

INTERVAL OF WEIGHT LOST (IN GRAMS)	NUMBER OF DRIVERS
$0 \leq w < 500$	1
$500 \leq w < 1\ 000$	2
$1\ 000 \leq w < 1\ 500$	3
$1\ 500 \leq w < 2\ 000$	8
$2\ 000 \leq w < 2\ 500$	6
$2\ 500 \leq w < 3\ 000$	15
$3\ 000 \leq w < 3\ 500$	5
Total	**40**

2.1.1 Write down the modal class of the data. (1)

2.1.2 Calculate the estimated mean weight-loss of the race car drivers. (3)

2.2 The recording of the weight lost in a second race was made a month later. The amount of weight lost in race 2 was *k* grams more than in race 1.
It is given that the maximum value of the ogive, representing race 2 was (3 504 ; 40) and the graph was grounded at (4 ; 0).

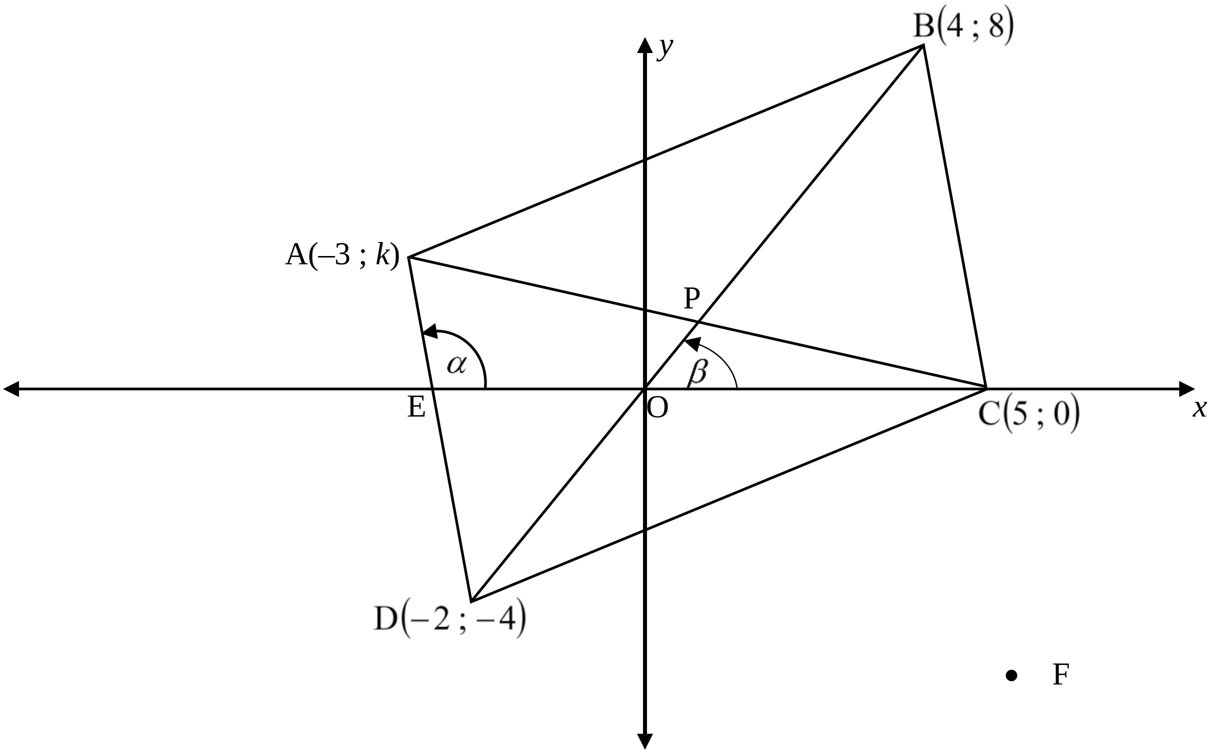
2.2.1 Sketch the ogive (cumulative frequency graph) representing race 2 in the ANSWER BOOK. (4)

2.2.2 How will the range of race 2 compare with the range of race 1? (1)

2.2.3 Determine the average weight lost in race 2. (2)
[11]

QUESTION 3

In the diagram below, $A(-3\,;k)$, $B(4\,;8)$, $C(5\,;0)$ and $D(-2\,;-4)$ are vertices of the parallelogram ABCD. Diagonals AC and BD bisect each other at P. The angles of inclination of AD and BD are α and β respectively. AD cuts the x-axis at E. F is a point in the fourth quadrant.



3.1 Determine the gradient of BC. (2)

3.2 If the distance between points $A(-3\,;k)$ and $B(4\,;8)$ is $\sqrt{65}$, calculate the value of k. (4)

3.3 Prove, using analytical geometry methods, that $BP \perp AC$. (3)

3.4 Calculate the coordinates of F if it is given that ACFD is a parallelogram. (2)

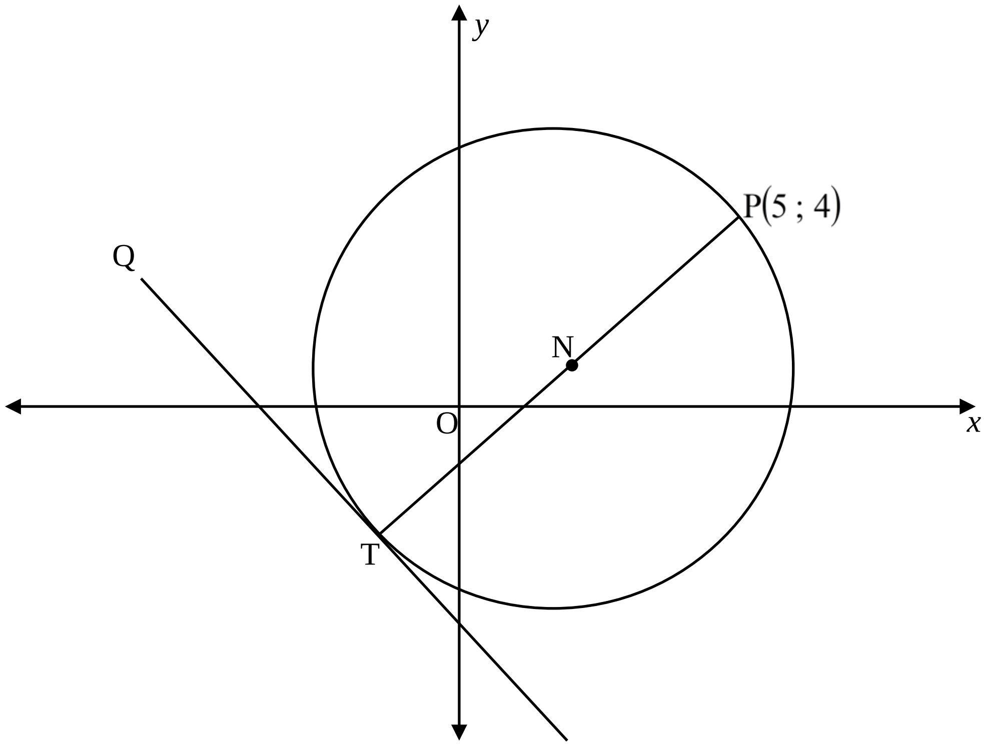
3.5 Calculate the size of $E\hat{D}O$ (correct to ONE decimal place). (6)

3.6 Calculate the area of ΔADC. (4)

[21]

QUESTION 4

In the diagram below, the equation of the circle with centre N is $x^2 + y^2 - 4x - 2y - 13 = 0$.
QT is a tangent to the circle at T. TNP is a diameter of the circle. Point $P(5\ ;\ 4)$ lies on the circle.



4.1 Write the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.2 Write down the coordinates of N and the length of NT. (2)

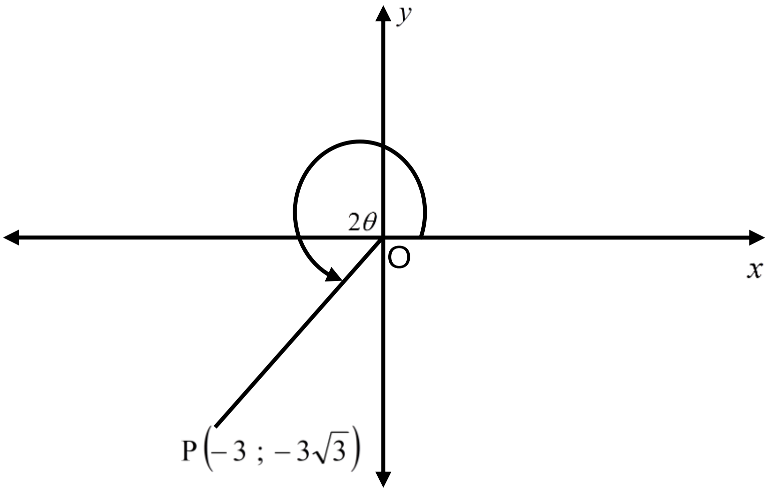
4.3 Determine the equation of the tangent QT in the form $y = mx + c$. (6)

4.4 The circle with centre $S(a\ ;\ b)$ touches the circle with centre N externally at T. QT is a tangent to both these circles. If NS = 3NT, determine the coordinates of S. (7)

[18]

QUESTION 5

5.1 In the diagram below, point $P\left(-3\ ;-3\sqrt{3}\right)$ and reflex angle 2θ are shown.



Determine, **without the use of a calculator**, the value of:

5.1.1 $\cos 2\theta$ (3)

5.1.2 $\sin \theta$ (3)

5.2 Simplify the following expression:

$\cos^2(180° + x) + \cos(-x)\ .\ \tan x\ .\ \cos(90° + x)$ (6)

5.3 Consider the equation $5\tan\theta - 6\cos\theta = 0$:

5.3.1 Show that the equation can be rewritten as $6\sin^2\theta + 5\sin\theta - 6 = 0$. (3)

5.3.2 Determine the general solution of $5\tan\theta - 6\cos\theta = 0$. (5)

5.4 Prove that:
$cos(2\alpha + 77°)\ cos(\alpha + 407°) - sin(\alpha + 47°)\ sin(2\alpha + 283°) = cos(\alpha + 30°)$ (5)

5.5 Solve for α and β:

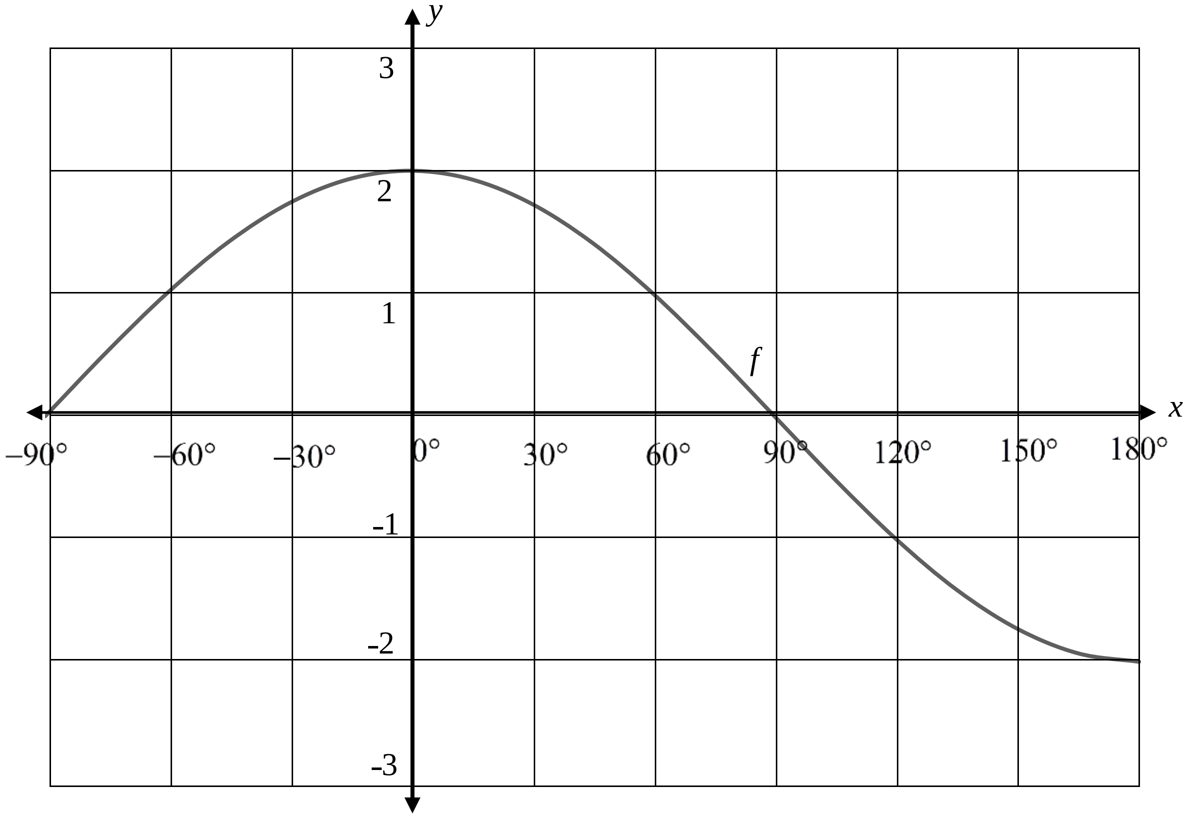
$\sin(3\alpha - \beta) = \dfrac{1}{\sqrt{2}}$ if $3\alpha - \beta \in [90°\ ;\ 270°]$

$\tan(2\alpha + \beta) = \dfrac{1}{\sqrt{3}}$ if $2\alpha + \beta \in [90°\ ;\ 270°]$

(4)
[29]

QUESTION 6

In the diagram below, the graph of $f(x) = 2\cos x$ is drawn for the interval $x \in [-90° ; 180°]$.



6.1 Draw the graph of $g(x) = -\tan\frac{3}{2}x$ for the interval $x \in [-90° ; 180°]$ on the grid provided in the ANSWER BOOK. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

6.2 Determine the period of g. (2)

6.3 Write down the values of x in the interval $x \in [-90° ;\ 180°]$ for which f is decreasing. (2)

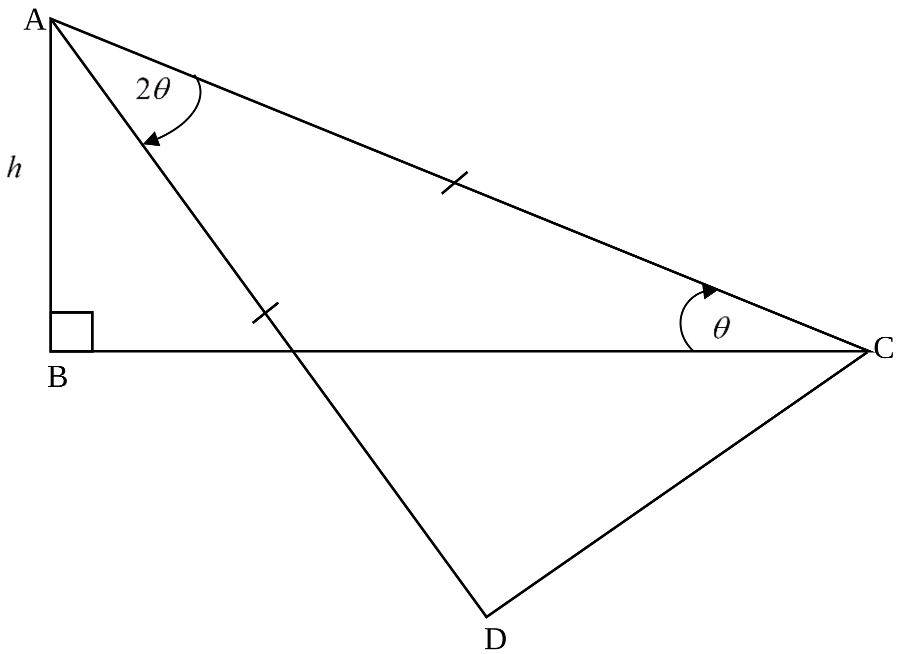
6.4 Use the graph of g to determine for which value(s) of x will $g(x) \geq 1$ for $x \in [-90° ; 180°]$. (4)

6.5 The function h is obtained by translating the graph of g, $30°$ to the right. Write down the equation of h. (2)

[14]

QUESTION 7

In the diagram below, AB is a pole anchored by two cables at C and D. B, C and D are in the same horizontal plane. The height of the pole is h and the angle of elevation from C to the top of the pole, A , is θ. $C\hat{A}D = 2\theta$ and AC = AD.

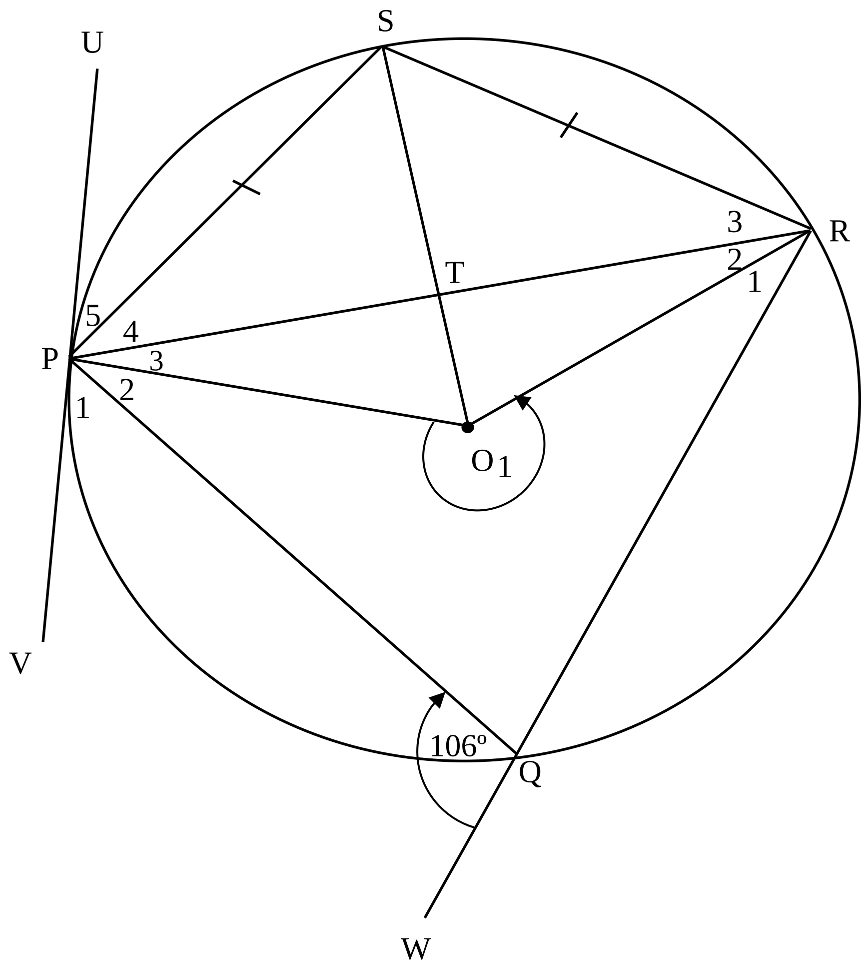


Determine CD, the distance between the two anchors, in terms of h. (7)

[7]

QUESTION 8

8.1 In the diagram below, points P, Q, R and S are points on a circle with centre O. UV is a tangent to the circle at P. PR and OS intersect at T and RQ is produced to W. $P\hat{Q}W = 106°$ and $SP = SR$.



Calculate, with reasons, the size of the following angles:

8.1.1 $P\hat{S}R$ (2)

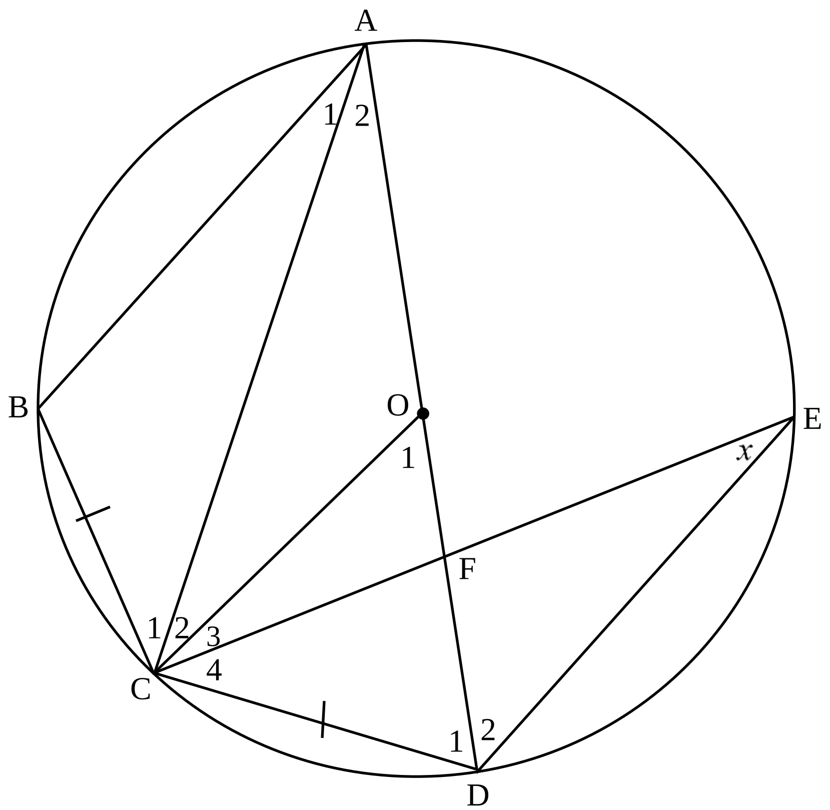
8.1.2 $\hat{R}_3$ (3)

8.1.3 $\hat{P}_5$ (2)

8.1.4 $\hat{O}_1$ (2)

8.1.5 $\hat{P}_3$ (2)

8.2 In the diagram below, A, B, C, D and E are points on a circle centred O. AC, OC and ED are drawn. Chords $BC = CD$. Let $C\hat{E}D = x$.



8.2.1 Determine, with reasons two other angles which are equal to x. (2)

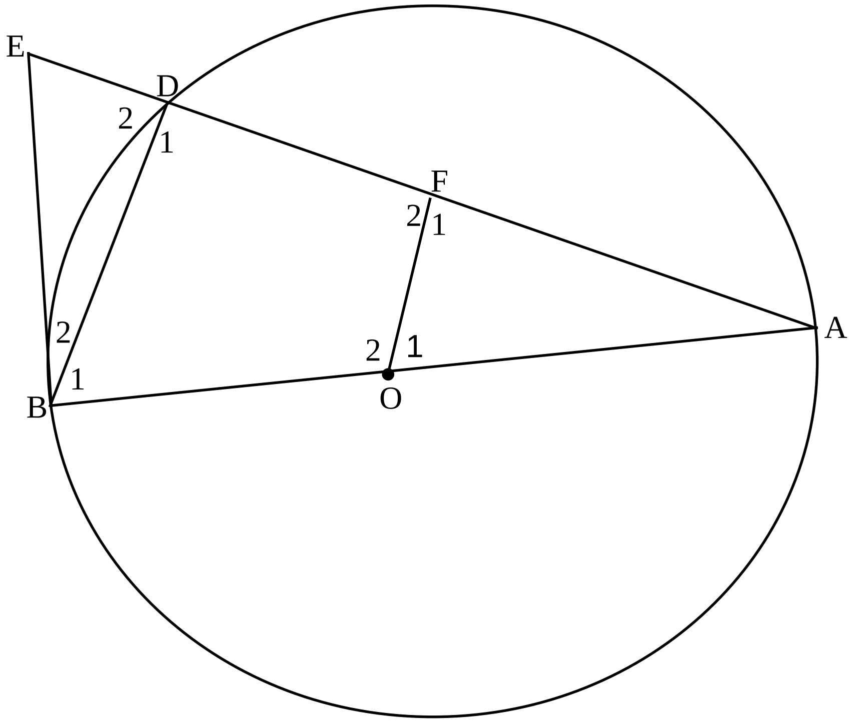
8.2.2 Determine $A\hat{B}C$ in terms of x. (3)

8.2.3 Prove AB || CO. (3)

[19]

QUESTION 9

In the diagram below, BOA is the diameter of the circle centred at O. BE is a tangent to the circle at B and EA cuts the circle at D. F is the midpoint of AD.



9.1 Prove, giving reasons that:

9.1.1 OBEF is a cyclic quadrilateral. (3)

9.1.2 $\Delta ADB \mid\mid\mid \Delta BDE$. (3)

9.1.3 OB is a tangent to the circle passing through B, D and E. (2)

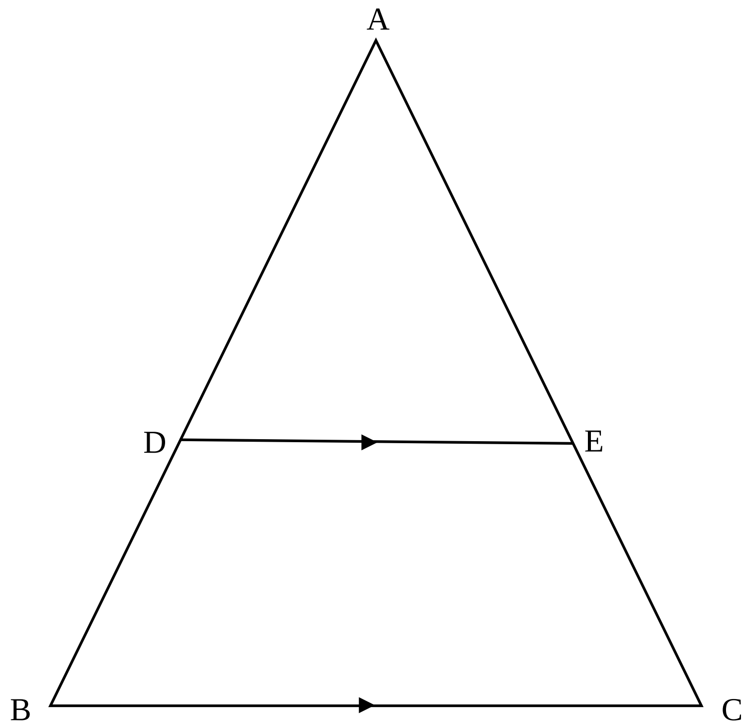
9.2 Prove: $OF^2 = \frac{AD \times DE}{4}$ (3)

[11]

QUESTION 10

10.1 In the diagram below, ΔABC is drawn. D is a point on AB and E is a point on AC such that DE || BC.

Use the diagram to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.

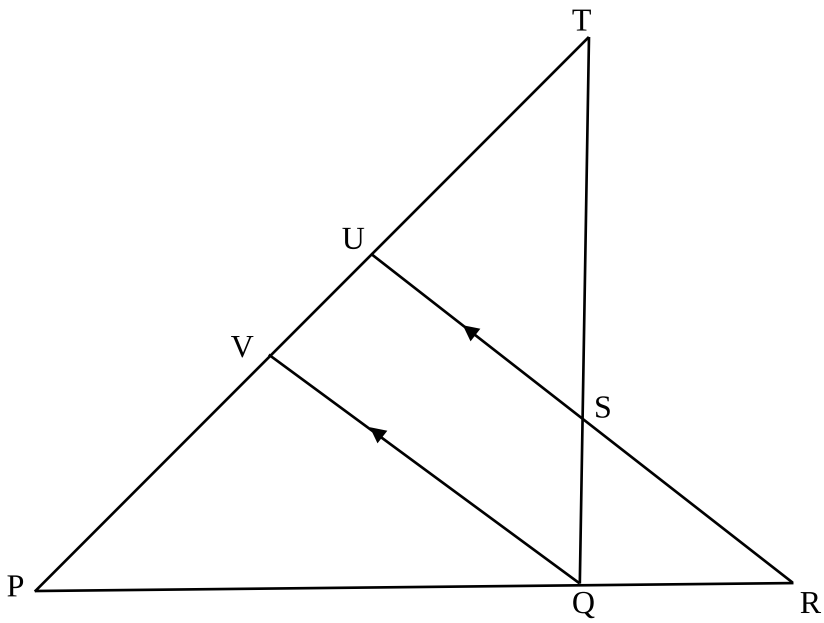


(5)

10.2 In the diagram below, ΔTPQ is drawn. PQ and US are produced to meet at R. UR and TQ intersect at S. SU || QV.

$\frac{TU}{UP} = \frac{2}{5}$ and $3QS = 2ST$.

Calculate, giving reasons $\frac{PQ}{QR}$.



(5)
[10]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}\ ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$