education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

JUNE 2023

MARKING GUIDELINE

MARKS: 150 marks

This question paper consists of 13 pages and an informati on sheet

NOTE:

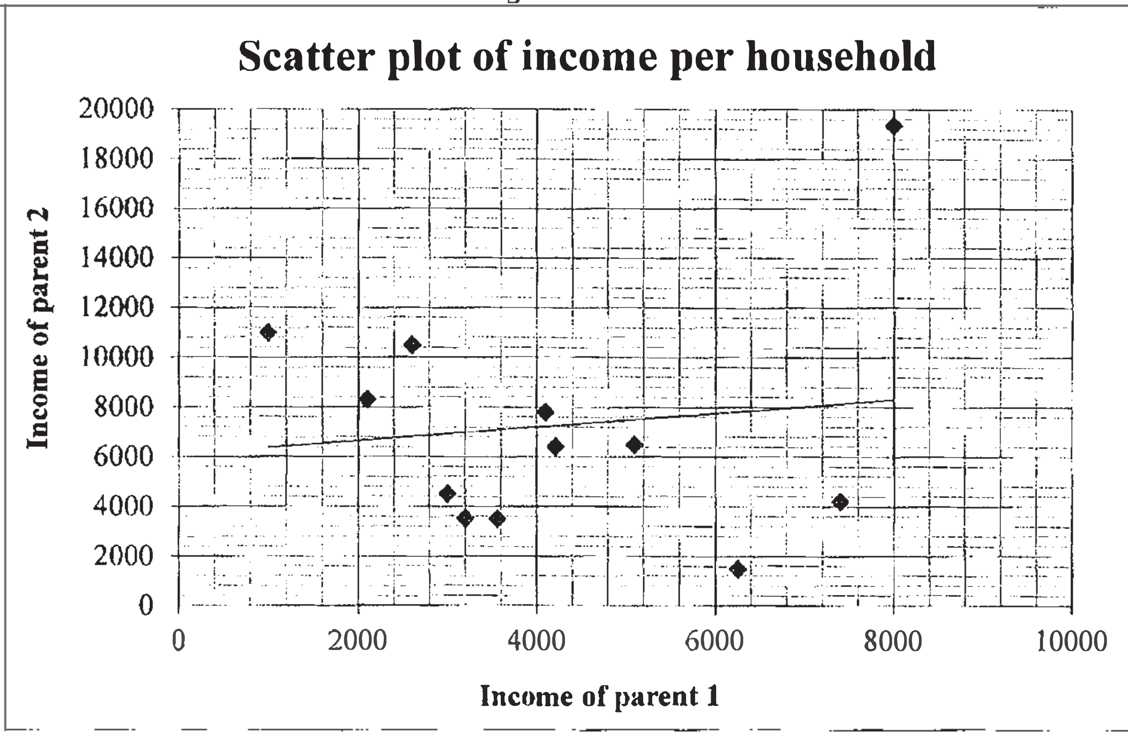
1. If a candidate answered a question TWICE, mark only the FIRST attempt.
2. If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
3. Consistent accuracy applies to ALL aspects of the marking memorandum.
4. Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

5. *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
6. *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
7. *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.*
8. *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*
9. Write neatly and legibly.

QUESTION/*VRAAG* 1

1.1	$\frac{87110}{12} = \text{R } 7259{,}17$	✓R 7259,17 (2)
1.2	SD = R4579,26 Above ONE standard deviation = mean+1SD $= \text{R } 7259{,}17 + \text{R}4579{,}26$ $= \text{R}11838{,}43$ Only ONE household	✓SD ✓boundary ✓answer (3)
1.3	$y = a + bx$ $a = 6102{,}11$ $b = 0{,}27$ $y = 6102{,}11 + 0{,}27x$	✓ $a = 6102{,}11$ $23{,}47$ ✓ $b = 0{,}27$ $92{,}85$ ✓ $y = 6102{,}11 + 0{,}27x$ (3)

<table>
<tr><td>1.4</td><td colspan="2">
</td></tr>
<tr><td></td><td></td><td>✓✓points correctly plotted
✓✓regression line
(4)</td></tr>
<tr><td>1.5</td><td>The gradient of the line will become smaller, causing the data to be more symmetrical about the regression line, and there will be no outlier.</td><td>✓✓answer
(2)</td></tr>
<tr><td></td><td></td><td>[14]</td></tr>
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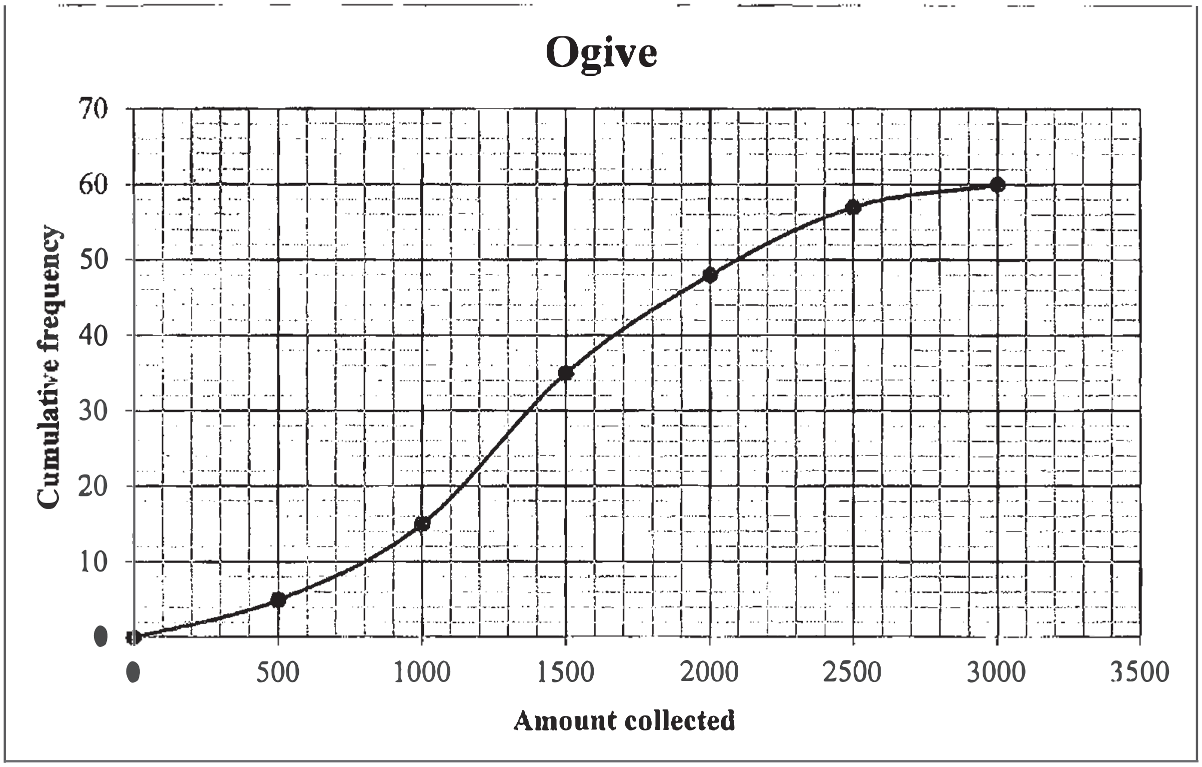
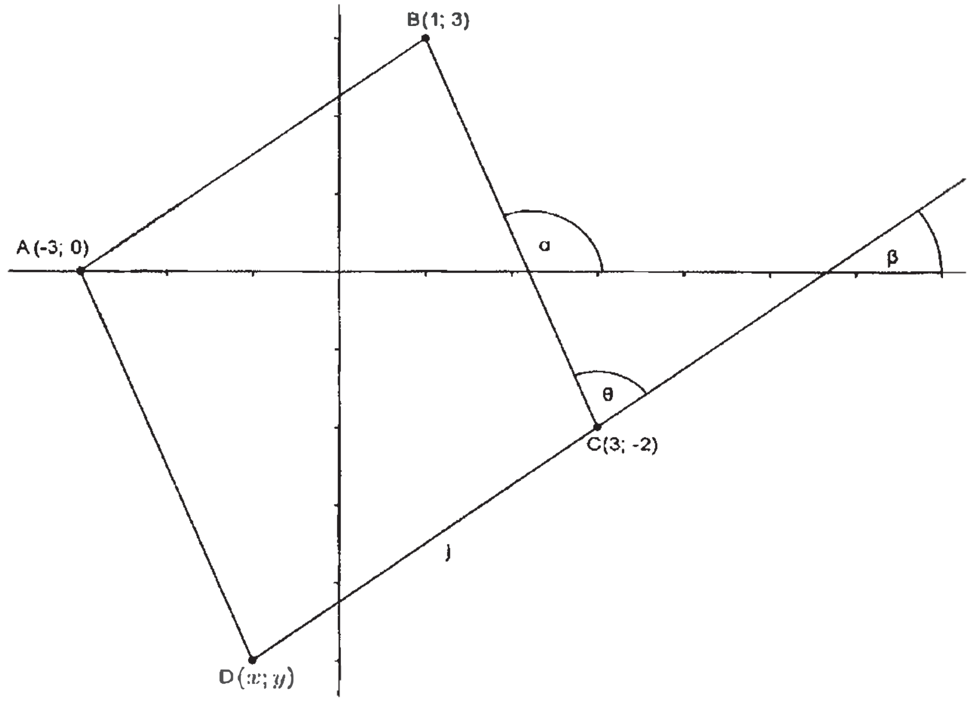
QUESTION/*VRAAG* 2



2.1	$1000 < x \leq 1500$	✓ answer (1)
2.2	$60 - 15 = 45$ parents	✓15 and 60 ✓answer (3)
2.2	a = lower quartile = 1000 b = median = 1350 c = upper quartile = 1850	✓method ✓Q1 ✓Q2 ✓ Q3 (4) **[7]**

QUESTION 3

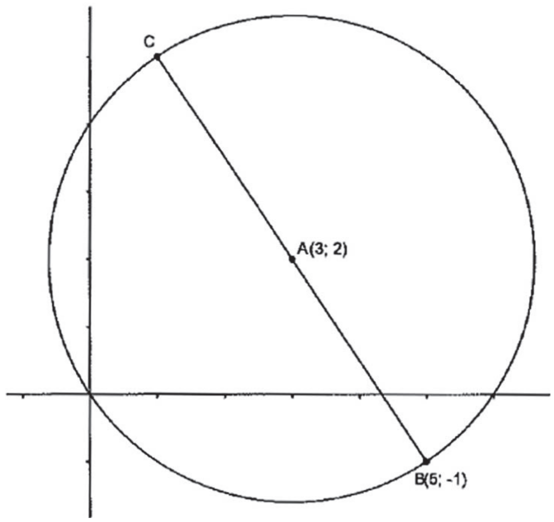


3.1	D(−1; −5)	✓✓ (2)
3.2	$m_{AB} = \frac{3-0}{1-(-3)} = \frac{3}{4}$ $m_{BC} = \frac{3-(-2)}{1-3} = \frac{5}{-2}$ $\frac{3}{4} \times \frac{5}{-2} = \frac{15}{-8} \neq -1$ $\therefore$ AB not perpendicular to BC, $\therefore$ ABCD is not a rectangle	m_{AB} ✓ m_{BC} ✓ $\neq -1$ ✓ Not rectangle✓(4)
3.3	$M_{AB} = \left(\frac{-3+1}{2}; \frac{0+3}{2}\right) = \left(-1; \frac{3}{2}\right)$	✓✓ (2)
3.4	$m_{AB} = \frac{3}{4}, \therefore m_{\perp} = -\frac{4}{3}$ $y - \frac{3}{2} = -\frac{4}{3}(x - (-1))$ $y = -\frac{4}{3}x - \frac{4}{3} + \frac{3}{2}$ $= -\frac{4}{3}x + \frac{1}{6}$	$m_{\perp}$ ✓ Subst ✓ ✓ (3)
3.5	$tan\alpha = -\frac{5}{2}$ $\alpha = -68{,}198 \ldots° + 180° = 111{,}80°$ $tan\beta = \frac{3}{4}$ (AB // CD) $\beta = 36{,}87°$	✓ ✓ ✓

	$\therefore \theta = 111{,}80° - 36{,}87°$ $= 74{,}93°$ (ext $\angle$ of Δ) $B\hat{C}D = 105{,}07°$ (angles on straight line)	✓ ✓ (5)
3.6	$BC^2 = (1-3)^2 + (3-(-2))^2$ $= 4 + 25 = 29$ $BC = \sqrt{29}$ $CD^2 = (3-(-1))^2 + (-2-(-5))^2$ $= 16 + 9 = 25$ $CD = 5$ Area $\Delta BCD = \frac{1}{2} BC.CD.\sin B\hat{C}D.$ $= \frac{1}{2}.\sqrt{29}.\ 5\ .\sin 105{,}07°$ $= 13$ units²	Subst in formula ✓ BC ✓ CD ✓ Subst in sine form ✓ ✓ (5)
		[21]

QUESTION 4

4.1

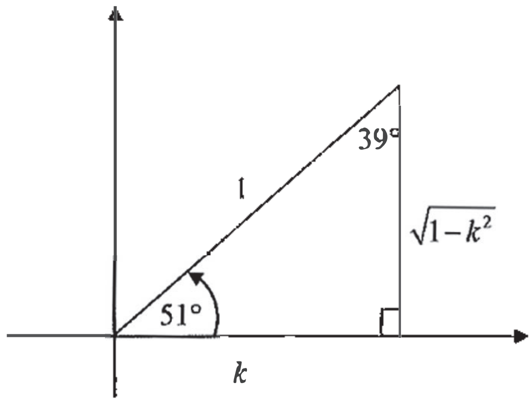


4.1.1	$r^2 = (5-3)^2 + (-1-2)^2$ $= 4 + 9 = 13$ $\therefore (x-3)^2 + (y-2)^2 = 13$	✓ ✓ ✓✓ (4)
4.1.2	C = (1; 5) (symmetry)	✓✓ (2)
4.1.3	$m_{AC} = \frac{5-2}{1-3} = \frac{3}{-2}$ $m_{tangent} = \frac{2}{3}$ $y - 5 = \frac{2}{3}(x-1)$ $y = \frac{2}{3}x - \frac{2}{3} + 5$ $= x + 4\frac{1}{3}$	✓ ✓ ✓ ✓ (4)
4.1.4	$r = \sqrt{13}$ and horizontal lines $\sqrt{13}$ from centre. $\therefore y = 2 + \sqrt{13}$ and $y = 2 - \sqrt{13}$	✓ ✓ (2)
4.1.5	$(x-3)^2 + (y-2)^2 = (4+\sqrt{13})^2$ Or $(x-3)^2 + (y-2)^2 = 57{,}84$	centre radius (2)
4.2	$x^2 + y^2 + 4y + 3 = 0$ $x^2 + y^2 + 4y + 2^2 = -3 + 2^2$ $x^2 + (y+2)^2 = 1$	✓

	Centres: A: (3; 0) and B: (0; -2) Distance between centres AB $= \sqrt{(3-0)^2+(2-0)^2} = \sqrt{13}$ $= 3{,}61$ Radii = 1 and 2 Sum of radii = 3 Sum of radii < Distance between centres $\therefore$ Circles do not intersect	✓ ✓ ✓ ✓ ✓	(6)
			[20]

QUESTION 5

5.1 $\cos 51° = \frac{k}{1}$



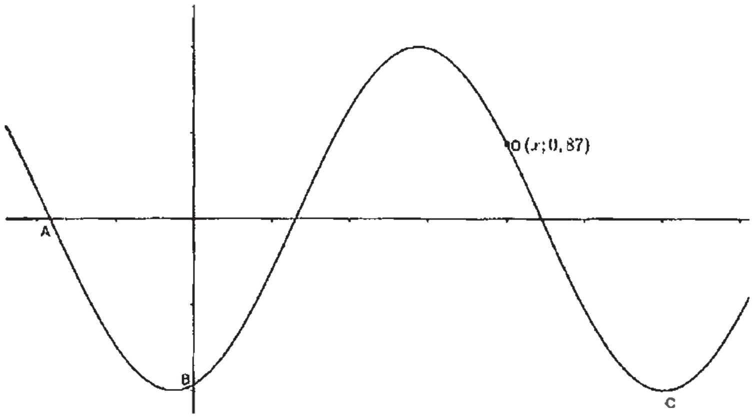
5.1.1	$y^2 = 1^2 - k^2$ $y = \sqrt{1-k^2}$ $\tan 219° = \tan(180° + 39°)$ $= \tan 39°$ $= \frac{k}{\sqrt{1-k^2}}$	✓ Pythagoras ✓$-\tan 39°$ ✓ answer (3)
5.1.2	$\sin(-411°) = \sin(-411° + 360°)$ $= \sin(-51°)$ $= -\sin 51°$ $= -\frac{\sqrt{1-k^2}}{1}$	✓ reduction ✓answer (2)
5.1.3	$\cos 9° = \cos(60° - 51°)$ $= \cos 60° \cos 51° + \sin 60° \sin 51°$ $= \left(\frac{1}{2}\right)(k) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{1-k^2}}{1}\right)$ $= \frac{1k}{2} + \frac{\sqrt{3(1-k^2)}}{2}$	✓compound angles ✓expansion ✓substitution ✓ substitution (4)

5.2	$\sin(45°+x)\cdot\sin(45°-x)$ $=(\sin 45\cos x+\cos 45\sin x)(\sin 45\cos x-\cos 45\sin x)$ $=\left(\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x\right)\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)$ $=\frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x$ $=\frac{1}{2}\cos 2x$	✓expansion ✓expansion ✓ $\frac{\sqrt{2}}{2}$ ✓ simplification ✓answer (5)
5.3	$\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x}=\tan x$ $LHS=\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x}$ $=\frac{\sin x+2\sin x\cos x}{1+\cos x+2\cos^2 x-1}$ $=\frac{\sin x+2\sin x\cos x}{\cos x+2\cos^2 x}$ $=\frac{\sin x(1+2\cos x)}{\cos x(1+2\cos x)}$ $=\tan x=RHS$	✓ $2\sin x\cos x$ ✓ $2\cos^2 x-1$ ✓simplification of denominator ✓common factor ✓common factor ✓ $\frac{\sin x}{\cos x}=\tan x$ (6)
5.4	$\sin 15°=\sin(45°-30°)$ $=\sin 45\cos 30-\cos 45\sin 30$ $=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ $=\frac{\sqrt{6}-\sqrt{2}}{4}$	✓ $\sin(45°-30°)$ ✓expansion ✓substitution ✓simplification $\sqrt{6}$ (4)

5.5	$\frac{(\sin x-\cos x)^2-1}{\sin^2 x-1}=2$ $\sin^2 x-2\sin x\cos x+\cos^2 x-1=2(\sin^2 x-1)$ $\sin^2 x-2\sin x\cos x+\cos^2 x-1=2\sin^2 x-2$ $-2\sin^2 x-2\sin x\cos x+2=0$ $-\sin^2 x-\sin x\cos x+1=0$ $-\sin^2 x - \sin x\cos x+\sin^2 x+\cos^2 x=0$ $\cos^2 x-\sin x\cos x=0$ $\cos x(\cos x-\sin x)=0$ $\cos x=0$ *or* $\cos x-\sin x=0$ $x=\pm 90°+360k$ $\cos x=\sin x$ $\tan x=1$ $x=45°+180k,\ k\in Z$	✓ $2\sin^2 x-2$ ✓ $\sin^2 x+\cos^2 x$ ✓standard form ✓common factor ✓ $x=\pm 90+360k \quad k\in Z$ ✓ $\tan x=1$ ✓ $x=45°+180k$ (7)
		[31]

QUESTION 6

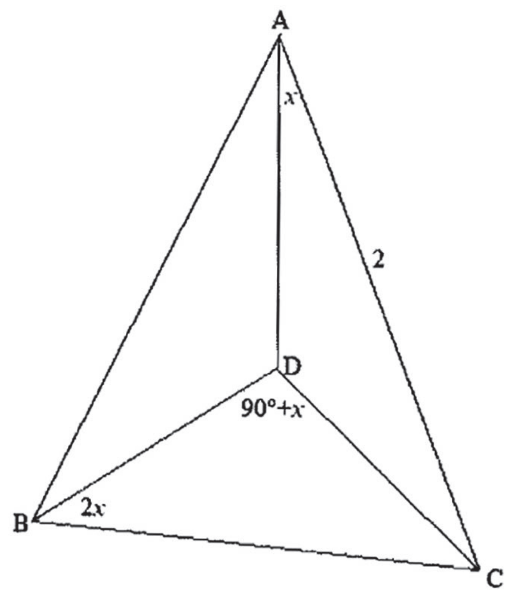
The graph of $f(x) = -2\cos(x + 15°)$is given.



6.1	Amplitude = 2	✓	(1)
6.2	Period = 360°	✓	(1)
6.3	Range of g(x): $y \in [0; 4]$	✓✓	(2)
6.4.1	∴ A = (−105°; 0)	✓✓	(2)
6.4.2	B = (0, −1,93)	✓✓	(2)
6.4.3	C = (165°; −2)	✓✓	(2)
6.4.4	$-2\cos(x + 15°) = 0{,}87$ $\cos(x + 15°) = -0{,}435$ $x + 15° = 115{,}785\ldots° + 360k, k \in Z$ $x = 100{,}79° + 360k$ ∴ D = (100,79°; 0,87)	 ✓ ✓ ✓	(3)
			[13]

QUESTION 7

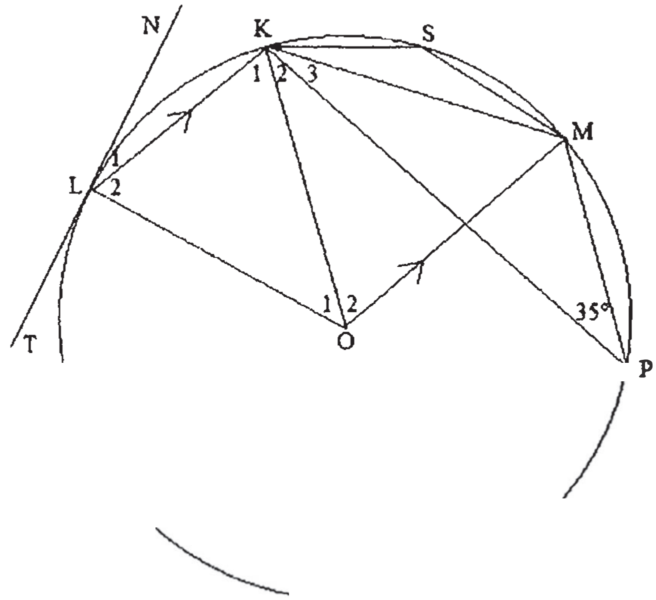
AD is a vertical pole and points B and C are in the same horizontal plane as D, the foot of the tower. $D\hat{A}C = x$, $B\hat{D}C = 2x$, $B\hat{D}C = 90° + x$ and AC = 2.



7.1	$\sin x = \frac{CD}{2}$ $CD = 2\sin x$ $\frac{BC}{\sin(90°+x)} = \frac{CD}{\sin 2x}$ $\frac{BC}{\cos x} = \frac{2\sin x}{2\sin x\cos x}$ $BC = \frac{2\sin x\,.\,\cos x}{2\sin x\cos x}$ $= 1$	✓ ✓ Subst in Sin-rule ✓ Cos x ✓ 2 sin$x$ cos$x$ ✓ Simplification ✓ (6)
7.2	$B\hat{C}D = 180° - (90° + x) - 2x$ int ∠s of Δ $= 90° - 3x$ In ΔBCD: $\frac{BD}{\sin(90°-3x)} = \frac{1}{\sin(90°+x)}$ $\frac{BD}{\cos 3x} = \frac{1}{\cos x}$ $\frac{\cos 3x}{\cos x} = 2\cos 2x - 1.$	✓ Subst in sine-form Co-functions✓ (3)
		[9]

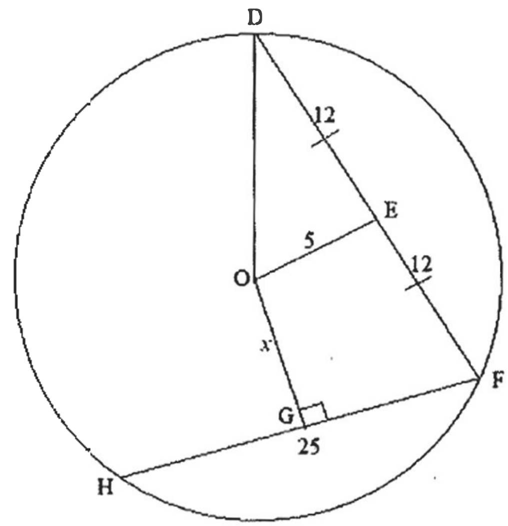
Question 8

8.1 In the diagram, O is the centre of the circle. KL // OM, NLT is a tangent to the circle at L.



8.1.1	$\hat{O}_2 = 70°$	$\angle$ at centre circle = $2\times\angle$ at circumf	S ✓ R✓ (2)
8.1.2	$\hat{K}_1 = 70°$ $\hat{L}_2 = 70°$ $\hat{O}_1 = 40°$	alt.$\angle$s, KL // MO $\angle$s opp = radii int $\angle$s of Δ	S + R ✓ S + R ✓ S + R ✓ (3)
8.1.3	$\hat{L}_1 = 20°$	radius $\perp$ tangent	S ✓ R✓ (2)
8.1.4	$\hat{S} = 135°$	opp $\angle$ of cyclic quad	S ✓ R✓ (2)

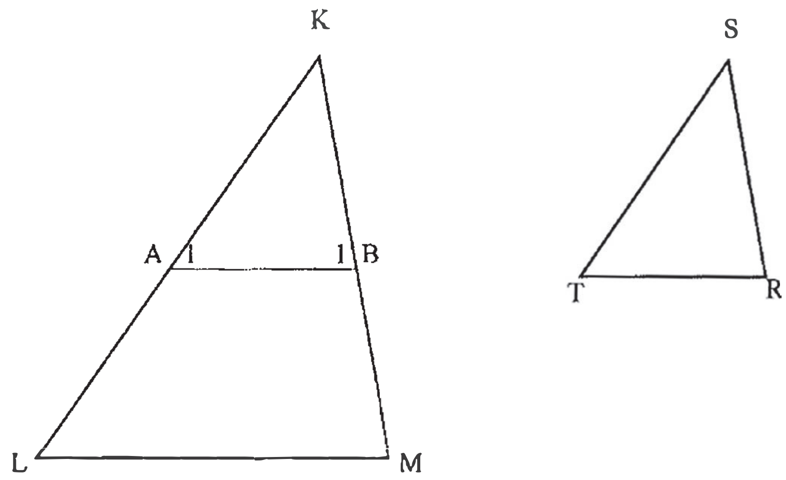
8.2



8.2.1	$O\hat{E}D = 90°$ $OD^2 = 12^2 + 5^2$ $= 169$ $OD = 13$	Centre circle midpoint chord Theorem of Pythagoras	S ✓ R✓ ✓ ✓ (4)
8.2.2	FG = 12,5	Centre circle ⊥ chord	S ✓ R✓ (2)
			[15]

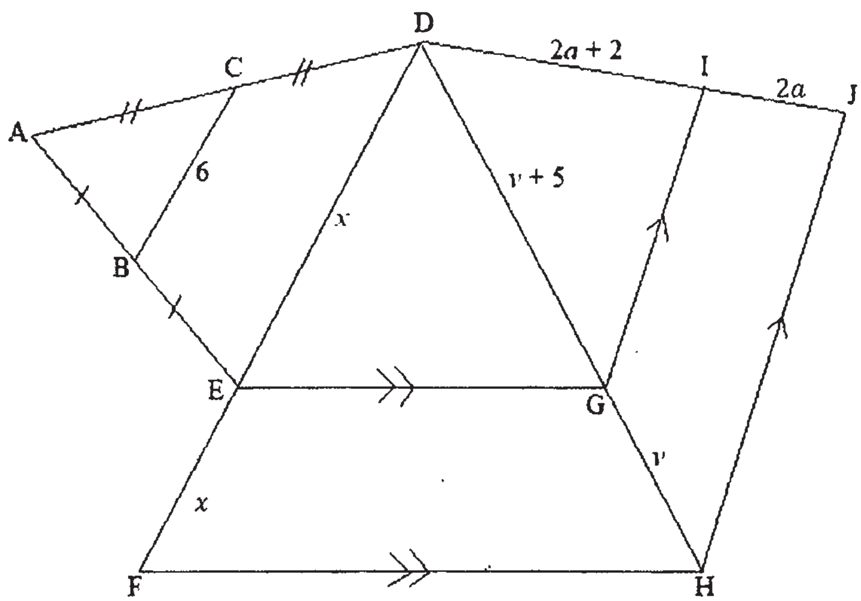
QUESTION 9

9.1 In ΔKLM and ΔSTR, $\hat{K} = \hat{S}$, $\hat{L} = \hat{T}$, $\hat{M} = \hat{R}$. Prove that $\frac{ST}{KL} = \frac{SR}{KM}$. (6)



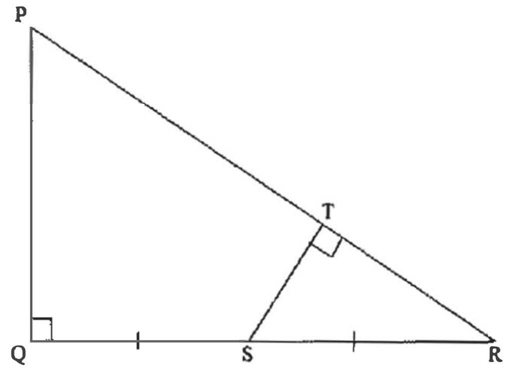
9.1	Construction		✓
	In ΔKAB and ΔSTR:		
	AK = ST	construction	
	KB = SR	construction	
	$\hat{K} = \hat{S}$	given	✓
	$\therefore \Delta KAB \equiv \Delta STR$	SAS	✓
	$\therefore \hat{A}1 = \hat{T}$		✓
	But $\hat{T} = \hat{L}$	given	
	$\therefore \hat{A}1 = \hat{T}$		✓
	$\therefore$ AB // LM	corresponding angles =	
	$\therefore \frac{KA}{KL} = \frac{KB}{KM}$	line // one side of Δ	✓
	$\therefore \frac{ST}{KL} = \frac{SR}{KM}$	AK = ST and KB = SR	(6)

9.2 In the sketch EG // FH and GI // HJ. AB = BE and AC = CD. BC = 6, DE = x, EF = $x - 3$, DG = $y + 5$, GH = y, DI = $2a + 2$ and IJ = $2a - 3$.



9.1	$x = 12$	midpt theorem	S ✓ R✓ (2)
9.2	$\frac{x}{x-3} = \frac{y+5}{y}$ $\frac{12}{9} = \frac{y+5}{y}$ $12y = 9y + 45$ $3y = 45$ $y = 15$	line // to onc side of Δ	S ✓ R✓ ✓ (3)
9.3	$\frac{2a+2}{2a-3} = \frac{y+5}{y}$ $\frac{2a+2}{2a-3} = \frac{20}{15} = \frac{4}{3}$ $6a + 6 = 8a - 12$ $2a = 18$ $a = 9$	line // to one side of Δ	S ✓ R✓ (3)
			[14]

QUESTION 10



10.1	In ΔPQR and ΔSTR $\hat{Q} = \hat{T}$ $\hat{R} = \hat{R}$ $\therefore \hat{P} = \hat{S}$ ΔPQR /// ΔSTR	 90° given common ∠ int ∠s of Δ A A A	 S + R ✓ S + R ✓ S + R ✓ R ✓ (4)
10.2	$\frac{PQ}{ST} = \frac{RQ}{RT} = \frac{PR}{RS}$ $\therefore PR.RT = RS.RQ$ but RS = QS $\therefore PR.RT = QS.RQ$	similarity	✓ ✓ (2)
			[10]

TOTAL: 150