Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2023

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 12 pages and an answer book of 22 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Write neatly and legibly. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

Between 05:00 and 06:00 on New Year's Day, 11 minibus taxis were stopped at a roadblock between King William's Town and East London. The following data set represents the number of passengers per minibus taxi.

18	26	25	18	16	12	10	8	18	17	8

1.1 Calculate the mean number of passengers per taxi. (2)

1.2 Calculate the standard deviation for this data set. (2)

1.3 Taxis having a number of passengers with one standard deviation above the mean could be regarded as overloaded. How many taxis were overloaded? (2)

1.4 If the number of passengers in a taxi is one standard deviation below the mean, the trip could be regarded as uneconomical. Calculate the percentage of taxis that are in this category. (2)

[8]

QUESTION 2

Working dads help working moms with housework.
The table below represents the number of hours spent per week in doing household work.

Hours	**Number of dads**
$0 < x \leq 5$	1
$5 < x \leq 10$	18
$10 < x \leq 15$	24
$15 < x \leq 20$	25
$20 < x \leq 25$	18
$25 < x \leq 30$	12
$30 < x \leq 35$	1
$35 < x \leq 40$	1

2.1 Complete the frequency table in the SPECIAL ANSWER BOOK and draw an ogive of the data on the grid provided. (4)

2.2 Use the graph to find an approximate median value. (2)

2.3 Write down the modal class. (1)

2.4 Calculate the approximate mean. (2)

2.5 Compare the mean, median and mode values. Explain what this means for the set of data. (3)

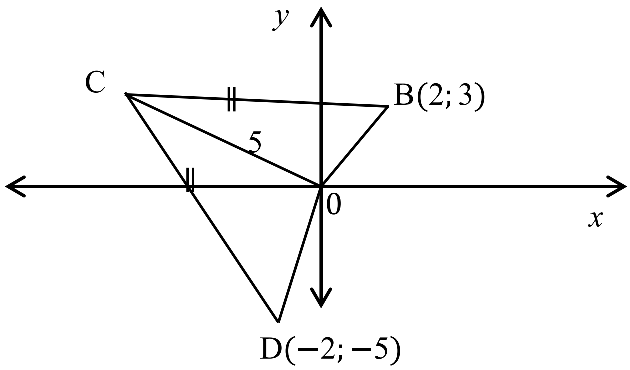
[12]

QUESTION 3

3.1 The straight-line $y = 3x - 3$ is perpendicular to the straight line which cuts the y-axis at $(0; 10)$ and passes through the point $\left(4\ ;\ \frac{p}{2}\right)$. Determine the value of p. (3)

3.2 The distance between the origin and point $P(-2\ ; p - 1)$ is $2p$ units. Calculate the value of p. (5)

3.3 The diagram below shows quadrilateral OBCD with vertices $\mathrm{O}(0\ ; 0)$, $\mathrm{B}(2\ ; 3)$, C and $\mathrm{D}(-2\ ;\ -5)$. The length of OC is 5 units and BC = DC.



3.3.1 Determine the gradient of BD. (2)

3.3.2 Determine the equation of the perpendicular bisector from C to BD in the form $y = mx + c$. (3)

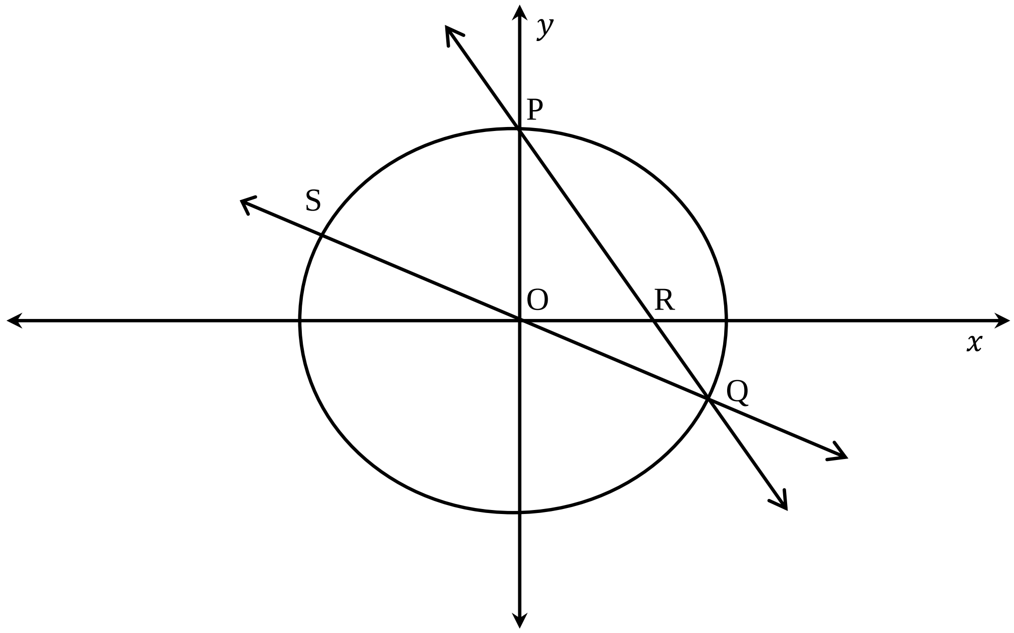
3.3.3 Determine the equation of the circle centred at O and passing through C. (2)

3.3.4 Determine the y-coordinate of point C. (6)

[21]

QUESTION 4

In the diagram below, circle $x^2 + y^2 = 16$ intersects the straight-line PQ, which is defined by $2x + y = 4$ at P and Q. R is the x-intercept of PQ.



4.1 Show that the coordinates of P and Q are $(0\,;4)$ and $(3{,}2\,;-2{,}4)$ respectively. (7)

4.2 QO produced cuts the circle at S. Determine the coordinates of S. (2)

4.3 Determine the equation of the circle with the centre at R and touches the y-axis. (4)

4.4 Determine the distance between the centres of the circles $x^2 + y^2 = 16$ and $(x-6)^2 + y^2 - y = 12$. (5)

[18]

QUESTION 5

5.1 If $5\cos\theta - 3 = 0$; $180^{\circ} < \theta < 360^{\circ}$ and $17\sin\alpha = 8$; $90^{\circ} < \alpha < 270^{\circ}$, determine, without the use of a calculator, the value of $\tan\alpha + \tan\theta$. (6)

5.2 If $\cos 42^{\circ} = p$, determine each of the following in terms of p:

5.2.1 $\sin 48^{\circ}$ (2)

5.2.2 $\sin(-2202^{\circ})$ (2)

5.2.3 $\cos 84^{\circ}$ (2)

5.3 Determine the value of the expression without the use of a calculator:

$$\frac{\tan 300^{\circ} + \cos(90^{\circ} + x)}{\sin(180^{\circ} - x) + 2\cos(-30^{\circ})}$$ (6)

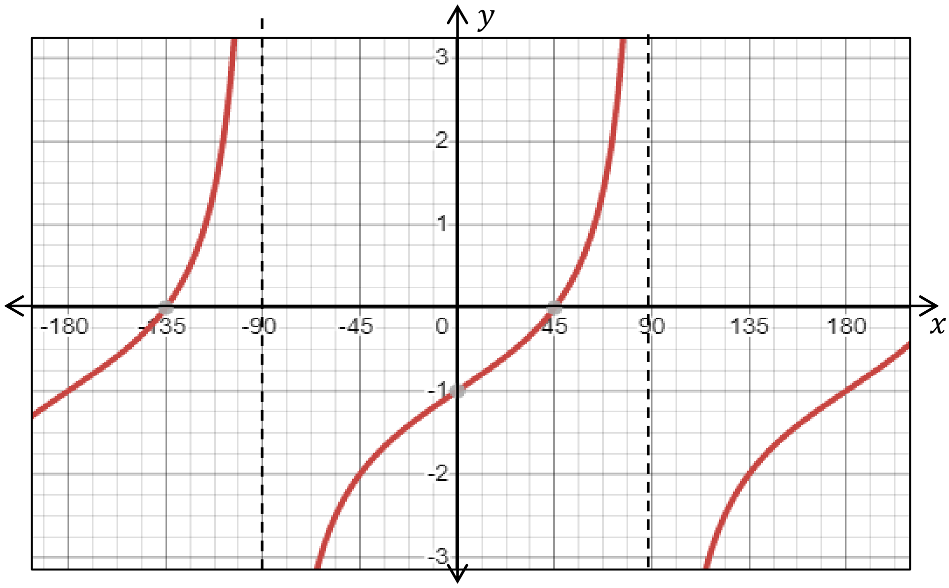
5.4 Prove the following identity: $\dfrac{1 - \sin 2x}{\cos 2x} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$ (5)

5.5 Determine the general solution of $\cos x - \sin x = \sqrt{2}$. (5)

[28]

QUESTION 6

In the diagram below, the function $f(x) = \tan x - 1$ is drawn for the interval $[-180^\circ; 180^\circ]$.



6.1 Draw the function $g(x) = \cos 2x$ in your SPECIAL ANSWER BOOK on the same set of axes. (3)

6.2 Write down the period of g. (1)

6.3 Write down the new equation in the form of $h(x) = \cdots$ if f is moved 3 units up. (1)

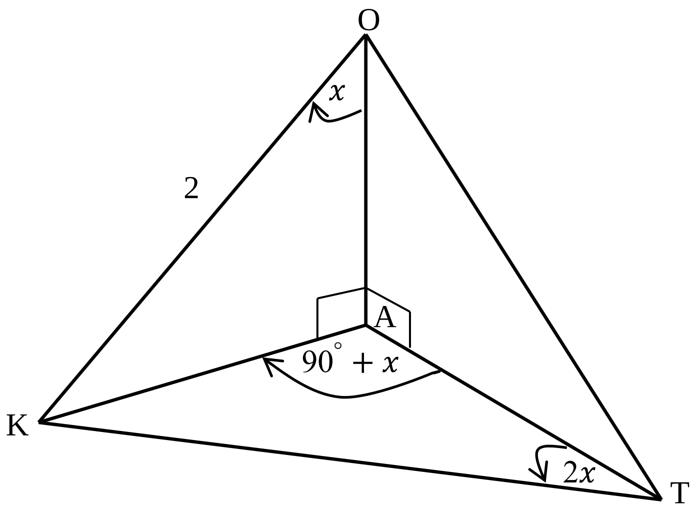
6.4 Use your graphs to determine the value(s) of x for which $\cos 2x \leq \tan x - 1$ for the interval $[-180^\circ; 0^\circ]$. (3)

6.5 Use your graph to solve the following equation: $\cos B + 1 = \tan\frac{1}{2}B$. (4)

[12]

QUESTION 7

In the figure below, OA is a vertical tower and the points K and T are in the same horizontal plane as A, the foot of the tower. $A\hat{O}K = x$, $K\hat{A}T = 90^{\circ} + x$, $K\hat{T}A = 2x$ and $OK = 2$ units.



7.1 Express AK in terms of a trigonometric function value of x in two different ways and hence or otherwise determine the length KT. (5)

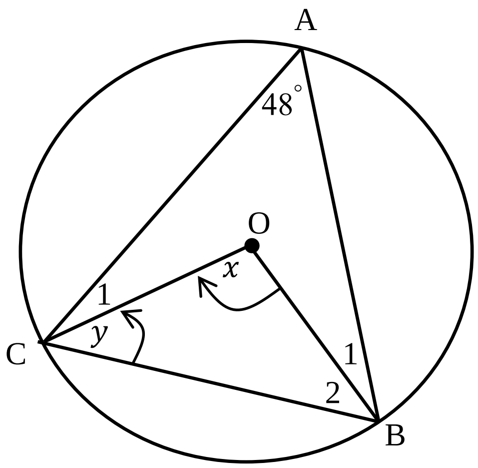
7.2 Show that: $AT = \dfrac{\cos 3x}{\cos x}$ (2)

7.3 Simplify $\dfrac{\cos 3x}{\cos x}$ to a trigonometric function of $\sin x$. (4)

[11]

QUESTION 8

8.1 In the diagram below, O is the centre of the circle passing through A, B and C.
$\hat{CAB} = 48^\circ$, $\hat{COB} = x$ and $\hat{C}_2 = y$.

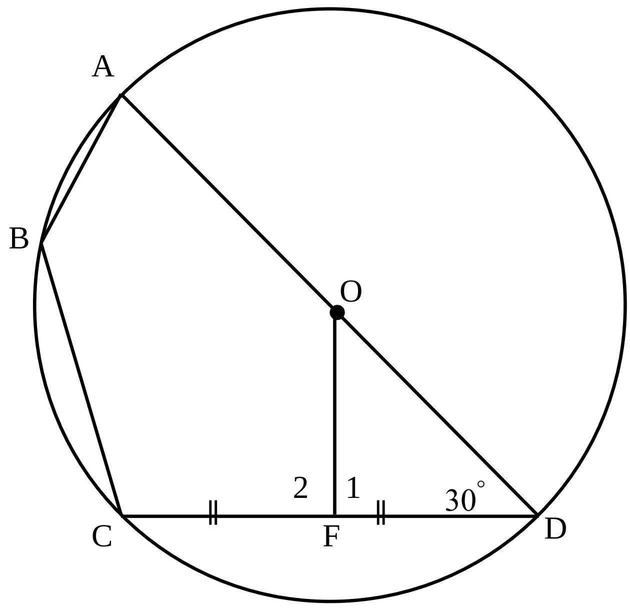


Determine, with reasons, the size of:

8.1.1 x (2)

8.1.2 y (2)

8.2 In the diagram below, O is the centre of the circle passing through A, B, C and D.
AOD is a straight line and F is the midpoint of chord CD. $\hat{ODF} = 30^\circ$.

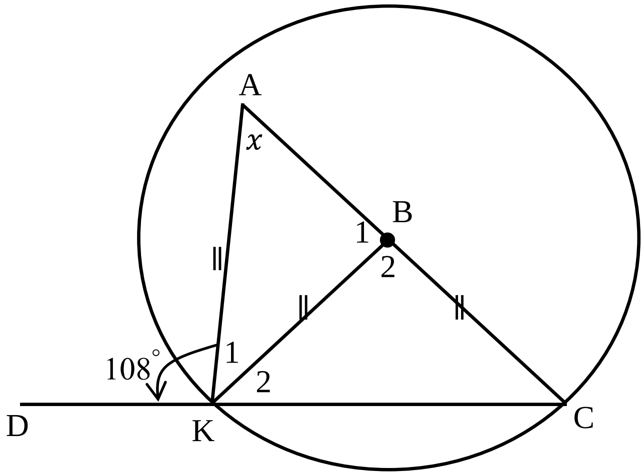


Determine, with reasons, the size of:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $\hat{ABC}$ (2)

8.3 In the diagram below, B is the centre of the circle. AK = KB = BC. $A\hat{K}D = 108^\circ$ and $\hat{A} = x$.



8.3.1 Express $\hat{B}_1$ in terms of x. (2)

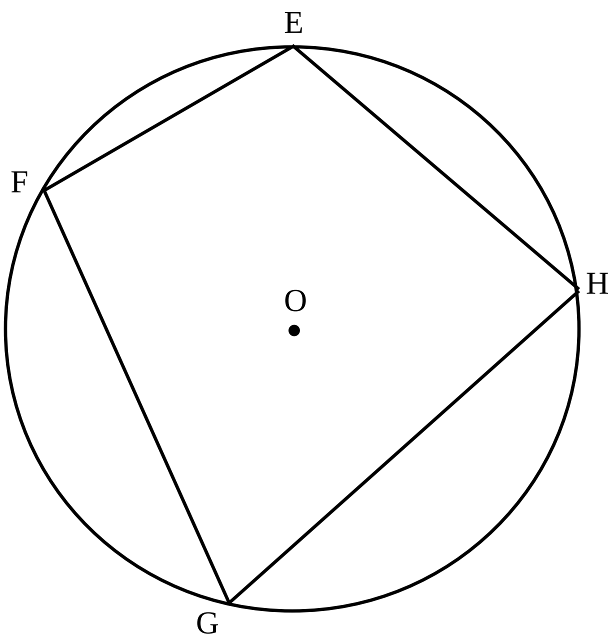
8.3.2 Show that $\hat{C} = \frac{x}{2}$. (3)

8.3.3 Solve for x. (4)

[17]

QUESTION 9

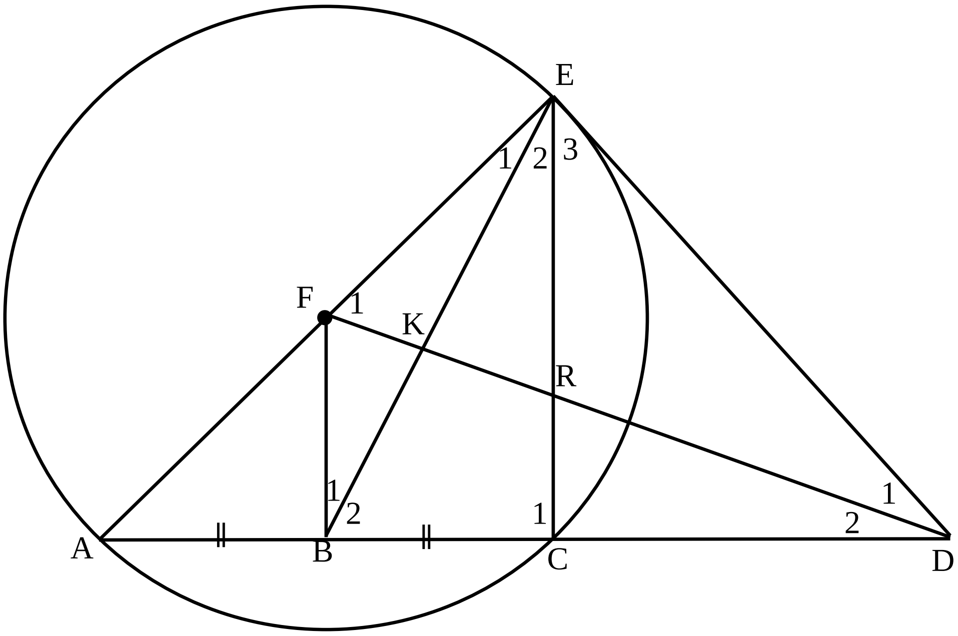
Refer to the diagram below. O is the centre of the circle. E, F, G and H are on the circumference of the circle. Prove the theorem that $\hat{E} + \hat{G} = 180^\circ$.



[6]

QUESTION 10

In the diagram below, ED is a tangent to the circle passing through A, C and E. F is the centre of the circle. AC is extended to meet ED at D and FB bisects AC. Straight-lines FD, BE and EC are drawn.



Prove, with reasons, that:

10.1 EFBD is a cyclic quadrilateral (4)

10.2 Δ BCE ||| ΔFED (6)

10.3 $BC = \dfrac{FA.CE}{ED}$ (3)

10.4 $BC = \dfrac{AC.FE}{AE}$ (4)

[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$$F=\frac{x\left[(1+i)^n-1\right]}{i} \qquad P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$T_n=a+(n-1)d$ $\qquad S_n=\frac{n}{2}\left(2a+(n-1)d\right)$

$T_n=ar^{n-1}$ $\qquad S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq 1$ $\qquad S_\infty=\frac{a}{1-r}$; $-1<r<1$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\qquad M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\qquad y-y_1=m(x-x_1)$ $\qquad m=\frac{y_2-y_1}{x_2-x_1}$ $\qquad m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\qquad a^2=b^2+c^2-2bc.\cos A$ $\qquad area\,\Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\qquad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\qquad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases} \qquad \sin 2\alpha=2\sin\alpha.\cos\alpha$$

$\bar{x}=\frac{\sum x}{n}$ $\qquad \sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$ $\qquad P(A)=\frac{n(A)}{n(S)}$ $\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y}=a+bx$ $\qquad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$