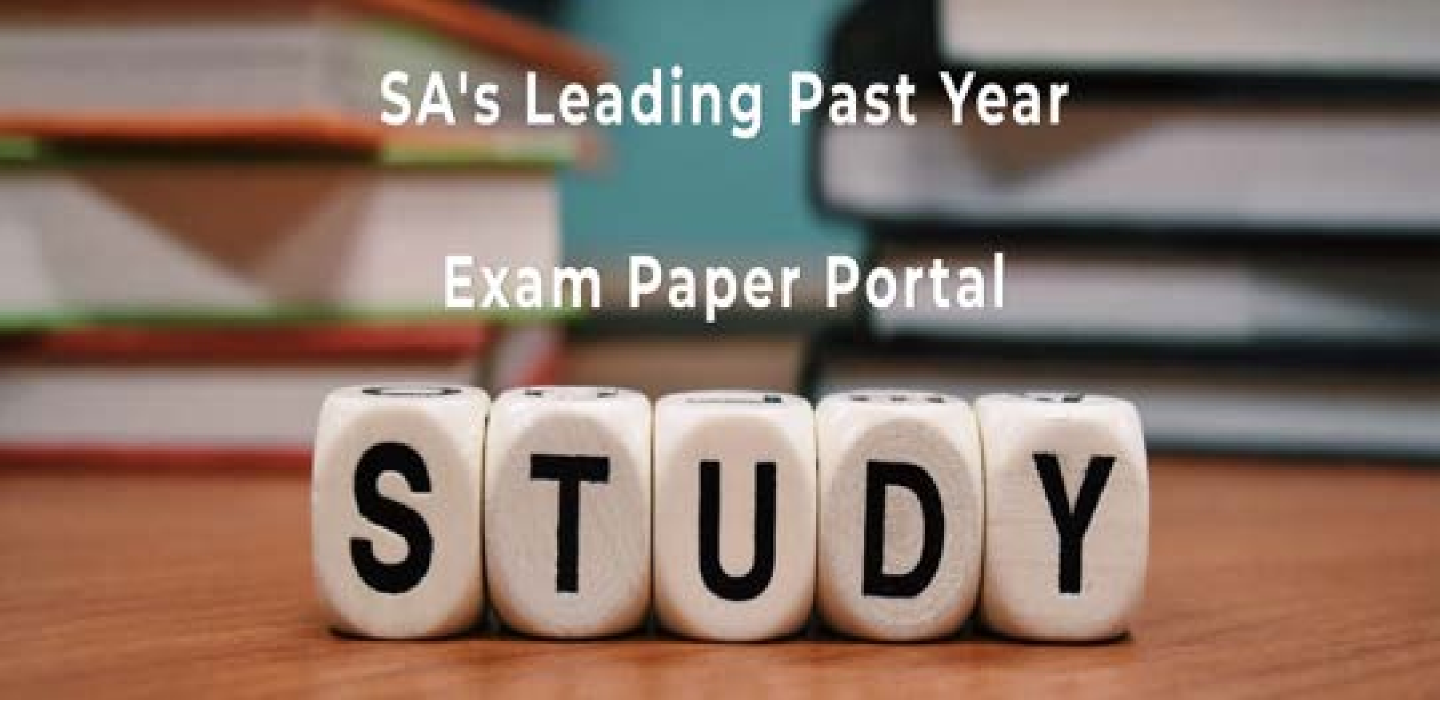
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STUDY

Province of the EASTERN CAPE EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2022

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 16 pages, including a 2-page information sheet.

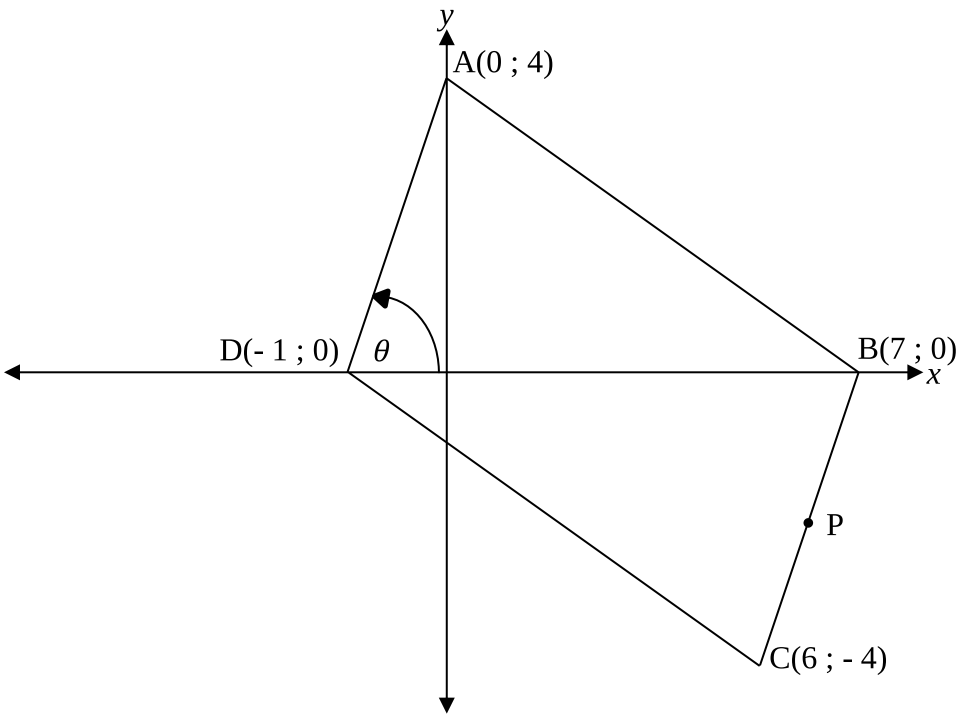
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

In the diagram below ABCD is a parallelogram with vertices A(0 ; 4), B(7 ; 0), C(6 ; - 4) and D(- 1 ; 0). θ is the inclination angle of line AD.



Determine:

1.1 The gradient of AD (2)

1.2 The inclination angle, θ, to the nearest degree (3)

1.3 The coordinates of P, the midpoint of BC (2)

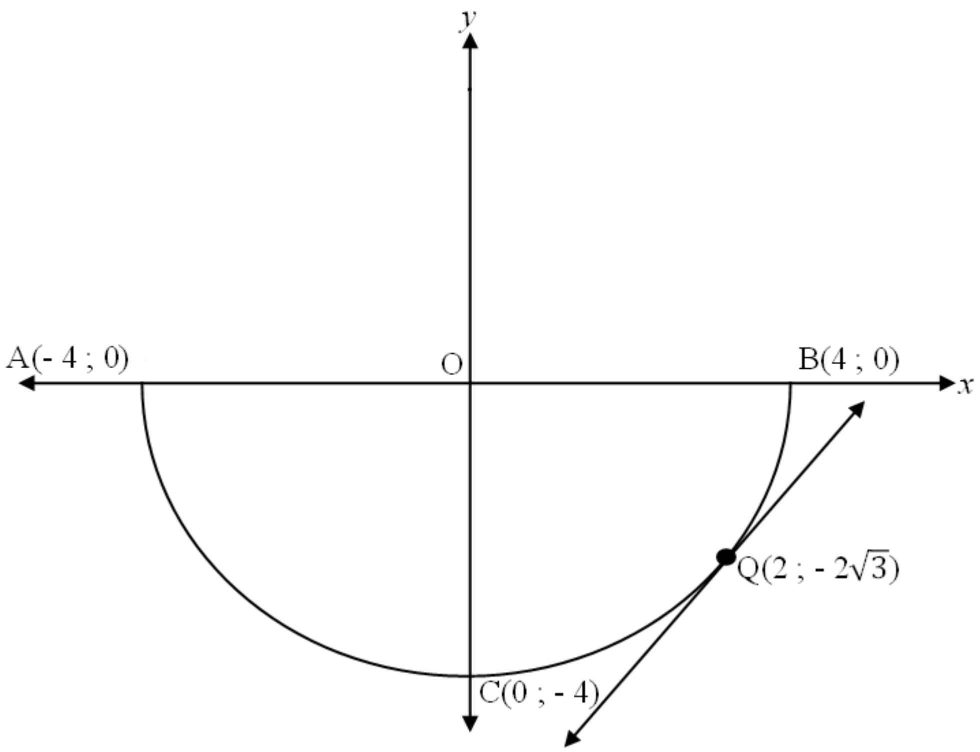
1.4 The length of AB and leave your answer in surd form (2)

1.5 The equation of the line perpendicular to BC and passing through the point P, in the form $y = ...$ (3)

[12]

QUESTION 2

2.1 In the diagram below a semi-circle is sketched, with a tangent line touching at point Q.



2.1.1 Determine the equation of the semi-circle. (3)

2.1.2 Determine the gradient of OQ. (2)

2.1.3 Hence, determine the equation of the tangent line in the form $y = ...$ if P(0;-5) is point on the tangent line. (2)

2.2 Given: $\frac{x^2}{25} + \frac{y^2}{16} = 1$

2.2.1 Sketch the given graph, in your SPECIAL ANSWER BOOK, clearly indicating all intercepts with the axis. (3)

2.2.2 Determine the length of the major axis. (1)

[11]

QUESTION 3

3.1 Given: $\hat{P} = 38{,}9^\circ$ and $\hat{S} = 153{,}2^\circ$

Determine the following:

3.1.1 $\sin P + \cos 2S$ (2)

3.1.2 $\sec\left(\frac{P}{3} + S\right)$ (3)

3.2 If $\tan 52^\circ = k$, determine the following in terms of k.

3.2.1 $\cos 52^\circ$ (3)

3.2.2 $\text{cosec } 38^\circ$ (1)

3.2.3 $\sin 232^\circ$ (3)

3.3 Solve for θ, rounded off to ONE decimal digit, if $\theta \in (0^\circ; 90^\circ)$:

$$\frac{1}{2}\text{cosec } 2\theta = 0{,}814$$

(4)

[16]

QUESTION 4

4.1 Complete the identity:

$$1+\cot^2\theta = \ldots$$ (1)

4.2 Simplify:

$$\frac{\cos(180°+\theta)\cdot\tan(360°-\theta)\cdot\cos^2(360°-\theta)}{\sin(180°-\theta)}+\cos^2\theta$$ (7)

4.3 Prove that:

$$\frac{\sec\theta+\text{cosec }\theta}{\sin\theta+\cos\theta}=\tan\theta+\cot\theta$$ (8)

[16]

QUESTION 5

Given $f(x)=\sin x$ and $g(x)=\cos x+1,\ x\in[0°;360°]$

5.1 On the same axes, given in your SPECIAL ANSWER BOOK, draw the graphs of $f(x)=\sin x$ and $g(x)=\cos x+1$. Clearly show the intercepts with the axes, turning points and endpoints. (7)

5.2 Write down the range of *g*. (2)

5.3 Write down the period of *f*. (1)

5.4 Use your graphs to determine for which values of *x* is $g(x)=f(x)$ (2)

[12]

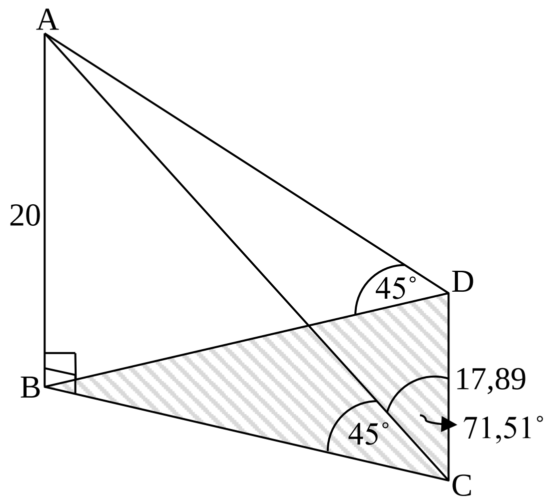
QUESTION 6

AB is a lamp pole anchored with steel cables at points C and D.
B, C and D are in the same horizontal plane.

AB = 20 units and CD = 17,89 units

$A\hat{D}B = A\hat{C}B = 45°$

$A\hat{C}D = 71,51°$



6.1 Determine the length of AC, one of the anchor cables. (2)

6.2 Determine the area of the ΔACD. (3)

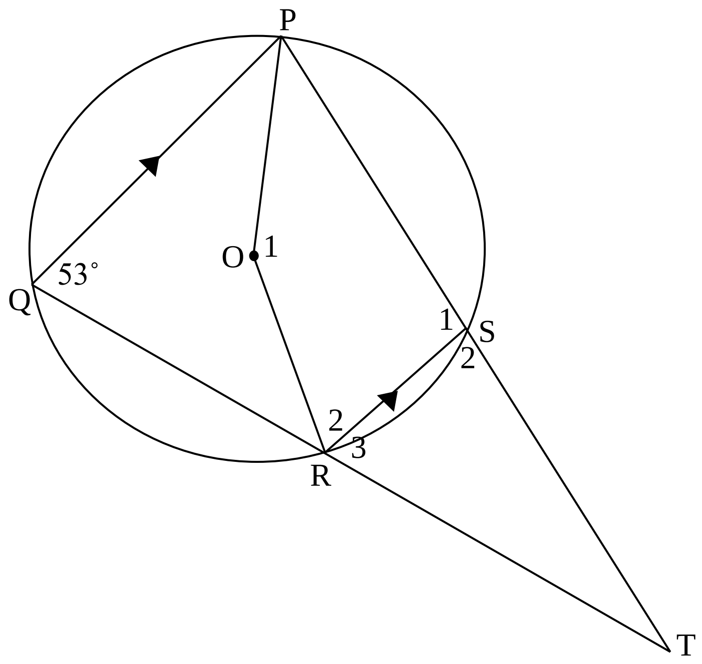
[5]

QUESTION 7

7.1 Fill in the missing word(s) in the following theorem statement:

"The angle subtended by an arc at the centre of the circle is … the size of the angle subtended by the same arc at the circumference of the circle." (1)

7.2 In the diagram below, O is the centre of the circle and PQRS is a cyclic quadrilateral QP ∥ RS.
PS and QR are produced to meet at T.
$\hat{Q} = 53°$



Write down the sizes of the following angles with reasons:

7.2.1 $\hat{O}_1$ (2)

7.2.2 $\hat{S}_2$ (2)

7.2.3 $\hat{P}$ (2)

[7]

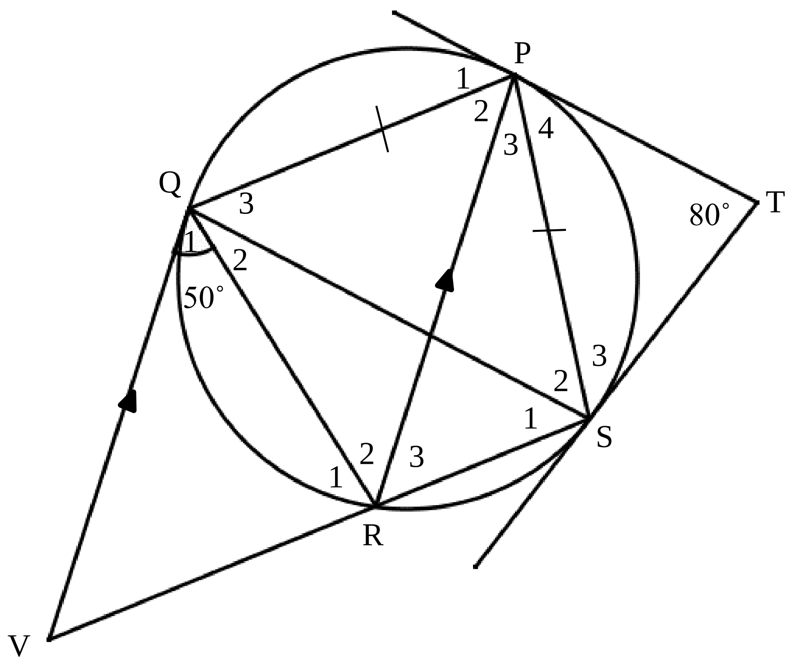
QUESTION 8

8.1 Fill in the missing word(s) in the following theorem statement:

"The angle between the tangent to a circle and the chord drawn from the point of contact is ... to the angle in the alternate segment." (1)

8.2 In the diagram below PQRS is a cyclic quadrilateral, with PS = PQ.
SR is produced to V, such that QV ∥ PR.
QV is a tangent to the circle at Q, with $\hat{Q}_1 = 50°$

TP and TS are tangents from the same point T, with $\hat{T} = 80°$



8.2.1 Give a reason why TP = TS. (1)

8.2.2 Hence, calculate the size of $\hat{P}_4$, with reasons. (4)

8.2.3 Calculate the size of the following angles, with reasons:

(a) $\hat{P}_2$ (2)

(b) $\hat{S}_2$ (2)

(c) $\hat{R}_2$ (2)

(d) $\hat{V}$ (2)

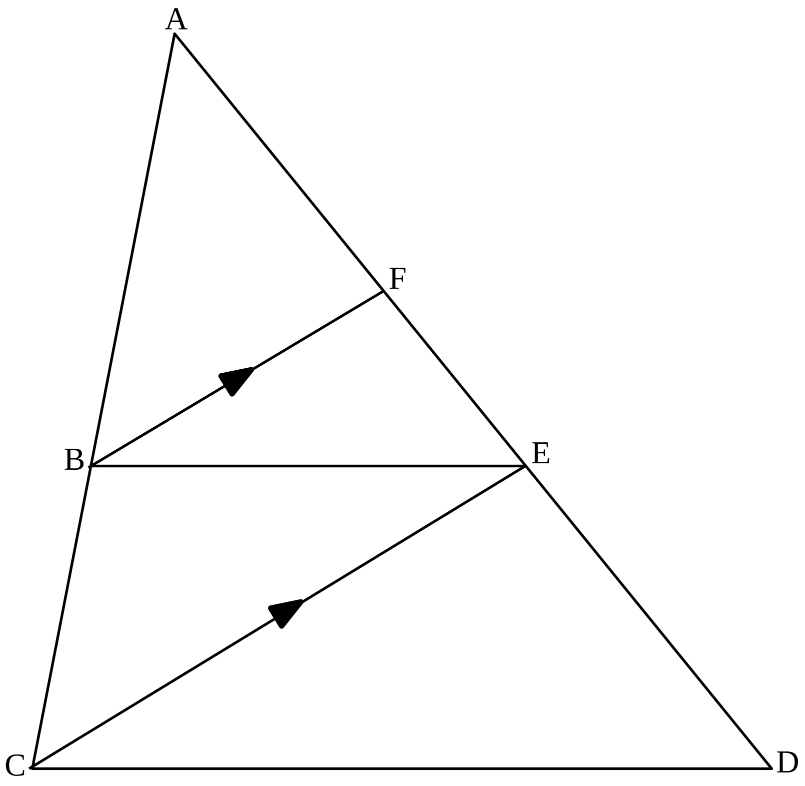
[14]

QUESTION 9

9.1 Fill in the missing word(s) in the following theorem statement:

"If a line divides two sides of a triangle in the same proportion, then the line is ... to the third side ." (1)

9.2 In the diagram below, $BF \parallel CE$; $AB : AC = 3 : 4$; $FE : ED = 3 : 4$ and $AE = 36$ cm.



Determine:

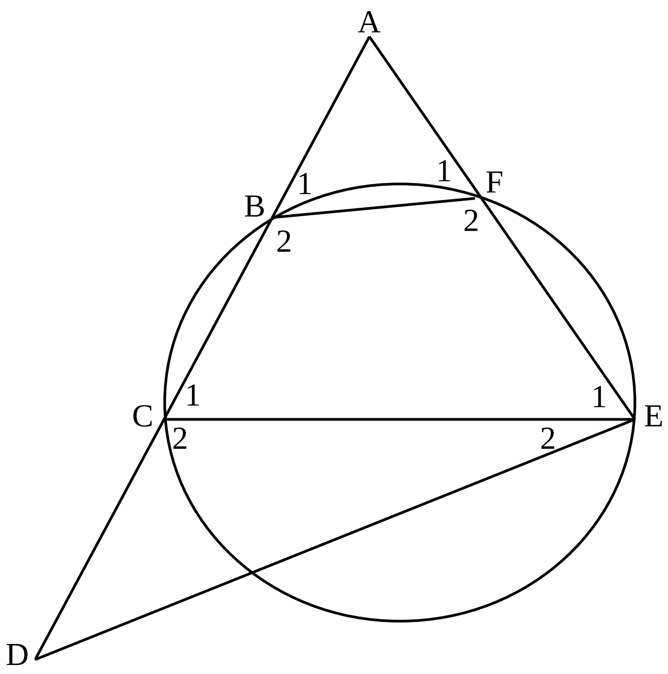
9.2.1 The length of AF (3)

9.2.2 The length of ED (3)

9.3 Fill in the missing word(s) in the following theorem statement:

"If the corresponding sides of two triangles are ..., then the triangles are equiangular." (1)

9.4 In the diagram below, BFEC is a cyclic quadrilateral. Sides CB and EF are extended to meet at A. Triangle ECD is formed.



9.4.1 Prove that $\Delta ABF \parallel\!| \Delta AEC$ (4)

9.4.2 Further if, AB = 25 cm; AF = 20 cm; FE = 40 cm and CD = 27 cm.

(a) Determine the length of BC. (3)

(b) Show that $BF \parallel DE$. (3)

(c) Prove that $\Delta AEC \parallel\!| \Delta ADE$. (3)

(d) Hence, determine DE : BF. (2)

[23]

QUESTION 10

10.1 A crack on a music CD forms an angle of 60˚ as it moves on the track, from B to A. The radius of the CD is 5 cm. FIGURE 1 represent the crack.

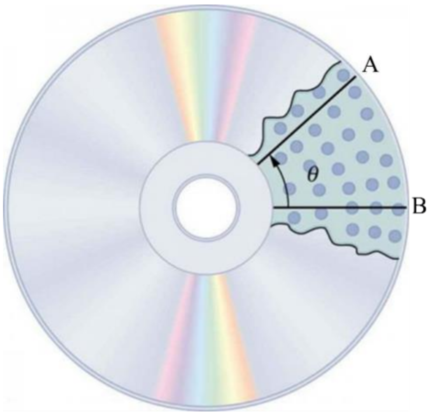
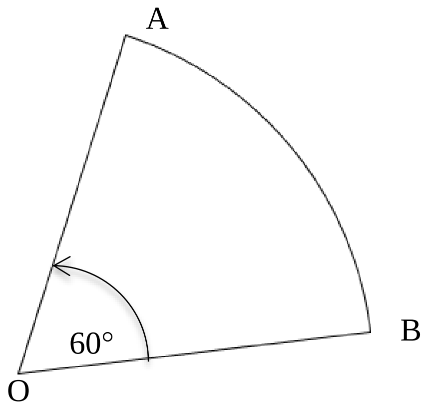


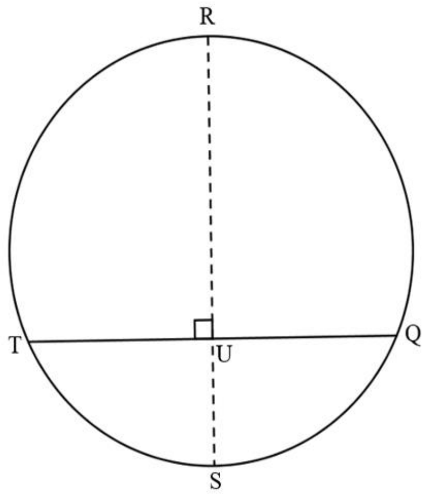
FIGURE 1



10.1.1 Calculate the arc length, AB, that the crack moved. (4)

10.1.2 Determine the area AOB, formed by the crack. (3)

10.2 A tub in a washing machine, with diameter 420 mm, rotates at 1 800 revolutions per minute. In the diagram alongside, SR represent the diameter of the tub, with S and R points on the circumference and TQ represents a chord of the tub (circle) with $TQ \perp SR$ at U.



Calculate:

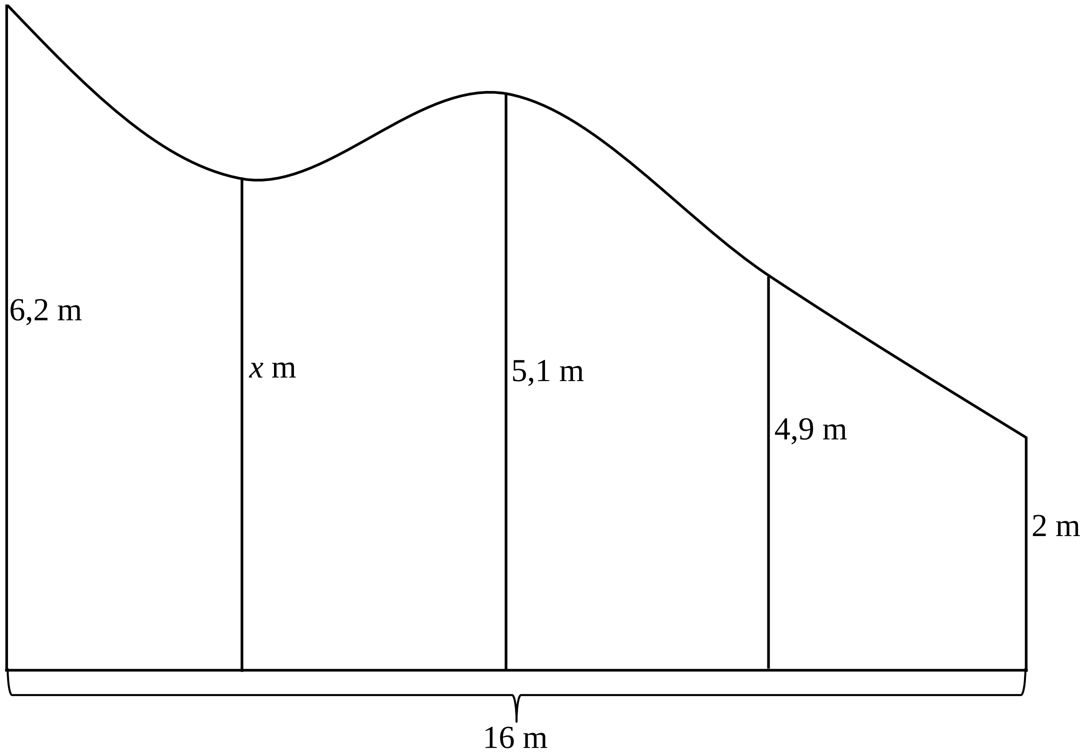
10.2.1 The angular velocity, of the tub, in radians per second (4)

10.2.2 The circumferential velocity, of the tub, in meters per second (4)

10.2.3 The length of US, if chord TQ = 250 mm. Give your final answer in cm (5)
[20]

QUESTION 11

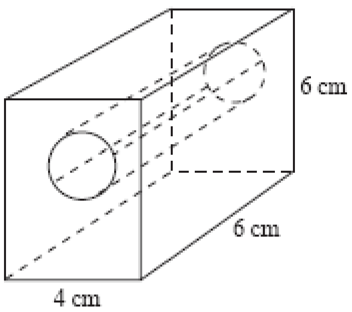
11.1 The irregular shape below, with ordinates 6,2, *x*, 5,1, 4,9 and 2 m, has a TOTAL area of 73,6 m^2. One side has a length of 16 m and has been divided into four equal parts.



Calculate the value of *x*. (5)

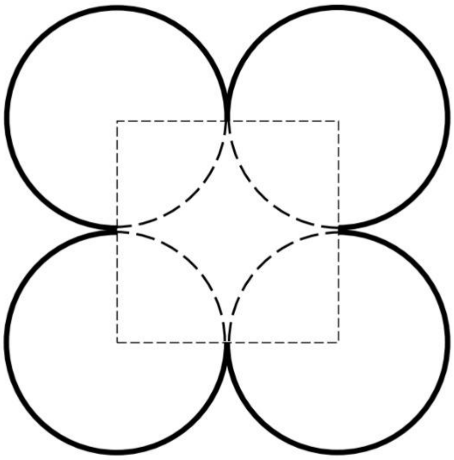
11.2 A block of wood has a cylinder cut out of it, through which an electrical cord needs to run. The cylinder has a diameter of 1,5 cm.

Volume of cylinder = $\pi r^2 h$
Volume of rectangular prism = $l \times b \times h$



Calculate the volume of the remaining block of wood after the cylinder has been cut out. (4)

11.3 A poster is made up of 4 identical circles. The circles have a radius of 15 cm. The poster must be pasted on a wall that has a circular space with a circumference of 200 cm. The midpoints of the circles form the vertices of a square. The diagram below illustrates what the poster should look like.



Determine if the poster can be pasted in the space that is available on the wall. (5)

[14]

TOTAL: 150

INFORMATION SHEET

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $x = -\dfrac{b}{2a}$ $\quad$ $y = \dfrac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1$ and $b > 0$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 + i)^n$ $\quad$ $A = P(1 - i)^n$

$i_{eff} = \left(1 + \dfrac{i}{m}\right)^m - 1$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$ $\quad$ $\int kx^n dx = k \cdot \dfrac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$

$\int \dfrac{1}{x} dx = \ln(x) + C, \quad x > 0$ $\quad$ $\int \dfrac{k}{x} dx = k \cdot \ln(x) + C, \quad x > 0$

$\int a^x dx = \dfrac{a^x}{\ln a} + C \quad , \quad a > 0$ $\quad$ $\int ka^{nx} dx = k \cdot \dfrac{a^{nx}}{n \ln a} + C \quad , \quad a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

In ΔABC:

$\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$ $\quad$ $a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$ $\quad$ $\text{Area} = \dfrac{1}{2} ab.\sin \mathrm{C}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $1 + \tan^2\theta = \sec^2\theta$ $\quad$ $\cot^2\theta = 1 + \mathrm{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360°n$ where n = rotation frequency

Circumferential velocity = $v = \pi Dn$ where D = diameter and n = rotation frequency

Circumferential velocity = $v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius and s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of the circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates