GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2022

11092

TECHNICAL MATHEMATICS

PAPER 2

TIME: 3 hours
MARKS: 150
15 pages + 2 information sheets and an answer book of 22 pages

TECHNICAL MATHEMATICS: Paper 2

11092E

X05

P.T.O.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 12 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.

4. Answers ONLY will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

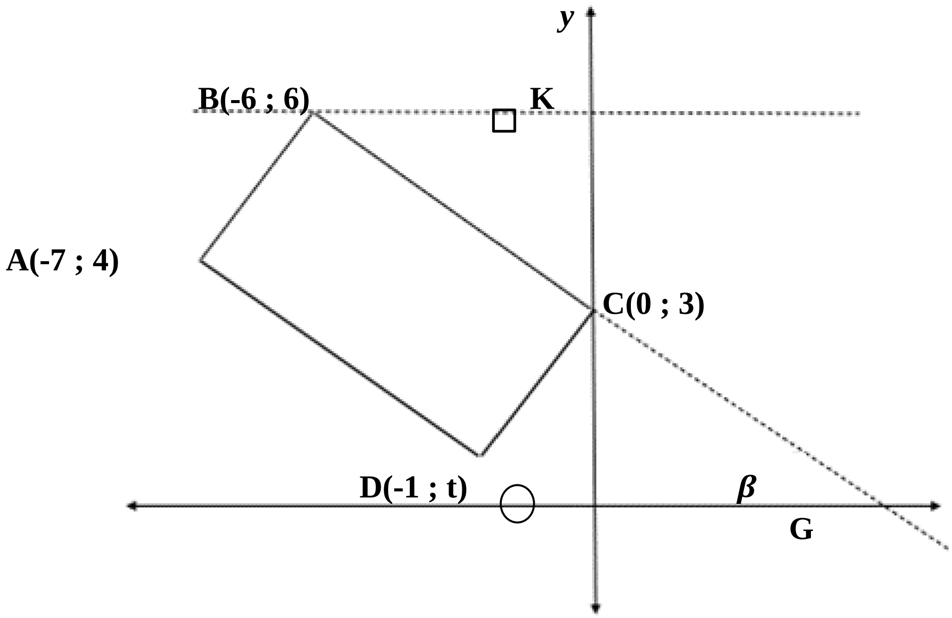
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Write neatly and legibly.

QUESTION 1

In the figure below, $A(-7; 4)$; $B(-6; 6)$; $C(0; 3)$ and $D(-1; t)$ are the vertices of a rectangle.



1.1 Calculate the length of the diagonal AC, leaving your answer in the simplest surd form. (2)

1.2 Write down the co-ordinates of K, where K is a point on the *y*-axis and BK is parallel to the *x*-axis. (1)

1.3 Determine M, the mid-point of diagonal AC. (2)

1.4 Determine the gradient of DC in terms of t. (2)

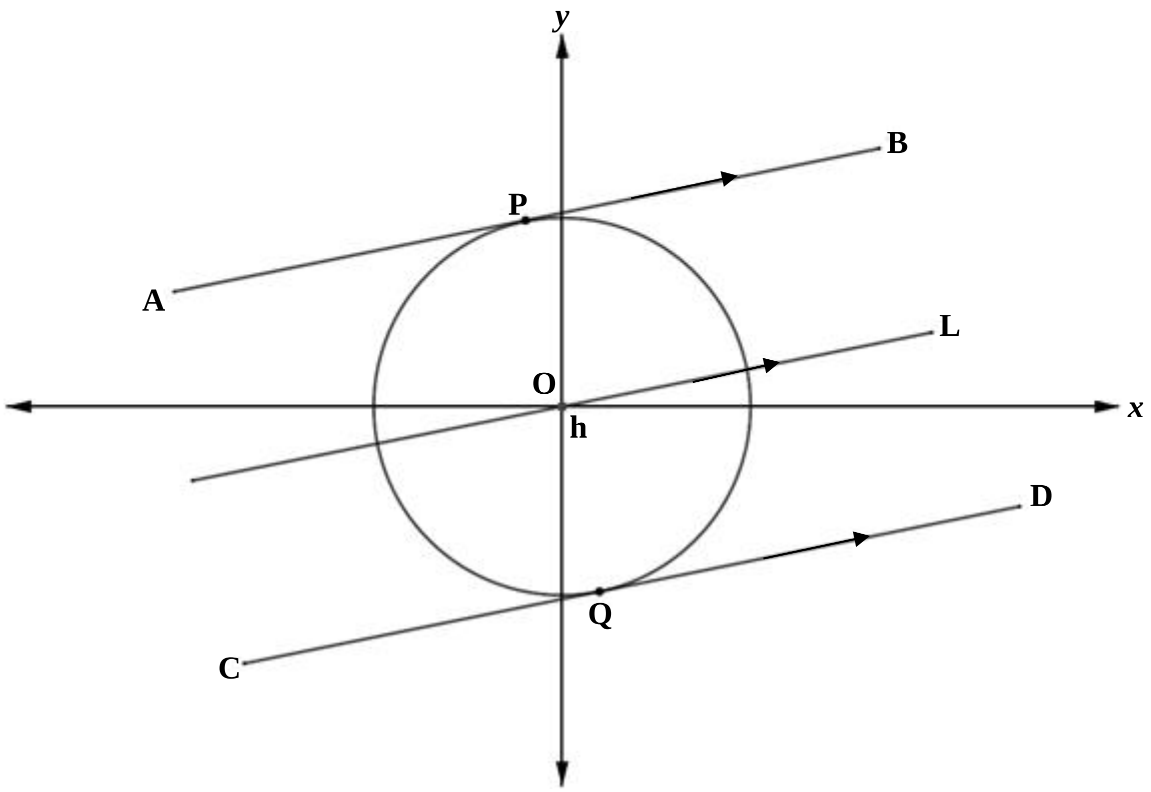
1.5 Calculate the value of t. (3)

1.6 Side BC is produced to cut the *x*-axis at G and makes an angle of β, as indicated. Determine the size of angle $C\hat{B}K$ (correct to ONE decimal place). (4)

[14]

QUESTION 2

2.1 The sketch below shows the circle defined by $x^2 + y^2 = 26$. APB and CQD are tangents to the circle at P and Q respectively. APB ∥ OL ∥ CQD.
The equation of OL is defined by $x - 5y = 0$.



2.1.1 Show that the points P, O and Q are collinear. (2)

2.1.2 Determine the coordinates of P. (6)

2.1.3 Write down the coordinates of Q. (1)

2.1.4 Hence, determine the equation of the tangent to the circle at Q. (2)

2.2 Draw the graph defined by:

$$\frac{x^2}{48} + \frac{y^2}{108} = \frac{1}{3}$$

Clearly show ALL the intercepts with the axes. (3)

[14]

QUESTION 3

3.1 If $\theta = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{6}$; determine the following, rounded-off to TWO decimal digits.

3.1.1 $cosec\theta$ (2)

3.1.2 $\cos(2\theta + a)$ (4)

3.2 Given: $13\cos\theta + 5 = 0$ and $\theta \in [180°; 360°]$.

With the aid of a diagram and WITHOUT the use of a calculator, determine:

3.2.1 $sin\theta$ (4)

3.2.2 $2tan\theta - 5sec^2\theta$ (3)

[13]

QUESTION 4

4.1 Complete: $\cot^2 x - \_\_\_\_\_\_ = -1$. (1)

4.2 Prove the following identity:

$$tanx.cotx - \frac{sinx}{cosecx} = \cos^2 x$$

(3)

4.3 Simplify the following to a single trigonometric ratio of x WITHOUT the use of a calculator.

$$\frac{\cos(180° - x).\tan(180° + x).\sin 240°}{\sin^2 x.\tan 210°}$$

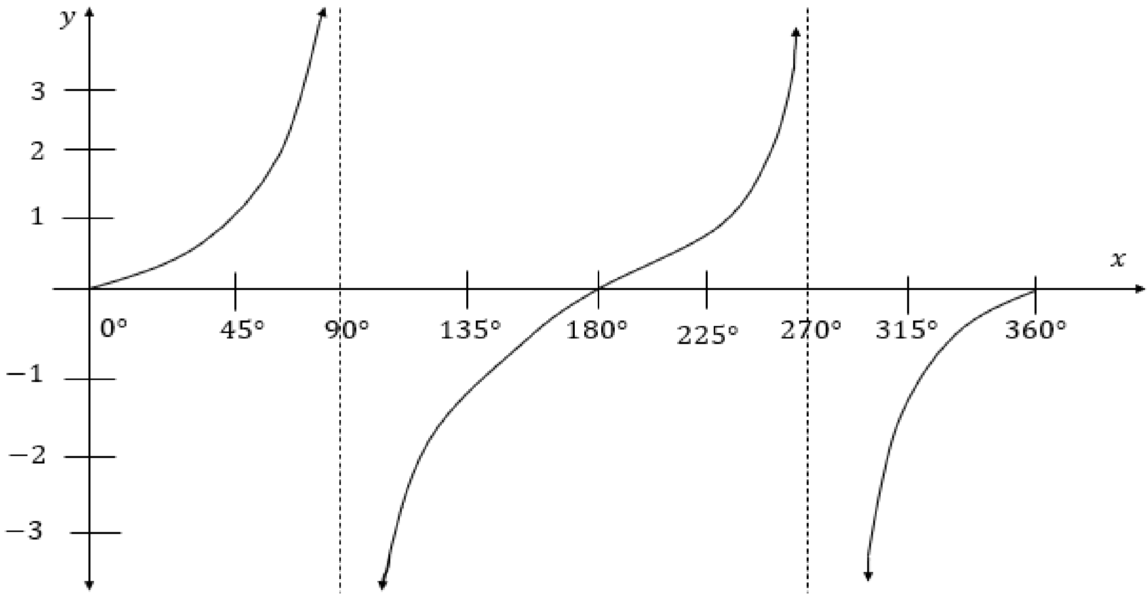
(7)

4.4 Solve for α if $2sin2\alpha = \sqrt{3}$ and $2a \in [0° ; 360°]$. (4)

[15]

QUESTION 5

In the diagram below, the graph of $f(x) = tanx$ is drawn for $x \in [0^o; 360^o]$.



5.1 Draw the graph of $g(x) = sin2x$ on the same set of axes in your ANSWER BOOK. (3)

5.2 Write down the amplitude of g . (1)

5.3 Write down the period of g. (1)

5.4 Write down the range of $k(x) = g(x) - 1$. (2)

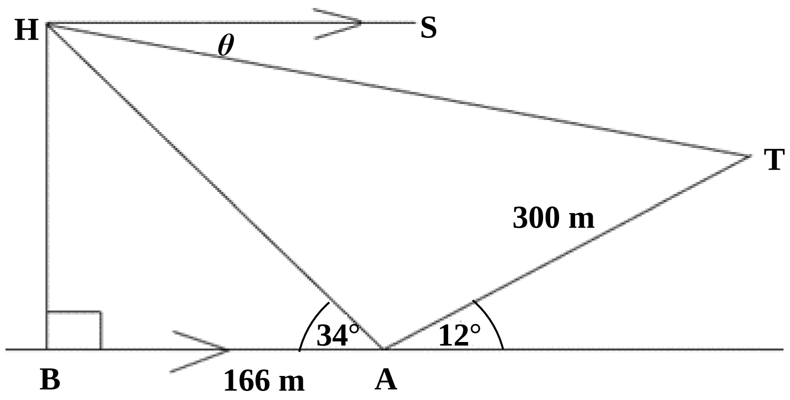
5.5 For which value(s) of x is $f(x) < 0$, where $x \in [0^o; 270^o]$? (2)

[9]

QUESTION 6

Beauty, standing at point A, looks up at an angle of 34^o to the top of a cliff HB, which is 166 m away from her. She turns around and walks in the opposite direction (away from HB) at an inclination of 12°, for 300 m to point T.

H, A, T and B is are on the same vertical plane.



6.1 Calculate $\mathrm{H\hat{A}T}$. (1)

6.2 Show that $\mathrm{AH} = 200$ m (rounded-off to the nearest metre). (2)

6.3 Calculate the length of HT (correct to the nearest metre). (4)

6.4 Calculate $\mathrm{A\hat{H}T}$ (correct to one decimal place). (3)

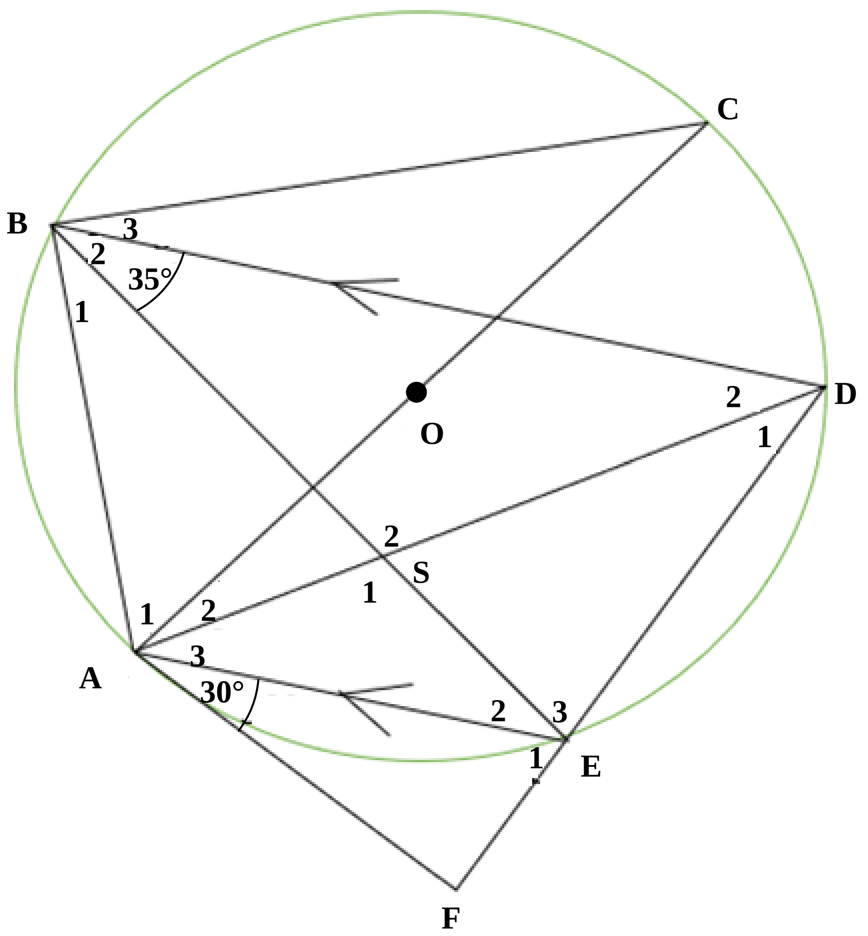
[10]

QUESTION 7

7.1 Complete the following theorem:

The line drawn from the centre of a circle to the midpoint of a chord … (1)

7.2 In the diagram below, A, B, C, D and E are points on the circumference of the circle with centre O. ABDE, is a cyclic quadrilateral. AF is a tangent to the circle at point A. DE is produced to meet point F. BD ∥ AE.
$E\hat{A}F = 30°$ and $D\hat{B}E = 35°$



7.2.1 Determine, with reasons, two angles that are equal to $F\hat{A}E$. (3)

7.2.2 Determine, stating reasons, the size of EACH in the following angles:

(a) $\hat{D}_2$ (2)

(b) $A\hat{B}C$ (2)

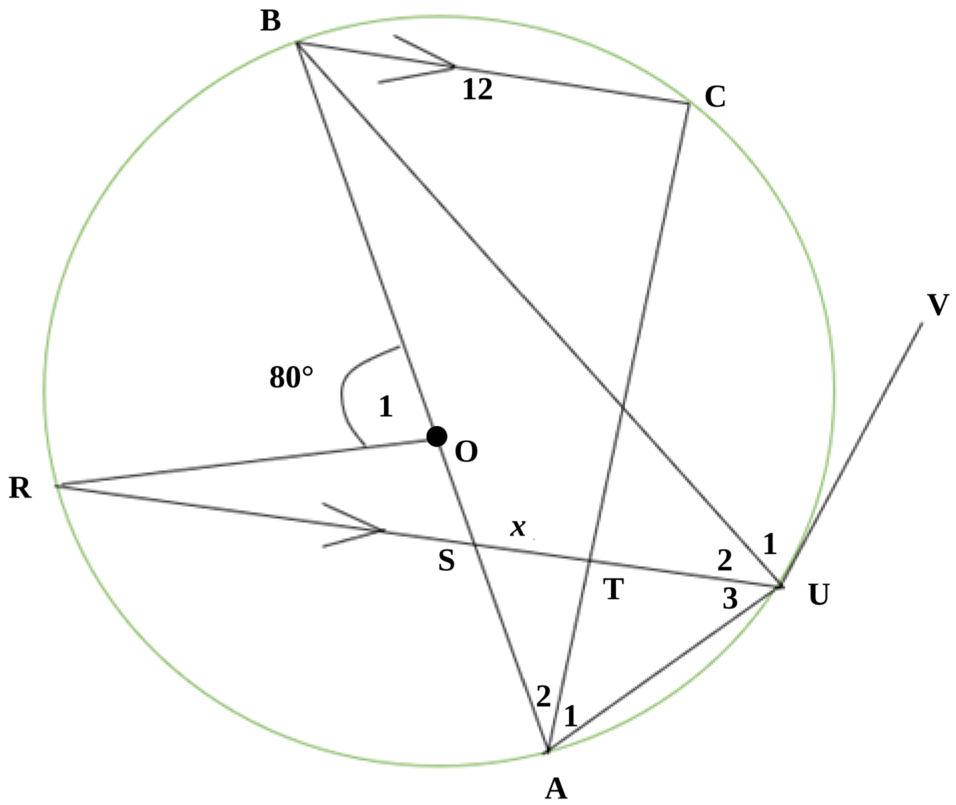
(c) $B\hat{A}E$ (2)

(d) $\hat{E}_1$ (3)

[13]

QUESTION 8

In the diagram below, A, R, B, C, and U are points on the circumference of a circle with centre O. UV is a tangent to the circle at point U. AB is a diameter of the circle and BC ∥ RU.
AT = 2, TC = 6; BC = 12 and ST = x. $\hat{O}_1 = 80°$.



8.1 Name TWO angles, with reason(s), that are equal to 90°. (3)

8.2 Calculate, with reasons, the numerical values of:

8.2.1 $\hat{U}_3$ (5)

8.2.2 $\hat{U}_1$, if $A\hat{B}U = 20°$ (3)

8.2.3 The length of ST (4)

[15]

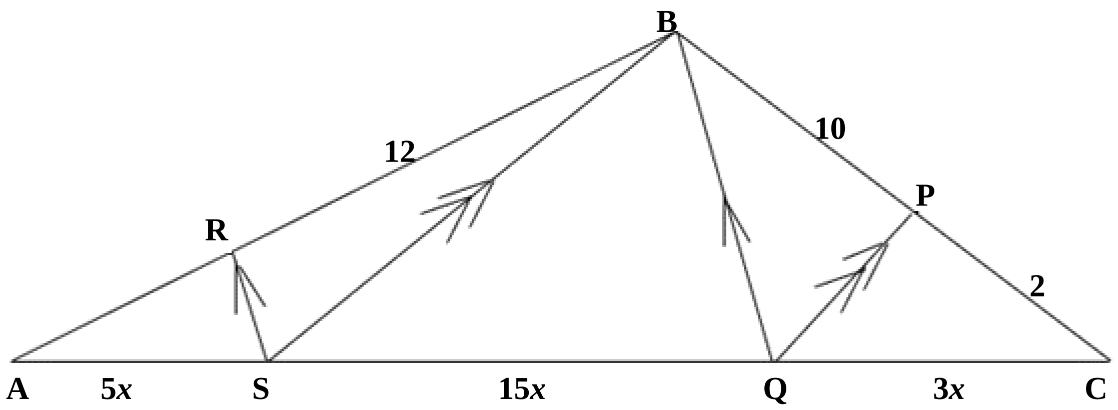
QUESTION 9

9.1 Complete the following theorem:

A line drawn parallel to one side of a triangle divides the other two sides … (1)

9.2 Roof trusses are highly engineered pieces of lumber connected by metal plates to form a web that supports a roof structure. The diagram below, not drawn to scale, is an example of one of these trusses.

RB = 12, AS = $5x$, SQ = $15x$, PB = 10, PC = 2 and QC = $3x$. RS ∥ BQ and PQ ∥ BS.



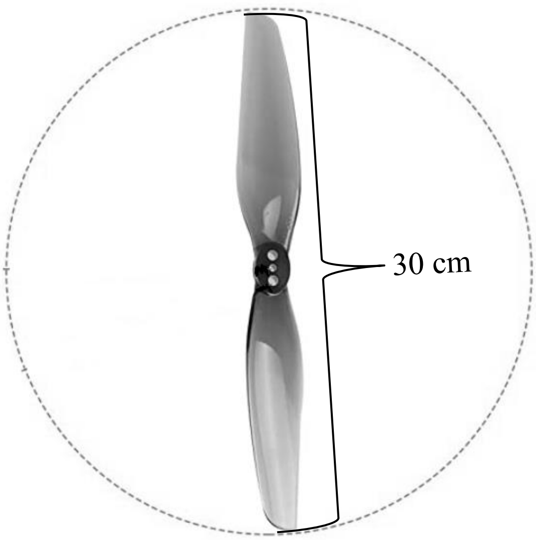
9.2.1 Calculate the length of AR. (4)

9.2.2 What type of a quadrilateral is BSQP? Give a reason for your answer. (2)

9.2.3 Hence, show that ΔPQC ||| ΔBSC, if PQ $= 2$ cm and BS $= 12$ cm. (4)
[11]

QUESTION 10

10.1 A drone has propeller blades where each blade has a length of 30 cm in diameter, rotating at 5 280 revolutions per minute.



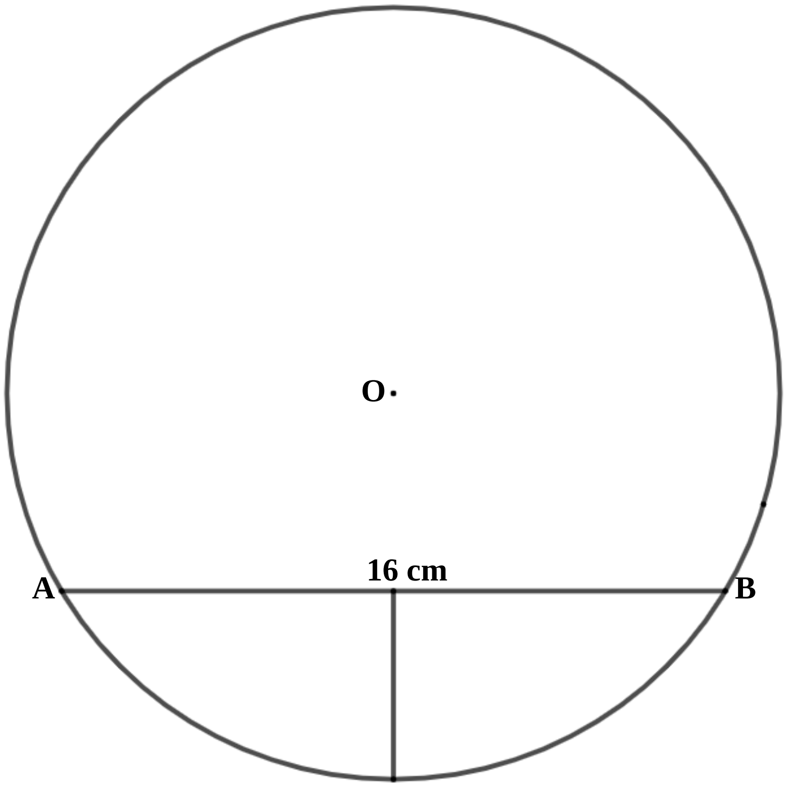
Calculate the following for one rotating blade:

10.1.1 The rotational frequency in revolutions per second (1)

10.1.2 The circumferential velocity in metres per second (3)

10.1.3 The angular velocity in radians per second (2)

10.2 Calculate the larger height of the two segments in a circle with centre O which has a chord AB of 16 cm and a diameter of 18 cm. (5)

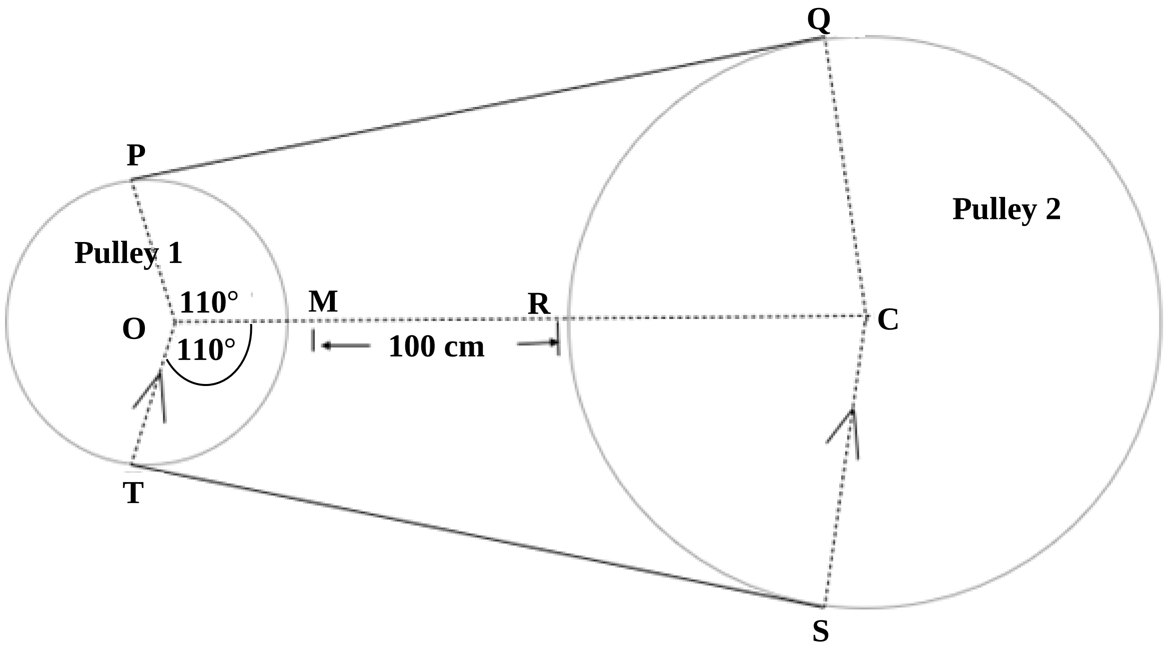


[11]

QUESTION 11

A pulley, in mechanics, is a wheel that carries a flexible rope, cord, cable, chain, or belt on its rim. Pulleys are used, singularly or in combination, to transmit energy and motion. Together with a lever, wedge, wheel, axle, and screw, the pulley is considered one of the five simple machines. Below is a picture of a set of pulleys.

Below is a diagram, not drawn to scale, of two pulleys with a belt connecting the two.
MR = 100 cm, TS = 220 cm, TS = PQ, $TO \parallel SC$ and $T\hat{O}C = 110°$.
Pulley 1 has a diameter of 30 cm and Pulley 2 has a diameter of 50 cm .



11.1 Calculate the magnitude of $S\hat{C}R$. (1)

11.2 Determine the length between the two centres of the pulleys. (2)

11.3 Convert $70°$ to radians. (2)

11.4 Calculate the reflex angle ($Q\hat{C}S$) of arc length QS if $S\hat{C}R = R\hat{C}Q$. (2)

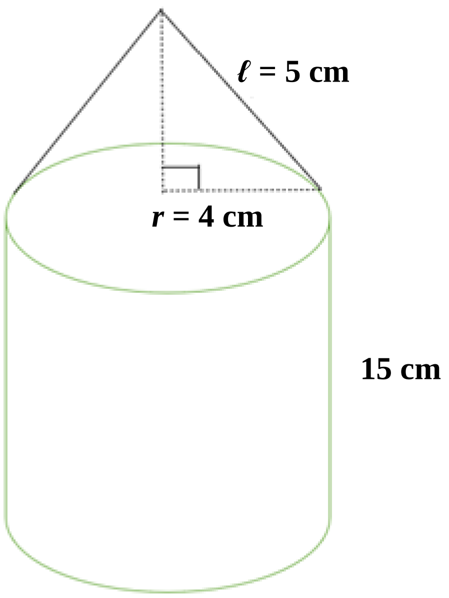
11.5 Calculate QS, the arc length of Pulley 2. (3)

11.6 Calculate the total length of the belt. (4)
[14]

QUESTION 12

12.1 A community fundraising team wants to design and make a model for a presentation to a sponsor for a water reservoir because of the shortage of water in the area. They plan to have water stored in the cylindrical part of the container which is open at the top and has a lid in the form of a cone on top. They are also planning to paint the entire container on the outside, from the base of the cylinder to the top of the cone. Below, is the diagram of the projected container with its measurements.

Curve area of cone = $\pi r\ell$
Outer surface area cylinder = $2\pi rh + 2\pi$
r^2 Volume of cylinder = $\pi r^2 h$
ℓ – slant height
r – radius
h – height



Calculate the:

12.1.1 Volume of the cylinder (2)

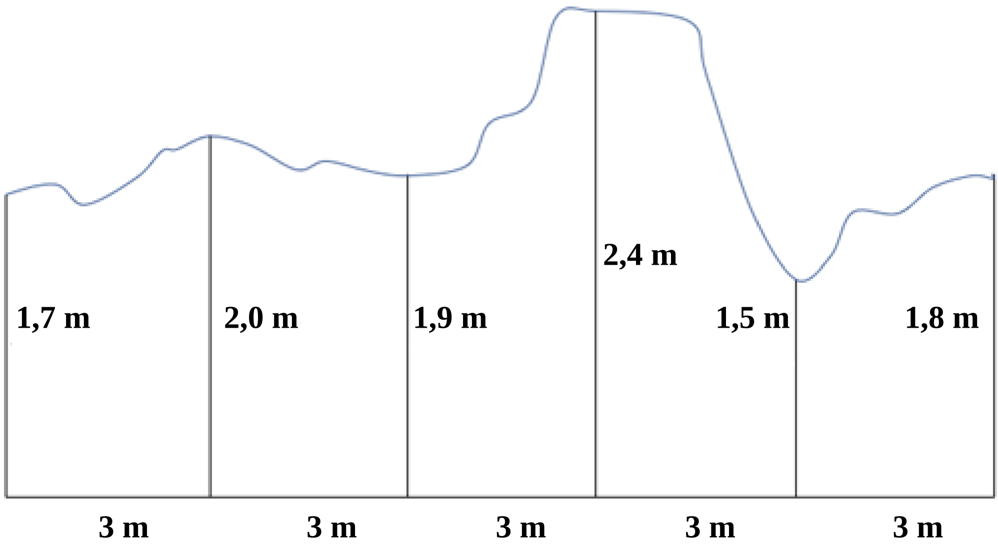
12.1.2 Surface area of:

(a) The curved area of the cone (2)

(b) The cylinder with the open top (3)

12.1.3 Total outer surface area that needs to be painted (1)

12.2 The diagram below shows a rectangular piece of sheet metal that needs to be cut to build a specific pipe when folded along the ordinates.



Use the mid-ordinate rule to calculate the area of the irregular shape/figure. (3)

[11]

TOTAL: 150

END

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \Leftrightarrow x = log_a b, \quad a > 0, a \neq 1$ and $b > 0$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$f^{/}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1 \qquad \int kx^n dx = k.\frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{x} dx = ln(x) + C, x > 0 \qquad \int \frac{k}{x} dx = k. ln(x) + C, x > 0$$

$$\int a^x dx = \frac{a^x}{ln\, a} + C, a > 0 \qquad \int ka^{nx} dx = k.\frac{a^{nx}}{n\, ln\, a} + C, a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \mathrm{cosec}^2\theta$$

$$\pi rad = 180^\circ$$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360^0 n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where *D* = diameter and *n* = rotation frequency

Arc length $s = r\theta$ where *r* = radius and θ = central angle in radians

Area of a sector = $\frac{rs}{2}$ where *r* = radius, *s* = arc length

Area of a sector $= \frac{r^2\theta}{2}$ where *r* = radius, θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where *h* = height of segment, *d* = diameter of circle and *x* = length of chord

$\mathrm{A_T} = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$ where *a* = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and *n* = number of ordinates

OR

$\mathrm{A_T} = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where *a* = equal parts, $o_n = n^{th}$ ordinate and *n* = number of ordinates