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**NATIONAL
SENIOR CERTIFICATE/
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GRADE/GRAAD 12

JUNE/JUNIE 2023

**TECHNICAL MATHEMATICS P1/TEGNIIESE WISKUNDE V1
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

MARKING CODES/NASIENKODES	
A	Accuracy/Akkuraatheid
CA	Consistent accuracy/Volgehoue akkuraatheid
M	Method/Metode
R	Rounding/Afronding
NPR	No penalty for rounding/Geen penalisering vir afronding nie
NPU	No penalty for units omitted/Geen penalisering vir eenhede weggelaat nie
S	Simplification/Vereenvoudiging
SF	Substitution in correct formula/Vervanging in korrekte formule

This marking guideline consists of 15 pages./
Hierdie nasienriglyn bestaan uit 15 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt to a question and not attempt the question again, then mark the crossed-out version should be marked.
- Consistent accuracy (CA) applies to ALL aspects of the marking guideline.
- Assuming answers/values to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.*
- *Indien 'n kandidaat 'n poging kanselleer en nie poog om die vraag weer te beantwoord dan moet die gekanselleerde antwoord gemerk word.*
- *Volgehoue akkuraatheid (CA) is van toepassing op ALLE aspekte van die nasienriglyn.*
- *Die aanvaarding van antwoorde / waardes om 'n probleem op te los is NIE aanvaarbaar NIE.*

QUESTION/VRAAG 1				
1.1	1.1.1	$x(3x-1)=0$ $\therefore x=0$ or/of $x=\frac{1}{3}$	$\checkmark x=0$ A $\checkmark x=\frac{1}{3}$ A	(2)
	1.1.2	$2x^2+13=5x$ $2x^2-15x+3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-15)\pm\sqrt{(-15)^2-4(2)(3)}}{2(2)}$ $x=7,29$ or / of $x=0,21$	$\checkmark$ Standard Form / <i>standaardvorm</i> A $\checkmark$ Formula / <i>Formule</i> A $\checkmark$ Substitution / <i>Vervanging</i> CA $\checkmark$ both values of x / <i>beide waardes van x</i> CA	(4)
	1.1.3	$(x-3)(x+4) \geq 0$ C.V / KW: -4 and / en 3 Solution / <i>Oplossing</i> $x \leq -4$ or / of $x \geq 3$ OR/OF 	$\checkmark$ Critical Values / <i>Kritiese waardes</i> A $\checkmark x \leq -4$ CA $\checkmark$ or / of $x \geq 3$ CA OR/OF $\checkmark$ Correct number line / <i>Korrekte getal lyn</i> CA	(3)

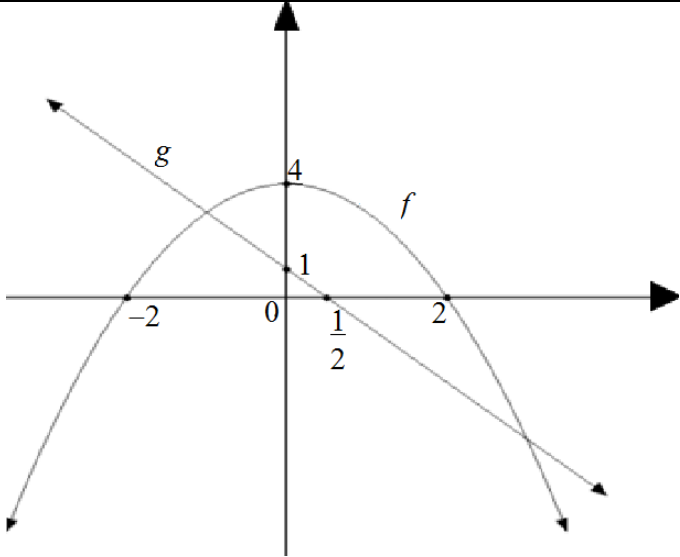
1.2	$y = x^2 - 11x + 36$ and/en $y = 2x - 6$ $x^2 - 11x + 36 = 2x - 6$ $x^2 - 13x + 42 = 0$ $(x - 7)(x - 6) = 0$ $x = 7$ or/of $x = 6$ $y = 2(7) - 6 = 8$ or / of $y = 2(6) - 6 = 6$ OR/OF $y = x^2 - 11x + 36$ $\frac{y+6}{2} = x$ $y = \left(\frac{y+6}{2}\right)^2 - 11\left(\frac{y+6}{2}\right) + 36$ $y = \left(\frac{y^2 + 12y + 36}{4}\right) - \left(\frac{11y + 66}{2}\right) + 36$ $4y = y^2 + 12y + 36 - 22y - 132 + 144$ $0 = y^2 - 14y + 48$ $(y - 8)(y - 6) = 0$ $y = 8$ or/of $y = 6$ $x = \frac{8+6}{2} = 7$ or/of $x = \frac{6+6}{2} = 6$	✓ Equating equations / <i>gelykstelling van vergl</i> A ✓ Simplification/Std form / <i>vereenvoudiging /std vorm</i> CA ✓ Factors/ Substitution / <i>faktore/vervanging</i> CA ✓ x-values/ <i>waardes</i> CA ✓ y-values/ <i>waardes</i> CA OR/OF ✓ Making x the subject and substitution / <i>maak x die onderwerp en vervanging</i> A ✓ Simplification/Std form / <i>vereenvoudiging/std vorm</i> CA ✓ Factors/ Substitution / <i>faktore/vervanging</i> CA ✓ y-values / <i>waardes</i> CA ✓ x-values/ <i>waardes</i> CA	(5)
1.3	1.3.1 $A = \frac{1}{2}h(a+b)$ $\frac{2A}{(a+b)} = h$	✓ h the subject / <i>die onderwerp</i> A	(1)

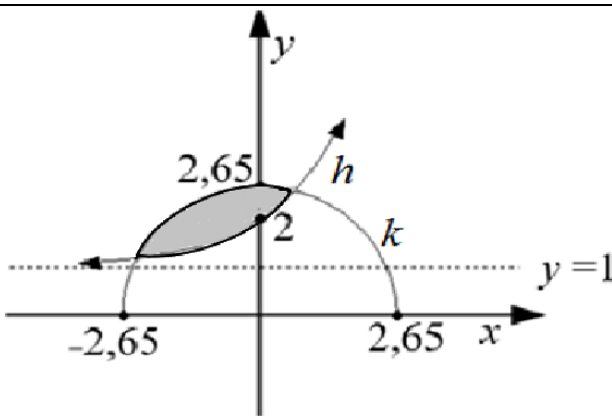
	1.3.2	$h = \frac{2A}{(a+b)}$ $= \frac{2(1,8064 \times 100)}{(15,24 + 20,32)}$ $= 10,16 \text{ cm}$ OR/OF $A = \frac{1}{2}h(a+b)$ $1,8064 \times 100 = \frac{1}{2}h(15,24 + 20,32)$ $\frac{180,64 \times 2}{35,56} = h$ $10,16 = h$	✓ Conversion /herleiding A ✓ Substitution /vervanging A ✓ Simplification/ vereenvoudiging A OR/OF ✓ Conversion /herleiding A ✓ Substitution /vervanging A ✓ Simplification /vereenvoudiging CA	(3)
1.4		$K = 89 - 16$ $= 73$ $73 = 2^6 + 0 + 0 + 2^3 + 0 + 0 + 2^0$ $K = 1001001_2$	✓ Difference /verskil A ✓ Method / metode A ✓ 1001001_2 CA	(3)
				[21]

QUESTION/VRAAG 2				
2.1	2.1.1	2 roots / wortels	✓ Answer /antwoord A	(1)
	2.1.2	$\Delta = b^2 - 4ac$ $= 0 - 4(1)(-121)$ $= 484$	✓ SF A ✓ Simplification / vereenvoudiging CA	(2)
	2.1.3	Roots are real, unequal, and irrational/ Wortels is reëel, ongelyk en irrasionaal	✓ Real, unequal and irrational / reëel, ongelyk en irrasionaal CA	(1)
2.2		$x^2 + px + 4 = 0$ $\Delta = b^2 - 4ac$ $= p^2 - 4(1)(4)$ $= p^2 - 16$ $\Delta \geq 0$ $p^2 - 16 \geq 0$ $(p + 4)(p - 4) \geq 0$ $p \leq -4$ or / of $p \geq 4$	✓ SF A ✓ Simplification/vereenvoudiging CA ✓ $\Delta \geq 0$ A ✓ Value(s) of p /waarde(s) van p CA	(4)
				[8]

QUESTION/VRAAG 3				
3.1	3.1.1	$\frac{m^6 n^7}{(m^2 n)^3} = \frac{m^6 n^7}{(m^6 n^3)}$ $= n^4$	✓ Exponential property/ <i>eksponensiële eienskap</i> A ✓ n^4 CA	(2)
	3.1.2	$\sqrt{98x^2} + \sqrt{32x^2}$ $= \sqrt{(49 \times 2)x^2} + \sqrt{(16 \times 2)x^2}$ $= 7x\sqrt{2} + 4x\sqrt{2}$ $= 11x\sqrt{2}$ OR/OF $\sqrt{98x^2} + \sqrt{32x^2}$ $= \sqrt{(2 \times 7^2)x^2} + \sqrt{(2 \times 2^4)x^2}$ $= 7x\sqrt{2} + 2^2 x\sqrt{2}$ $= 11x\sqrt{2}$	✓ Factors / <i>faktore</i> A ✓ Simplification / <i>vereenvoudiging</i> CA ✓ Simplification / <i>vereenvoudiging</i> CA OR/OF ✓ Prime factors/ <i>priemfaktore</i> A ✓ Simplification/ <i>vereenvoudiging</i> CA ✓ Simplification / <i>vereenvoudiging</i> CA	(3)
	3.1.3	$\frac{1}{2} \log_2 16 + \log_3 27$ $= \frac{1}{2} \log_2 2^4 + \log_3 3^3$ $= \frac{4}{2} \log_2 2 + 3 \log_3 3$ $= 2 \times 1 + 3 \times 1$ $= 2$	✓ Log property/ <i>eienskap</i> A ✓ Log property / <i>eienskap</i> A ✓ Simplification / <i>vereenvoudiging</i> CA	(3)
3.2	3.2.1	$(x+1)^3 = 64$ $(x+1)^3 = 4^3$ $x+1 = 4$ $\therefore x = 3$ OR/OF $(x+1)^3 = 64$ $(x+1)(x^2 + 2x + 1) = 64$ $x^3 + 2x^2 + x^2 + x + 2x + 1 - 64 = 0$ $x^3 + 3x^2 + 3x - 63 = 0$ $(x-3)(x^2 + 3x + 21) = 0$ $\therefore x = 3$	✓ Exponential property / <i>eksponensiële eienskap</i> A ✓ Equal exponent / <i>gelyke eksponent</i> A ✓ $x = 3$ CA OR/OF ✓ Expanded form / <i>uitgebreide vorm</i> A ✓ Factors / <i>faktore</i> A ✓ $x = 3$ CA	(3)

	3.2.2	$\log x + 1 = \log (x + 9)$ $\log x + \log 10 = \log (x + 9)$ $\log 10x = \log (x + 9)$ $10x = x + 9$ $9x = 9$ $x = 1$ OR/OF $\log x + 1 = \log (x + 9)$ $\log x - \log (x + 9) = -1\log 10$ $\log \frac{x}{(x + 9)} = \log 10^{-1}$ $\frac{x}{(x + 9)} = \frac{1}{10}$ $10x = x + 9$ $9x = 9$ $x = 1$	$\checkmark \log \text{ law/wet}$ A $\checkmark \log \text{ law /wet}$ A $\checkmark \text{ Simplification / vereenvoudiging}$ CA $\checkmark x = 1$ CA OR/OF $\checkmark \log \text{ law /wet}$ A $\checkmark \log \text{ law/wet}$ A $\checkmark \text{ Simplification / vereenvoudiging}$ CA $\checkmark x = 1$ CA	(4)
3.3	$m - (3 - i) = ni + 5$ $m - 3 + i = ni + 5$ $m - 3 = 5 \text{ and/en } i = ni$ $m = 8 \text{ and/en } n = 1$	$\checkmark \text{ Simplification / vereenvoudiging}$ A $\checkmark \text{ Value of } m \text{ /waarde van } m$ CA $\checkmark \text{ Value of } n \text{ /waarde van } n$ CA	(3)	
3.4	$z = 1 - 5i$ $r = \sqrt{(1)^2 + (-5)^2}$ $= \sqrt{26}$ $= 5,1$ $\tan \theta = -\frac{5}{1}$ $\text{ref / verw.}\angle: \theta = 78,7^\circ$ $\theta = 360^\circ - 78,7^\circ = 281,3^\circ$ $\therefore z = 3,74 \text{ cis } 281,3^\circ$	$\checkmark \text{ SF}$ A $\checkmark r = 5,1$ CA $\checkmark \tan \theta = -5$ CA $\checkmark \text{ Ref /verw } \theta = 78,7^\circ$ CA $\checkmark z = 5,1 \text{ cis } 281,3^\circ$ CA	(5)	
			[23]	

QUESTION /VRAAG 4				
4.1.1	$y = 4$ $0 = -x^2 + 4$ $x^2 = 4$ $x = \pm 2$		$\checkmark y = 4$ A $\checkmark$ Equating to 0 /gelyk stel aan 0 A $\checkmark x = \pm 2$ CA	(3)
4.1.2	$x = 0$ and/en $y = 4$		$\checkmark x = 0$ A $\checkmark y = 4$ A	(2)
4.1.3	y – int/ afsnit $y = 1$ x – int/ afsnit $0 = -2x + 1$ $-1 = -2x$ $x = \frac{1}{2}$		$\checkmark y = 1$ A $\checkmark y = 0$ A $\checkmark x = \frac{1}{2}$ CA	(3)
4.1.4			f : $\checkmark x$ – intercepts/afsnitte CA $\checkmark y$ – intercept /afsnit CA $\checkmark$ Turning point/draaipunt CA $\checkmark$ Shape /vorm A g : $\checkmark x$ and/en y -intercepts/afsnitte CA $\checkmark$ Shape/vorm A	(6)
4.1.5	$x \in \mathbb{R}$ OR/OF $-\infty \leq x \leq \infty$ OR/OF $x \in [-\infty; \infty]$		$\checkmark$ Critical values /kritiese waardes CA $\checkmark$ Correct notation / korrekte notasie A	(2)
4.2	4.2.1	$r = \sqrt{7}$ OR / OF 2,65	$\checkmark$ Value of r /waarde van r A	(1)

	4.2.2	$y = 1$	✓ Equation of asymptote/vergelyking van asimptoot A	(1)
	4.2.3	$h(0) = 2^0 + 1$ $\therefore y = 2$	✓ y-intercept /afsnit A	(1)
	4.2.4	$-\sqrt{7} \leq x \leq \sqrt{7}$ OR / OF $x \in [-\sqrt{7}; \sqrt{7}]$ OR / OF $-2,65 \leq x \leq 2,65$ OR / OF $x \in [-2,65; 2,65]$	✓ Critical values /kritiese waardes CA ✓ Correct notation / korrekte notasie A	(2)
	4.2.5		<h>: ✓ Asymptote /asimptoot CA ✓ Shape /vorm A ✓ y-intercept /afsnit CA k: ✓ x-and/en y- int/afsnit CA ✓ Shape /vorm A </h>	(5)
	4.2.6	Shaded area on the graph / Geskakeerde gedeelte op die grafiek	✓ Shaded area / Geskakeerde gedeelte	(1)
4.3	4.3.1	$0 = \frac{8}{x} + 2$ $-2 = \frac{8}{x}$ $-2x = 8$ $x = -4$	✓ $y = 0$ A ✓ Simplification / vereenvoudiging CA ✓ $x = -4$ CA	(3)
	4.3.2	$y = 2$	✓ $y = 2$ A	(1)
	4.3.3	$x \in \mathbb{R}, x \neq 4$	✓ $x \in \mathbb{R}, x \neq 4$ CA	(1)
				[32]

QUESTION /VRAAG 5			
5.1	$A = P(1 - i)^n$ $= 300\,000(1 - 0,5)^{10}$ $= 292\,000$ There will be 292 000 people in the town./ <i>Daar sal 292 000 mense in die dorp wees.</i>	✓ F CA ✓ SF CA ✓ 292 000 CA	(3)
5.2	$A = P(1 + i)^n$ $75\,000 = 5\,000(1 + 9,5)^n$ $\frac{75\,000}{5\,000} = (1 + 9,5)^n$ $15 = (1,95)^n$ $\log_{1,95} 15 = n$ $4,0550 = n$	✓ F CA ✓ SF CA ✓ Simplification / <i>vereenvoudiging</i> CA ✓ log form / <i>vorm</i> CA ✓ $n = 4$ CA	(5)
5.3	$A = P(1 + i)^n$ $= 200\,000 \left(1 + \frac{7,5\%}{12}\right)^{36}$ $= R250\,289,23$ Amount after withdrawal / <i>Bedrag na onttrekking</i> $R250\,289,23 - R50\,000 = R200\,289,23$ Value of the investment at the end of 5 years / <i>Waarde van die belegging aan die einde van 5 jaar</i> $A = P(1 + i)^n$ $= R200\,289,23 \left(1 + \frac{6\%}{4}\right)^{4 \times 2}$ $= R225\,624,33$	✓ SF CA ✓ R250 289,23 CA ✓ M - R50 000 A ✓ SF CA ✓ R225 757,96 CA	(5)
			[13]

QUESTION /VRAAG 6				
6.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3(x+h) - (1 - 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x - 3h - 1 + 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{-3h}{h}$ $= \lim_{h \rightarrow 0} -3$ $= -3$		✓ Definition /definisie A ✓ Substitution /vervanging CA ✓ Simplification / vereenvoudiging CA ✓ Simplification / vereenvoudiging CA ✓ $f'(x) = -3$ CA	(5)
6.2	6.2.1	$y = \frac{2}{x^3} - 15x + 7m$ $= 2x^{-3} - 15x + 7m$ $\frac{dy}{dx} = -6x^{-2} - 15$	✓ $2x^{-3}$ A ✓ $-6x^{-2}$ CA ✓ -15 CA	(3)
	6.2.2	$D_x \left[9 + 2x^{-1} + \sqrt[3]{x^{27}} \right]$ $= D_x \left[9 + 2x^{-1} + x^9 \right]$ $= -2x^{-2} + 9x^8$	✓ x^9 OR / OF $x^{\frac{27}{3}}$ A ✓ $-2x^{-2}$ CA ✓ $9x^8$ CA	(3)
6.3	6.3.1	$g(x) = \frac{x^2}{20} - \frac{7x}{20} + 15$ $g'(x) = \frac{1}{10}x - \frac{7}{20}$	✓ $\frac{1}{10}x$ A ✓ $-\frac{7}{20}$ A	(2)
	6.3.2	$g'(5) = \frac{1}{10}(5) - \frac{7}{20}$ $= \frac{3}{20}$	✓ Substitution /vervanging CA ✓ Simplification / vereenvoudiging CA	(2)
				[15]

QUESTION / VRAAG 7			
7.1	$h(x) = x^3 - 16x$ $0 = x(x^2 - 16)$ $0 = x(x - 4)(x + 4)$ $x = 0$ or/of $x = -4$ or/of $x = 4$ $A(-4; 0)$ $B(4; 0)$	✓ Substitution by 0 / <i>vervangende deur 0</i> A ✓ Factors/ Substitution / <i>faktore/vervangende</i> CA ✓ Coordinates of A / <i>koördinate van A</i> CA ✓ Coordinates of B / <i>koördinate van B</i> CA	(4)
7.2	$h(x) = x^3 - 16x$ $h'(x) = 3x^2 - 16$ $0 = 3x^2 - 16$ $\frac{16}{3} = x^2$ $x = \pm 2,31$ $h(2,31) = (2,31)^3 - 16(2,31) = -24,63$ $h(-2,31) = (-2,31)^3 - 16(-2,31) = 24,63$ $D(-2,31; -24,63)$ $E(2,31; 24,63)$	✓ Derivative / <i>afgeleide</i> A ✓ $h'(x) = 0$ A ✓ Both x values / <i>beide x-waardes</i> CA ✓ y-coordinate of D / <i>y-koördinaat van D</i> CA ✓ y-coordinate of E / <i>y-koördinaat van E</i> CA	(5)
7.3	$x \leq -2,31$ or/of $x \geq 2,31$ OR / OF $-\infty < x \leq -2,31$ or/of $2,31 \leq x < \infty$ OR / OF $x \in (-\infty; -2,31]$ or/of $x \in [2,31; \infty)$	✓ $-2,31$ CA ✓ $2,31$ CA ✓ correct notation / <i>korrekte notasie</i> A	(3)
			[12]

QUESTION / VRAAG 8			
8.1	$2b + 2h = 80$ $2b = 80 - 2h$ $b = 40 - h$	✓ Formula / formule A ✓ Simplification / vereenvoudiging CA	(2)
8.2	$V = l \times h \times b$ $= (20 - 2h)(40 - h) \times h$ $= 2h^3 - 100h^2 + 800h$	✓ Formula / formule A ✓ $(20 - 2h)$ CA ✓ SF CA	(3)
8.3	$V = 2h^3 - 100h^2 + 800h$ $\frac{dV}{dh} = 6h^2 - 200h + 800$ $0 = 6h^2 - 200h + 800$ $h = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(6)(800)}}{2(6)}$ $h = 28,69$ or / of $h = 4,64$ $V(28,69) = 2(28,69)^3 - 100(28,69)^2 + 800(28,69)$ $= -12129,21\text{cm}^3$ $V(4,64) = 2(4,64)^3 - 100(4,64)^2 + 800(4,64)$ $= 1758,83\text{cm}^3$ The value of h is / Die waarde van h is 4,64	✓ Derivative / afgeleide A ✓ Derivative / afgeleide = 0 A ✓ SF CA ✓ Both values of h / beide waardes van h CA ✓ Calculating both V / berekening beide V CA ✓ Choosing/Kies $h = 4,64$ CA	(6)
			[11]

QUESTION 9				
9.1	9.1.1	$\int (2x^2 + x) dx$ $= \frac{2x^3}{3} + \frac{x^2}{2} + C$	$\checkmark \frac{2}{3} x^3$ A $\checkmark \frac{x^2}{2}$ A $\checkmark C$ A	(3)
	9.1.2	$\int \frac{16x^6 - 4x^2}{2x} dx$ $\int (8x^5 - 2x) dx$ $= \frac{8x^6}{6} - x^2$ $= \frac{4x^6}{3} - x^2 + C$	$\checkmark 8x^5$ A $\checkmark -2x$ A $\checkmark \frac{8x^6}{6}$ CA $\checkmark -x^2 + C$ CA	(4)
	9.1.3	$\int_0^2 x^3 dx$ $= \frac{x^4}{4} \Big _0^2$ $= \frac{2^4}{4}$ $= 4$	$\checkmark \frac{x^4}{4}$ A $\checkmark$ Substitution / vervanging CA $\checkmark$ Simplification / vereenvoudiging CA	(3)
9.2	$A = \int_{-0,5}^{-2} (x^2 + 3x) dx$ $= \left[\frac{x^3}{3} + \frac{3}{2} x^2 \right]_{-0,5}^{-2}$ $= \left(\frac{(-0,5)^3}{3} + \frac{3}{2} (-0,5)^2 \right) - \left(\frac{(-2)^3}{3} + 2(-2)^2 \right)$ $A = \frac{1}{3} - \frac{16}{3} = -5$ $\therefore A = 5 \text{ square units / vierkante eenhede}$		$\checkmark$ Definite integral formula / Bepaalde integral formule CA $\checkmark$ Integral / Integraal CA $\checkmark \checkmark$ Substitution / vervanging CA $\checkmark$ Area CA	(5)
				[15]
			TOTAL/TOTAAL:	150