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**MARKING MEMORANDUM
CONTROLLED TEST 1
01 MARCH 2023**

GRADE 12

MATHEMATICS

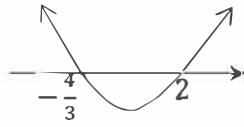
MARKS: 50

This Marking Guidelines consists of 6 pages including this cover page.

IMPORTANT NOTES AND INFORMATION

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Accept any other Mathematically valid attempt which yields a correct answer and credit full marks.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

Q#	Suggested Solutions	Descriptors
1.1.1	$x(2x - 3) = 0$ $x = 0 \text{ or } x = \frac{3}{2}$	✓ $x = 0$ ✓ $x = \frac{3}{2}$ (2)
1.1.2	$0 = 3x^2 - 5x - 11$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-11)}}{2(3)}$ $x = -1,25 \text{ or } x = 2,92$	✓ correct substitution ✓ $x = -1,25$ ✓ $x = 2,92$ (3)
1.1.3	$x(3x + 4) - 2(3x + 4) \leq 0$ $(x - 2)(3x + 4) \leq 0$ Critical values: $x = 2 ; x = -\frac{4}{3}$ $-\frac{4}{3} \leq x \leq 2$	 ✓ factors/ method ✓ critical values ✓ solution (3)
1.2	For real roots: $8 - p^3 \geq 0 ; p \neq 0$ $-p^3 \geq -8$ $p^3 \leq 8$ $p \leq 2$	✓ $8 - p^3 \geq 0$ ✓ $p \neq 0$ ✓ $p \leq 2$ (3)
Subtotal		[11]

QUESTION 2

Q#	Suggested Solutions	Descriptors
2.1	$11 ; 20 ; 33 ; 50$ 1st differences: $9 ; 13 ; 17$ 2nd differences: $4 ; 4$ $\therefore 2a = 4$ $3a + b = 9$ $a + b + c = 11$ $a = 2$ $3(2) + b = 9$ $2 + 3 + c = 11$ $b = 3$ $c = 6$ $\therefore T_n = 2n^2 + 3n + 6$	✓ $a = 2$ ✓ $b = 3$ ✓ $c = 6$ ✓ $T_n = 2n^2 + 3n + 6$ (4)
2.2	1st differences: $9 ; 13 ; 17$ $a = 9 ; d = 4$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $= \frac{n}{2}[2(9) + (n - 1)4]$ $= \frac{n}{2}[14 + 4n]$ $= 7n + 2n^2$	✓ correct substitution ✓ simplification (2)
Subtotal		[6]

QUESTION 3

Q#	Suggested Solutions	Descriptors
3.1	$r = \frac{x+1}{3}$ $-1 < \frac{x+1}{3} < 1$ $-3 < x + 1 < 3$ $-4 < x < 2, \text{ but } x \neq -1$	✓ $-1 < \frac{x+1}{3} < 1$ ✓ $-4 < x < 2$ ✓ $x \neq -1$ (3)
3.2	$S_{\infty} = \frac{a}{1-r}$ $= \frac{5(1+1)}{\frac{3}{1-1}}$ $= 10$	✓ correct substitution ✓ answer (2)
Subtotal		[5]

QUESTION 4

Q#	Suggested Solutions	Descriptors
4.1	$(2x - 5) - (x + 2) = (x + 2) - (1 - x)$ $x - 7 = 2x + 1$ $-8 = x$	✓ method ✓ simplification ✓ $x = -8$ (3)
4.2	$9 ; -6 ; -21$ $a = 9, d = -15, n = 100$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_{100} = \frac{100}{2}[2(9) + (100 - 1)(-15)]$ $= -73\ 350$	✓ correct substitution ✓ answer (2)
4.3	$\sum_{n=0}^{m-1} (24 - 15n) = S_{100} + 73\ 320$ $= -73\ 350 + 73\ 320$ $\frac{m}{2}[2(24) + (m - 1)(-15)] = -30$ $m(48 - 15m + 15) = -60$ $63m - 15m^2 = -60$ $0 = 15m^2 - 63m - 60$ $0 = 5m^2 - 21m - 20$ $0 = (5m + 4)(m - 5)$ $m = 5 \text{ or } m \neq -\frac{4}{5}$	✓ RHS = -30 ✓ $\frac{m}{2}[2(24) + (m - 1)(-15)]$ ✓ standard form ✓ $m = 5$ and rejection (4)
Subtotal		[9]

QUESTION 5

Q#	Suggested Solutions	Descriptors
5.1	$p = 3$ $q = -1$	✓ $p = 3$ ✓ $q = -1$ (2)
5.2	$g(x) = -x + k$ $-1 = -(-3) + k \quad \dots \text{at point } (-3; -1)$ $-4 = k$	✓ correct substitution ✓ $-4 = k$ (2)

<p>5.3</p> $h(x) = -2(-x - 4)$ $= 2x + 8$ <p>For inverse:</p> $x = 2y + 8$ $x - 8 = 2y$ $\frac{x-8}{2} = y$ $\therefore y = \frac{x}{2} - 4 \Rightarrow h^{-1}(x) = \frac{x}{2} - 4$	<p><input checked="" type="checkbox"/> $h(x) = 2x + 8$</p> <p><input checked="" type="checkbox"/> swapping x and y</p> <p><input checked="" type="checkbox"/> $y = \frac{x}{2} - 4$</p> <p style="text-align: right;">(3)</p>
<p>5.4</p>	<p><input checked="" type="checkbox"/> correct graph shapes</p> <p><input checked="" type="checkbox"/> x- and y-intercepts interchanging</p> <p><input checked="" type="checkbox"/> correct point of intersection</p> <p><input checked="" type="checkbox"/> axis of symmetry passes through the correct point of intersection</p> <p style="text-align: right;">(4)</p>
Subtotal	[11]

QUESTION 6

Q#	Suggested Solutions	Descriptors
6.1	$f(x) = ax^2$ $2 = a(-1)^2$ $\therefore a = 2$	$g(x) = b^x$ $2 = b^{-1}$ $b = \frac{1}{2}$ <p style="text-align: right;">(2)</p>
6.2	$x \leq 0$ or $x \geq 0$	<input checked="" type="checkbox"/> any one of the correct conditions (1)
6.3	$x \leq -1$ OR $x \in (-\infty; -1]$ OR $-\infty < x \leq -1$	<input checked="" type="checkbox"/> correct critical value <input checked="" type="checkbox"/> correct notation (2)
6.4.1	$A' (-4; 2)$	<input checked="" type="checkbox"/> $(-4; 2)$ (1)
6.4.2	$A'' (2; -4)$	<input checked="" type="checkbox"/> <input checked="" type="checkbox"/> $(2; -4)$ (2)
Subtotal		[8]

TOTAL = 50 MARKS

QUESTION 1

1.1 Solve for x .

$$1.1.1 \quad x(2x - 3) = 0 \quad (2)$$

$$1.1.2 \quad 0 = 3x^2 - 5x - 11 \quad (3)$$

$$1.1.3 \quad x(3x + 4) - 2(3x + 4) \leq 0 \quad (3)$$

1.2 The roots of a quadratic equation, in terms of p , are given as:

$$x = \frac{4 \pm \sqrt{8 - p^3}}{p} \quad (3)$$

Determine the value(s) of p for the roots to be real.

[11]

QUESTION 2

The first four (4) terms of a quadratic pattern are: 11 ; 20 ; 33 ; 50 ; ...

2.1 Determine the general term of this pattern in the form $T_n = an^2 + bn + c$. (4)

2.2 Prove that the sum of the first n first-differences of this quadratic pattern can be given by $S_n = 2n^2 + 7n$. (2)

[6]

QUESTION 3

A convergent geometric series is given by: $\frac{5(x+1)}{3} + \frac{5(x+1)^2}{9} + \frac{5(x+1)^3}{27} + \dots$

3.1 Calculate the values of x . (3)

3.2 If $x = 1$, calculate the sum to infinity, S_{∞} . (2)

[5]

QUESTION 4

$(1 - x)$; $(x + 2)$ and $(2x - 5)$ are the first three (3) terms of an arithmetic sequence.

4.1 Determine the value of x . (3)

4.2 If the first three (3) terms of this pattern are: 9; -6; -21; ..., calculate the numerical value of the sum of the first 100 terms, S_{100} . (2)

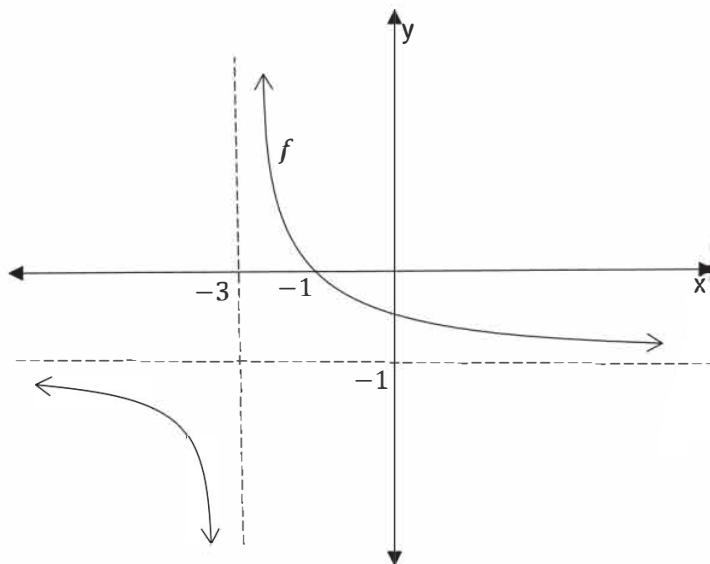
4.3 Hence, or otherwise, calculate m if:

$$\sum_{n=0}^{m-1} (24 - 15n) = S_{100} + 73\,320 \quad (4)$$

[9]

QUESTION 5

The graph of $f(x) = \frac{2}{x+p} + q$ is sketched below:

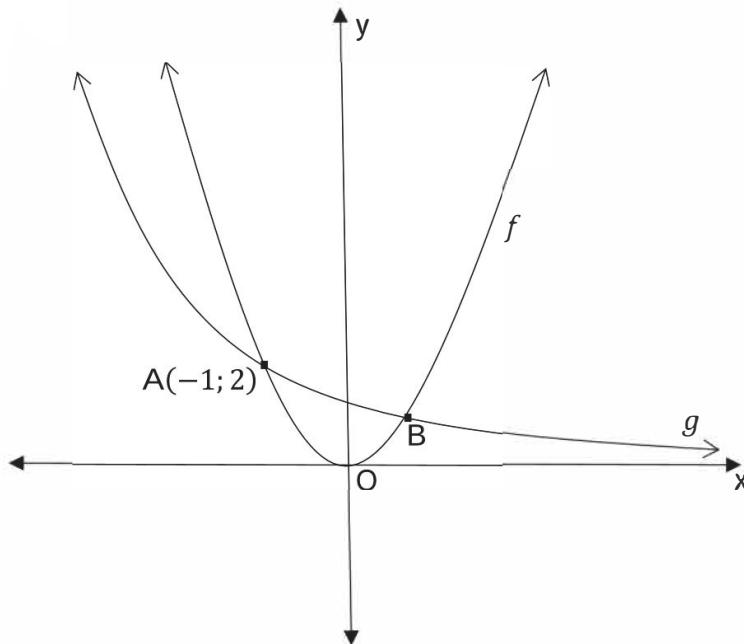


- 5.1 Write down the values of p and q (2)
- 5.2 The straight line $g(x) = -x + k$ is one of the axes of symmetry of the graph of f . Determine the value of k . (2)
- 5.3 If $h(x) = -2[g(x)]$, determine the equation of the inverse of h , h^{-1} , in the form $h^{-1}(x) = mx + c$. (3)
- 5.4 Draw a neat sketch of the graphs of h and h^{-1} on the same set of axes. Clearly show all intercepts with axes, point of intersection and the axis of symmetry. (4)

[11]

QUESTION 6

The graphs of $f(x) = ax^2$ and $g(x) = b^x$ are sketched on the same set of axes. Points $A(-1; 2)$ and B are points of intersection of f and g . The graph of f has the turning point at the origin:



- 6.1 Calculate the values of a and b . (2)
 - 6.2 The inverse of f is NOT a function. Write down at least one condition which can be used to restrict the domain of f such that its inverse will be a function. (1)
 - 6.3 For which value(s) of x , where $x \in (-\infty; 0]$, will $g(x) \leq f(x)$? (2)
 - 6.4 If $h(x) = g(x + 3)$, write down the coordinates of ...
 - 6.4.1 A' , the new coordinates of A on the graph of h . (1)
 - 6.4.2 A'' , the new coordinates of A on the graph of h^{-1} , the inverse of h (2)
- [8]

TOTAL = 50 MARKS

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$A = P(1+i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1-r} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum f_i x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$