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GRADE/GRAAD 12

JUNE/JUNIE 2023

**TECHNICAL MATHEMATICS P2/TEGNIESE WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

NOTE:

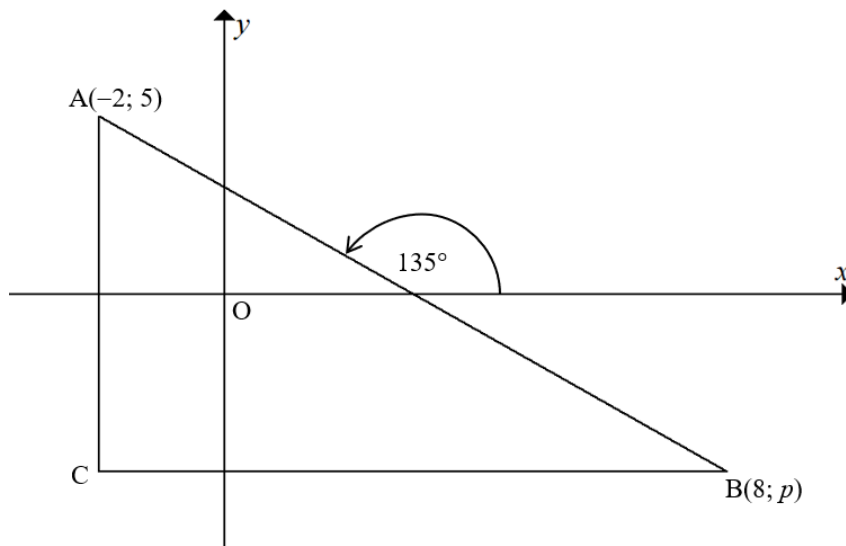
- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method/Metode
A	Accuracy/Akkuraatheid
AO	Answer only/Slegs antwoord
CA	Consistent accuracy/Deurlopende akkuraatheid
F	Formula/Formule
I	Identity/Identiteit
R	Rounding/Afronding
S	Simplification/Vereenvoudiging
ST	Statement/Bewering
RE	Reason/Rede
ST RE	Statement and correct reason/Bewering en korrekte rede
SF	Substitution correctly in correct formula/Korrekte vervanging in die korrekte formule
NPU	No penalty for omitting units/Geen penalisering vir eenhede uitgelaat

QUESTION/VRAAG 1

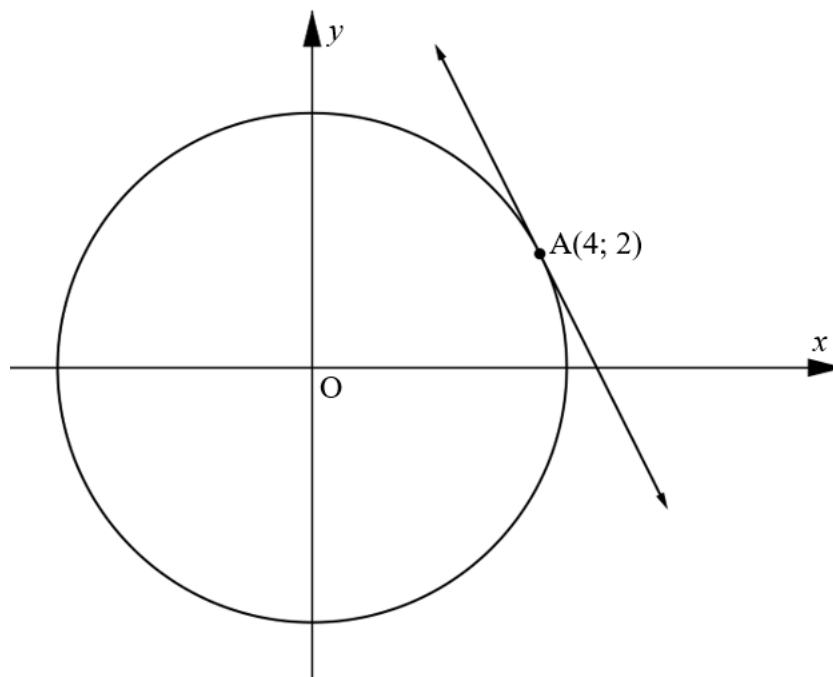


1.1	$m_{AB} = \tan 135^\circ = -1$ AO: Full marks / Volpunte	✓ M ✓ S A (2)
1.2	$m_{AB} = \frac{p-5}{8+2}$ $\therefore \frac{p-5}{8+2} = -1$ $\therefore p-5 = -10$ $\therefore p = -5$	✓ M ✓ S A ✓ S A (3)
1.3	$M_{AB} = \left(\frac{-2+8}{2}; \frac{5-5}{2} \right) = (3; 0)$	✓ x-value A ✓ y-waarde A (2)
1.4	$y = -5$	✓ A (1)
1.5	$C(-2; -5)$	✓ x-value A ✓ y-waarde A (2)

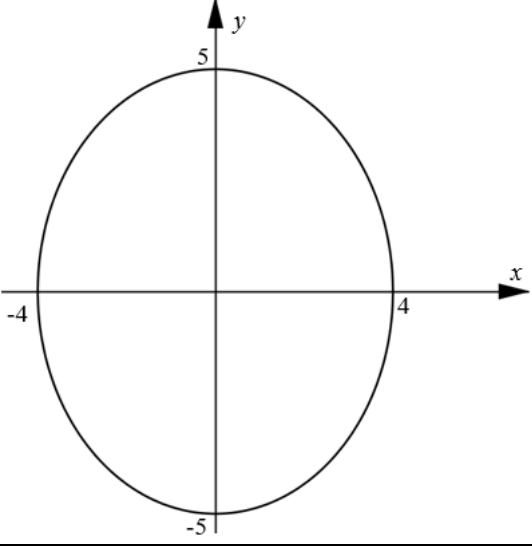
1.6	$m_{CM} = \frac{-5-0}{-2-3} = 1$ $m_{AB} \times m_{CM} = -1 \times 1 = -1$ $\therefore CM \perp AB$	✓ grad CA ✓ product/ produk A ✓ conclusion/ gevolgtrekking (3)
1.7	grad of line = $m_{CM} = 1$ $y - y_1 = m(x - x_1)$ OR/OF $y = mx + c$ $y - 5 = 1(x - (-2))$ $5 = 1(-2) + c$ $\therefore y = x + 7$	✓ grad. CA ✓ SF A ✓ equation/vergelyking CA (3)
		[16]

QUESTION/VRAAG 2

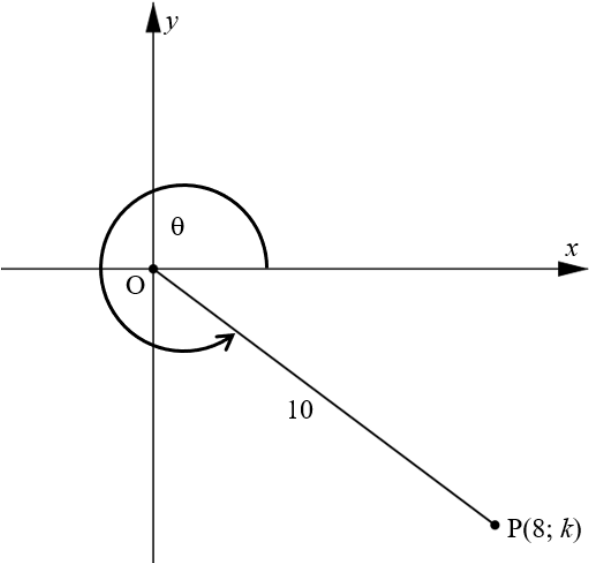
2.1



2.1.1	$r = \sqrt{20} = 2\sqrt{5}$	✓ A (1)
2.1.2	$xx_1 + yy_1 = r^2$ $\therefore x(4) + y(2) = 20$ $\therefore 2y = -4x + 20$ $\therefore y = -2x + 10$ OR/OF $\therefore m_{\text{radius}} = \frac{2}{4} = \frac{1}{2}$ $\therefore m_{\text{tangent/raaklyn}} = -2$ $y - y_1 = m(x - x_1)$ $\therefore y - 2 = -2(x - 4)$ $\therefore y = -2x + 10$	✓ F ✓ SF A ✓ S ✓ equation / vergl CA OR/OF ✓ grad. radius A ✓ grad. tan / raaklyn CA ✓ SF A ✓ equation / vergl CA (4)
2.1.3	$(-4; -2)$	✓ x-value A ✓ y-waarde A (2)

2.2		✓ elliptical shape / elliptiese vorm A ✓ x-intercepts/<i>afsnitte</i> A ✓ y-intercepts/<i>afsnitte</i> A (3)
		[10]

QUESTION/VRAAG 3

3.1		
3.1.1	$\cos \theta = \frac{8}{10} = \frac{4}{5}$	✓ A (1)
3.1.2	$k^2 + 8^2 = 10^2$ <i>Pythagoras</i> $\therefore k^2 = 36$ $\therefore k = -6$ <i>4th quadrant / 4de kwadrant</i>	✓ M ✓ S ✓ value of / waarde van k (3)
3.1.3	$\frac{\tan q}{\operatorname{cosec} q} = \frac{-6/8}{10/-6}$ $= \frac{9}{20}$	✓ tan ratio / verh. A ✓ cosec ratio / verh. A ✓ S CA (3)
3.2	$3\cos x - 1 = -1,5$ $3\cos x = -0,5$ $\therefore \cos x = -0,1666\dots$ <i>Ref / Verw $\angle = 80,41^\circ$</i> $\therefore x = 180^\circ - 80,41^\circ$ OR/OF $x = 180^\circ + 80,41^\circ$ $\therefore x = 99,59^\circ$ OR/OF $x = 260,41^\circ$	✓ S A ✓ Ref / Verw $\angle$ CA ✓ Quadrants / Kwadrante A ✓ values of x / waardes van x CA (4)
		[11]

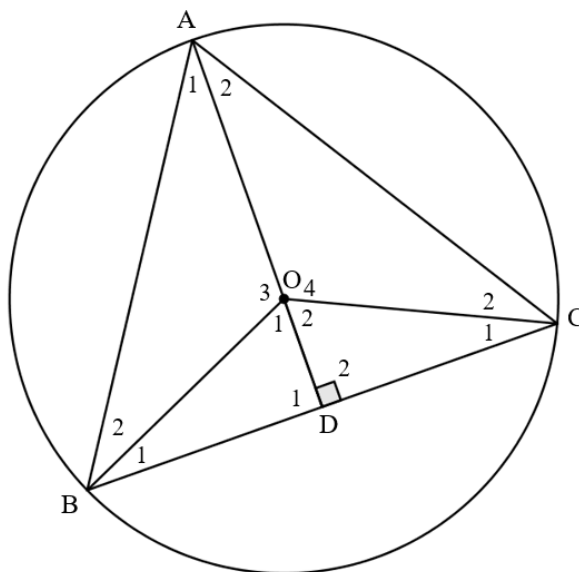
QUESTION/VRAAG 4

4.1	$(1 + \cos x)(1 - \cos x) = 1 - \cos^2 x$ $= \sin^2 x$	✓ S A ✓ I (2)
4.2	$\frac{\cos^2(2\pi - x) \tan^2 x}{\sin(180^\circ + x) \operatorname{cosec}(180^\circ - x)}$ $= \frac{\cos^2 x \frac{\sin^2 x}{\cos^2 x}}{(-\sin x)(\operatorname{cosec} x)}$ $= \frac{\cos^2 x \frac{\sin^2 x}{\cos^2 x}}{(-1)}$ $= -\sin^2 x$	✓ $\cos^2 x$ ✓ $\frac{\sin^2 x}{\cos^2 x}$ ✓ $-\sin x$ ✓ $\operatorname{cosec} x$ ✓ -1 ✓ $-\sin^2 x$ (6)
4.3	$\text{LHS} / \text{LK} = \cot x + \tan x$ $= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ $= \frac{\cos^2 x + \sin^2 x}{(\sin x)(\cos x)}$ $= \frac{1}{(\sin x)(\cos x)}$ $= \operatorname{cosec} x \square \sec x = \text{RHS} / \text{RK}$	✓ $\frac{\cos x}{\sin x}$ ✓ $\frac{\sin x}{\cos x}$ ✓ S ✓ $\cos^2 x + \sin^2 x = 1$ (4)
		[12]

QUESTION/VRAAG 5

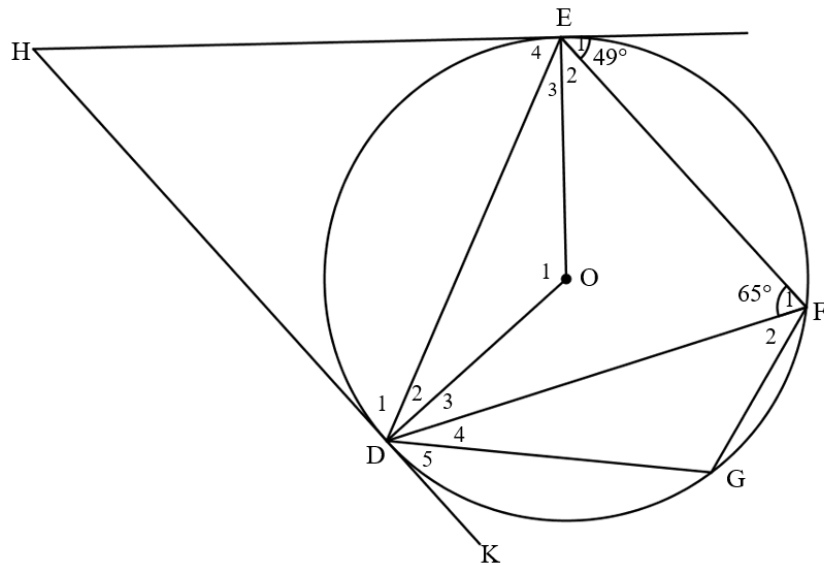
	$f(x) = \cos 2x$ and $g(x) = \sin(x - 30^\circ)$ for $x \in [0^\circ; 180^\circ]$	
5.1	$\text{Period}_f = \frac{360^\circ}{2} = 180^\circ$	✓ A (1)
5.2	$\text{Amplitude}_g = 1$	✓ A (1)
5.3		f: ✓ y-intercept at / y-afsnit by 1 ✓ x-intercepts at 45° and 135° / x-afsnitte by 45° en 135° ✓ turning point at / draaipunt by $(90^\circ; -1)$ ✓ End point at / eindpunt by $(180^\circ; 1)$ g: ✓ y-intercept at / y-afsnit by $-0,5$ ✓ x-intercept at 30° / x-afsnit by 30 ✓ turning point at / draaipunt by $(120^\circ; 1)$ ✓ End point at / eindpunt by $(180^\circ; 0,5)$ (8)
5.4.1	$45^\circ \leq x \leq 135^\circ$	✓ $45^\circ \leq x$ CA ✓ $x \leq 135^\circ$ CA (2)
5.4.2	$135^\circ \leq x < 180^\circ$	✓ $135^\circ \leq x$ CA ✓ $x < 180^\circ$ CA (2)
		[14]

QUESTION/VRAAG 7



7.1	BD = 5,5 cm (line from centre $\perp$ to chord / lyn vanuit midpt $\perp$ koord)	✓ ST ✓ RE (2)	A
7.2	$OB^2 = OD^2 + BD^2$ (Pythagoras) $OB^2 = 3^2 + 5,5^2 = 39,25$ $\therefore OB \approx 6,26$ cm	✓ ST ✓ OB length / lengte (2)	CA
7.3	In $\triangle ABD$ and/en $\triangle ACD$: BD = DC (line from centre $\perp$ to chord / lyn vanuit midpt $\perp$ koord) AD is common / gemeenskaplik $\hat{D}_1 = \hat{D}_2 = 90^\circ$ $\therefore \triangle ABD \equiv \triangle ACD$ (S $\angle$ S)	✓ ST A ✓ ST A ✓ ST A ✓ RE (4)	
7.4	$\sin B_1 = \frac{OD}{OB} = \frac{3}{6,26}$ $\therefore \hat{B}_1 \approx 28,64^\circ$ OR/OF any alternative trig ratio/ enige alternatiewe trig. verhouding	✓ trig ratio / verh. ✓ size of angle / grootte van hoek (2)	
7.5	$\hat{O}_1 = 61,36^\circ$ (Int $\angle$ of $\triangle$ / Binne $\angle$ e van $\triangle$) $\hat{O}_2 = 61,36^\circ$ (Congruency / Kongruensie) $\therefore \hat{A} = 61,36^\circ$ ($\angle$ at centre = $2 \times \angle$ at circumference / middelpts $\angle$ = $2 \times$ Omtreks $\angle$)	✓ ST CA ✓ ST CA ✓ ST CA ✓ RE (4)	
			[14]

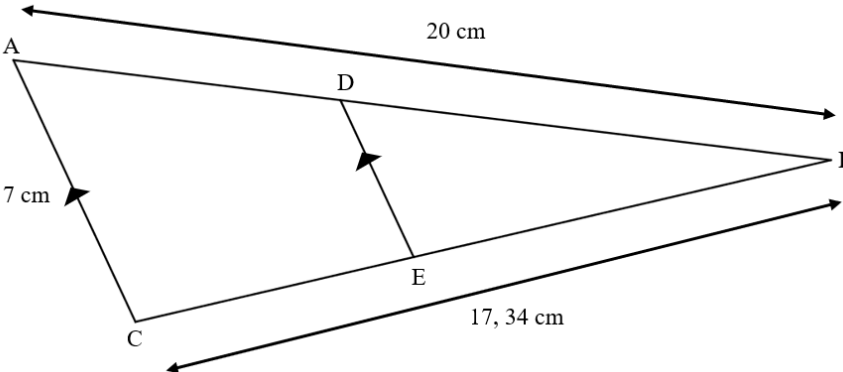
QUESTION/VRAAG 8



8.1	Tangents from a common point / raaklyne vanuit dieselfde punt	✓ A	(1)
8.2.1	$\hat{D}_1 = 65^\circ$ (tan – chord th / raaklyn – koord st)	✓ ST ✓ RE	A (2)
8.2.2	$\hat{D}_2 = 25^\circ$ (Radius $\perp$ Tangent / Raaklyn)	✓ ST ✓ RE	A (2)
8.2.3	$\hat{D}_3 = 66^\circ$ ($\angle$ on str line / $\angle$ e op reguit lyn)	✓ ST ✓ RE	A (2)
8.2.4	$\hat{D}_4 = 114^\circ$ (opp. $\angle$ s of cyclic quad / teenoorst. $\angle$ e van kdvh)	✓ ST ✓ RE	A (2)
8.2.5	$\hat{D}_5 = 66^\circ$ (tan – chord th / raaklyn – koord st)	✓ ST ✓ RE	A (2)

8.3	$\hat{H} = 50^\circ$ (Int $\angle$ s of Δ / Binne $\angle$ e van Δ) $\therefore \hat{H} + \hat{F} \neq 180^\circ$ $\therefore$ EHDF is not cyclic/is nie siklies nie (Opp. $\angle$ s NOT supplementary / Teenoorst. $\angle$ e is NIE supplementêr NIE) OR/OF $\hat{H} = 50^\circ$ (Int $\angle$ s of Δ / Binne $\angle$ e van Δ) $\therefore \hat{H} \neq \hat{F}$ $\therefore$ EHDF is not cyclic/is nie siklies nie (Ext $\angle$ NOT equal to opp. int $\angle$ Buite $\angle$ NIE gelyk aan teenoorst. binne $\angle$ NIE)	✓ ST ✓ RE ✓ RE CA OR / OF ✓ ST ✓ RE ✓ RE CA (3)
		[14]

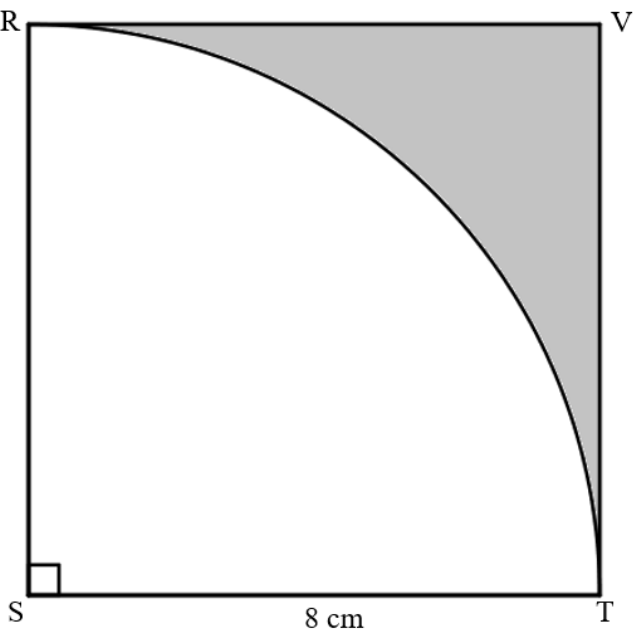
QUESTION/VRAAG 9

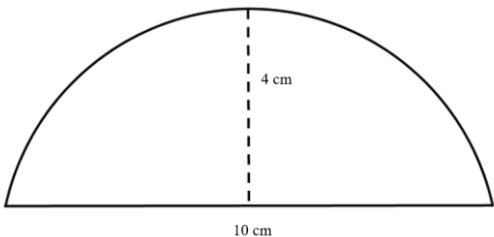
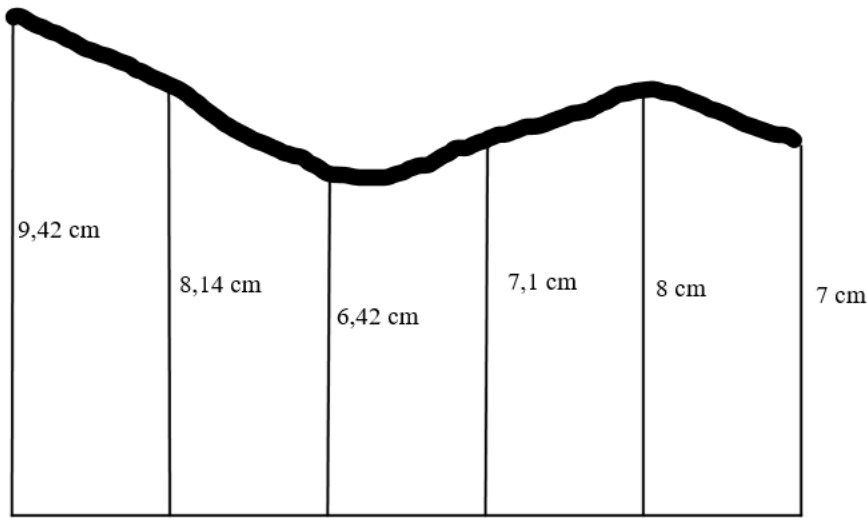
		
9.1	$2k + 3k = 20$ for some value k / vir enige waarde k . $\therefore 5k = 20$ $\therefore k = 4$ $\therefore AD = 8 \text{ cm}$ and/en $DB = 12 \text{ cm}$ OR/OF $AD = \frac{2}{5} \times 20 = 8 \text{ cm}$ $DB = \frac{3}{5} \times 20 = 12 \text{ cm}$	✓ setup equation / vergelyking opstel ✓ value of k / waarde van k ✓ AD and/en DB lengths / lengtes OR/OF ✓ fraction $\frac{2}{5}$ ✓ AD length / lengte ✓ DB length / lengte (3)
9.2	$\frac{BE}{BC} = \frac{DB}{BA}$ (Prop th, $DE \parallel AC$ / Ewer st, $DE \parallel AC$) $\frac{BE}{17,34} = \frac{12}{20}$ $\therefore BE \approx 10,40 \text{ cm}$	✓ ST A ✓ RE ✓ BE length / lengte (3)
9.3	In $\triangle BDE$ and/en $\triangle BAC$: $\hat{B}$ is common / gemeenskaplik $\hat{D} = \hat{A}$ corr. $\angle$ s / ooreenk $\angle$ e; $DE \parallel AC$ $\hat{E} = \hat{C}$ corr. $\angle$ s / ooreenk. $\angle$ e; $DE \parallel AC$ OR/OF Int $\angle$s of $\triangle$ / Binne $\angle$e van $\triangle$ $\triangle BDE \parallel \triangle BAC (\angle, \angle, \angle)$	✓ ST A ✓ ST RE A ✓ ST RE A Last mark for last statement and reason OR for final reason $\angle \angle \angle$ (3)
9.4	$\frac{DE}{AC} = \frac{BA}{BD}$ ($\triangle BDE \parallel \triangle BAC$) $\frac{DE}{7} = \frac{20}{12}$ $\therefore DE \approx 11,67 \text{ cm}$	✓ ST RE A ✓ ST CA ✓ DE length / lengte (3)
		[12]

QUESTION/VRAAG 10

10.1	$108 \text{ km/h} = \frac{108 \text{ km}}{1 \text{ h}} \times \frac{1\,000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 30 \text{ m/s}$	✓ conversion factors / herleidingsfaktore ✓ answer / antwoord (2)
10.2	$v = \pi D n$ $30 \text{ m/s} = \pi \times (0,25 \text{ m}) \times n$ $n = \frac{30}{0,25\pi}$ $n \approx 38,20 \text{ rev/s}$	✓ F ✓ conversion / herleiding ✓ SF A ✓ S ✓ answer / antwoord (5)
10.3	$\omega = 2\pi n$ $= 2\pi \times 38,20$ $\approx 240,02 \text{ rad/s}$	✓ F ✓ SF CA ✓ answer / antwoord (3)
10.4	$s = vt \quad \text{OR/OF} \quad D = S \times T$ $= 30 \times (10 \text{ min} \times 60 \text{ s})$ $= 18\,000 \text{ m}$ $= 18 \text{ km}$	✓ F ✓ SF CA ✓ answer / antwoord (3)
10.5	$n = \frac{\text{number of revolutions/aantal revolusies}}{\text{time/tyd}}$ $38,20 = \frac{20}{t}$ $t \approx 0,52 \text{ sec}$	✓ SF CA ✓ answer / antwoord (2)
		[15]

QUESTION/VRAAG 11

	 A square RSTV with side length 8 cm. A quarter-circle arc is drawn from vertex R to vertex T, with its center at vertex S. The area between the arc and the side RV is shaded gray.	
11.1.1	$s = r\theta$ $RT = 8 \times \frac{\pi}{2}$ $RT = 4\pi \text{ cm}$ $\approx 12,57 \text{ cm}$	✓F ✓ SF A ✓ RT length / lengte (3)
11.1.2	$\text{Area} = \frac{rs}{2}$ $= \frac{8 \times 4\pi}{2}$ $= 16\pi \text{ cm}^2$ $\approx 50,27 \text{ cm}^2$ OR/OF $\text{Area} = \frac{r^2\theta}{2}$ $= \frac{8^2 \times \frac{\pi}{2}}{2}$ $= 16\pi \text{ cm}^2$ $\approx 50,27 \text{ cm}^2$	✓F ✓ SF A ✓ Area OR/OF ✓F ✓ SF A ✓ Area (3)
11.1.3	Shaded area/Gearseerde area = Area_{square/vierkant} – Area_{sector/sektor} $= 8 \times 8 - 16\pi$ $\approx 13,73 \text{ cm}^2$	✓M ✓ area of square/ app van vierkant ✓ shaded area/ gearseerde app

11.2		
	$4h^2 - 4dh + x^2 = 0$ $4(4)^2 - 4d(4) + (10)^2 = 0$ $164 - 16d = 0$ $d = 10,25$ $\therefore r = 5,125 \text{ cm}$	✓F ✓SF A ✓S ✓ diameter / middellyn ✓ radius (5)
11.3		
	$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$ $113,61 = a \left(\frac{9,42 + 7}{2} + 8,14 + 6,42 + 7,1 + 8 \right)$ $113,61 = a(37,87)$ $\therefore a = 3 \text{ cm}$ OR/OF $A_T = a(m_1 + m_2 + m_3 + \dots + m_{n-1})$ $113,61 = a \left(\frac{9,42 + 8,14}{2} + \frac{8,14 + 6,42}{2} + \frac{6,42 + 7,1}{2} + \frac{7,1 + 8}{2} + \frac{8 + 7}{2} \right)$ $113,61 = a(37,87)$ $\therefore a = 3 \text{ cm}$	✓F ✓SF A ✓S ✓ value/waarde of a OR/OF ✓F ✓SF A ✓S ✓ value of a (4)
		[18]
	TOTAL/TOTAAL:	150