NATIONAL
SENIOR CERTIFICATE

GRADE 12

JUNE 2023

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 16 pages, including 2-page information sheet and a special answer book.

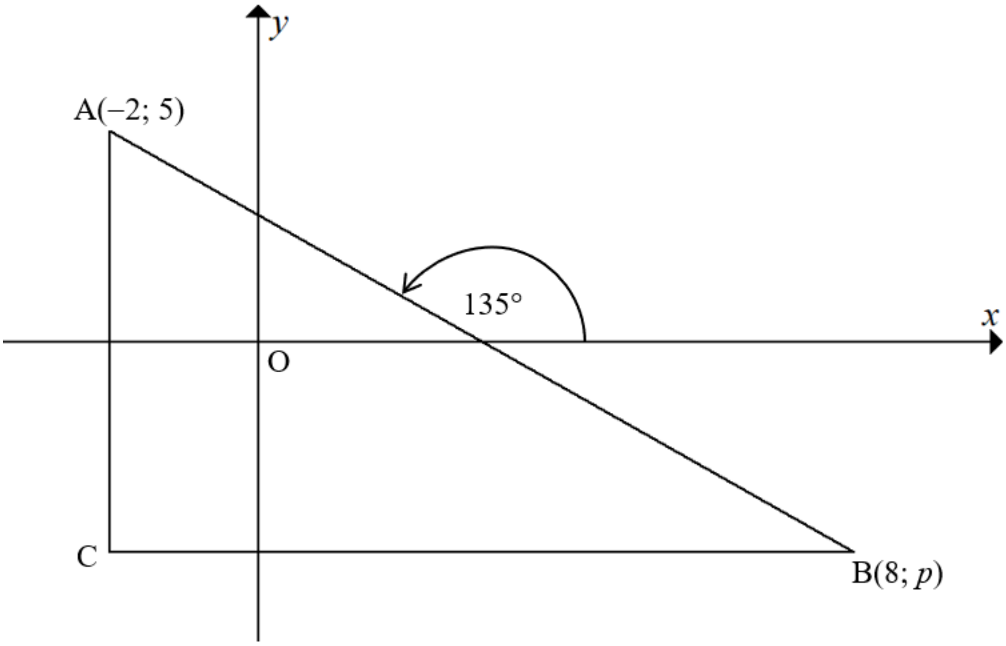
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

In the diagram below ABC is a triangle with vertices A(−2; 5); B(8; p) and C.
The inclination angle of AB is 135°.
AC is parallel to the y-axis and BC is parallel to the x-axis.

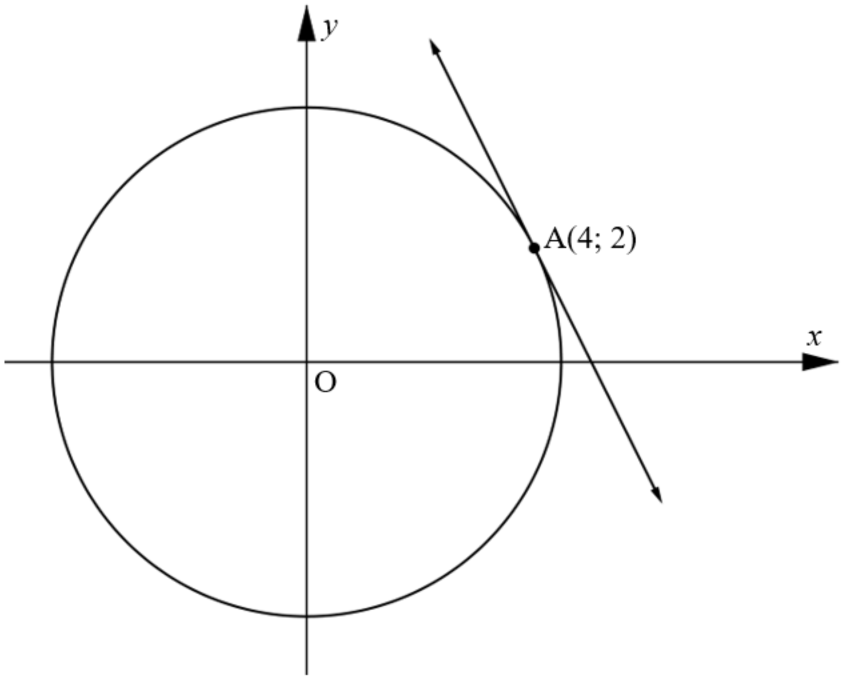


1.1	Determine the gradient of AB.	(2)
1.2	Show that $p = -5$.	(3)
1.3	Determine the coordinates of M, the midpoint of AB.	(2)
1.4	Write down the equation of BC.	(1)
1.5	Write down the coordinates of C.	(2)
1.6	Show that $CM \perp AB$.	(3)
1.7	Determine the equation of the straight-line parallel to CM and which passes through point A.	(3)

[16]

QUESTION 2

2.1 The diagram below shows the circle with equation $x^2 + y^2 = 20$.
A is a contact point of a tangent to the circle.



2.1.1 Write down the radius of the circle in simplified surd form. (1)

2.1.2 Determine the equation of the tangent to the circle at point A in the form $y = ...$ (4)

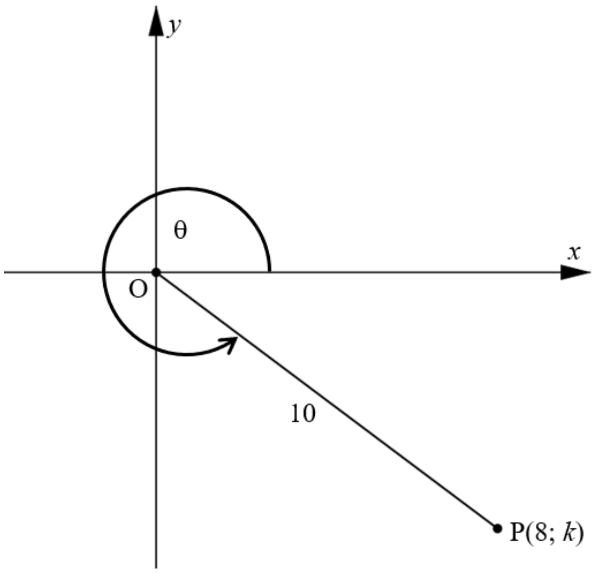
2.1.3 Write down the coordinates of another point where the line AO intersects with the circle. (2)

2.2 Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{25} = 1$. Clearly indicate the intercepts. (3)

[10]

QUESTION 3

3.1 In the diagram below, $P(8;k)$ is a point on the Cartesian plane.
OP forms a reflex angle q with the positive x-axis with OP equal to 10 units.



Determine the value of the following, WITHOUT using a calculator:

3.1.1 $\cos\theta$ (1)

3.1.2 k (3)

3.1.3 $\dfrac{\tan\theta}{\text{cosec}\theta}$ (3)

3.2 Determine the values of x, if $3\cos x - 1 = -1,5$ for $x \in [0°;360°]$ (4)
[11]

QUESTION 4

4.1 Simplify:

$$(1+\cos x)(1-\cos x)$$ (2)

4.2 Simplify:

$$\frac{\cos^2(2\pi - x)\tan^2 x}{\sin(180° + x)\operatorname{cosec}(180° - x)}$$ (6)

4.3 Prove that:

$$\cot x + \tan x = \operatorname{cosec} x \cdot \sec x$$ (4)

[12]

QUESTION 5

Given the functions defined by $f(x) = \cos 2x$ and $g(x) = \sin(x - 30°)$ for $x \in [0°;180°]$.

5.1 Write down the period of *f*. (1)

5.2 Write down the amplitude of *g*. (1)

5.3 On the same axes given in your SPECIAL ANSWER BOOK draw the graphs of *f* and *g*. Clearly show the turning points, endpoints, and the intercepts with the axes. (8)

5.4 Use your graphs to determine for which values of *x* is:

5.4.1 $f(x) \leq 0$ (2)

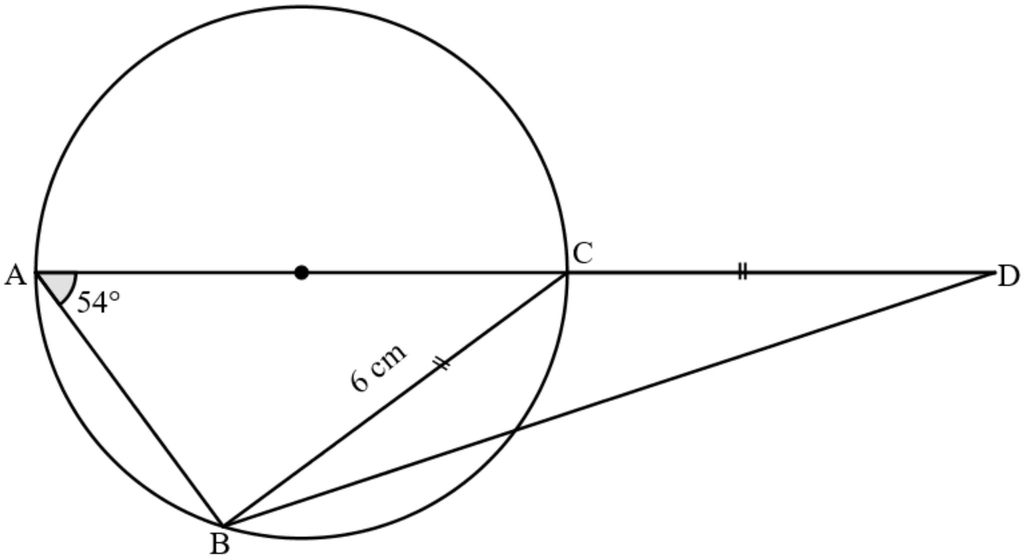
5.4.2 $f(x).g(x) \geq 0$ in the second quadrant (2)

[14]

QUESTION 6

6.1 Write down the cosine rule for ΔPQR. (1)

6.2 In the diagram below, AC is the diameter of the circle ABC.
AC is produced to D such that DC = CD = 6 cm.
$\hat{A} = 54^\circ$



Determine:

6.2.1 The size of $A\hat{B}C$, stating a reason (2)

6.2.2 The size of $B\hat{C}D$, stating a reason (2)

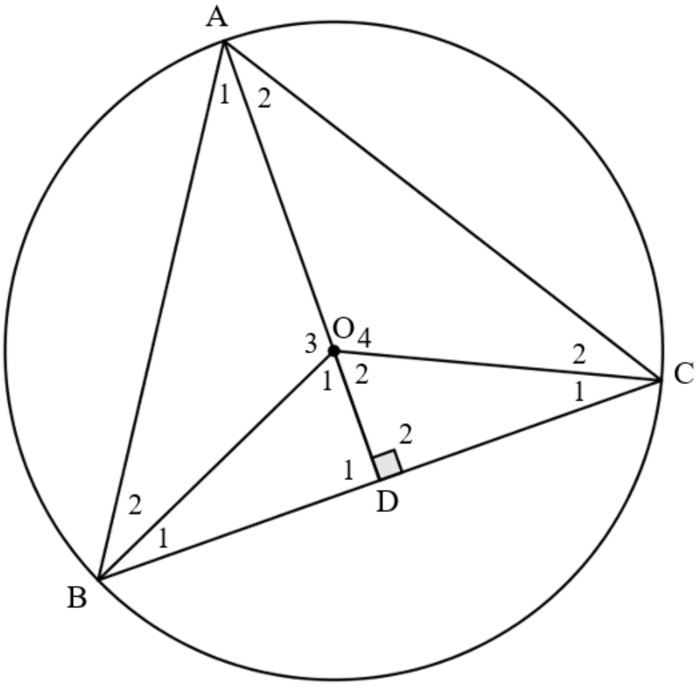
6.2.3 The length of BD (4)

6.2.4 The length of the diameter, AC (2)

6.2.5 The area of ΔABC (3)
[14]

QUESTION 7

In the diagram below, ABC is a circle with centre O.
OD = 3 cm, BC = 11 cm and $OD \perp BC$.
BO, AO and OC are joined.



7.1 Determine the length of BD, stating a reason. (2)

7.2 Calculate the length of OB. (2)

7.3 Show that $\Delta ABD \equiv \Delta ACD$. (4)

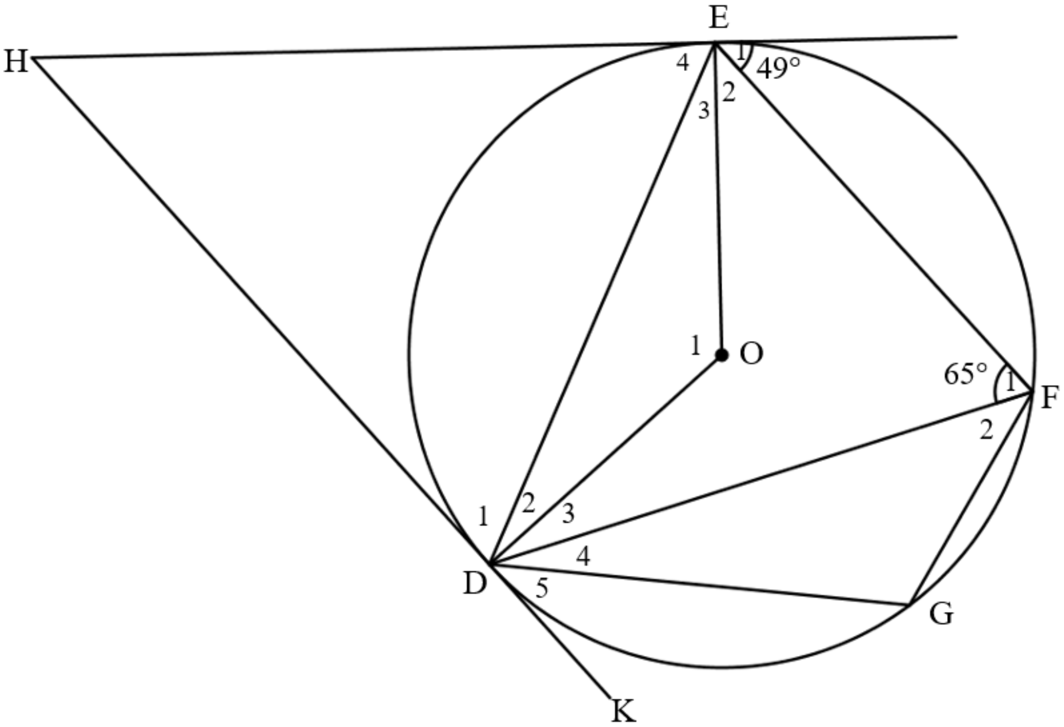
7.4 Calculate the size of $\hat{B}_1$. (2)

7.5 Hence, calculate the size $\hat{A}$, stating reasons. (4)

[14]

QUESTION 8

In the diagram below, a circle with centre O is given.
HE and HD are tangents to the circle such that $\hat{E}_1 = 49°$ and $\hat{F}_1 = 65°$.



8.1 Give a reason why HD = HE. (1)

8.2 Determine, stating reasons, the size of the following angles:

8.2.1 $\hat{D}_1$ (2)

8.2.2 $\hat{D}_2$ (2)

8.2.3 $D\hat{E}F$ (2)

8.2.4 $\hat{G}$ (2)

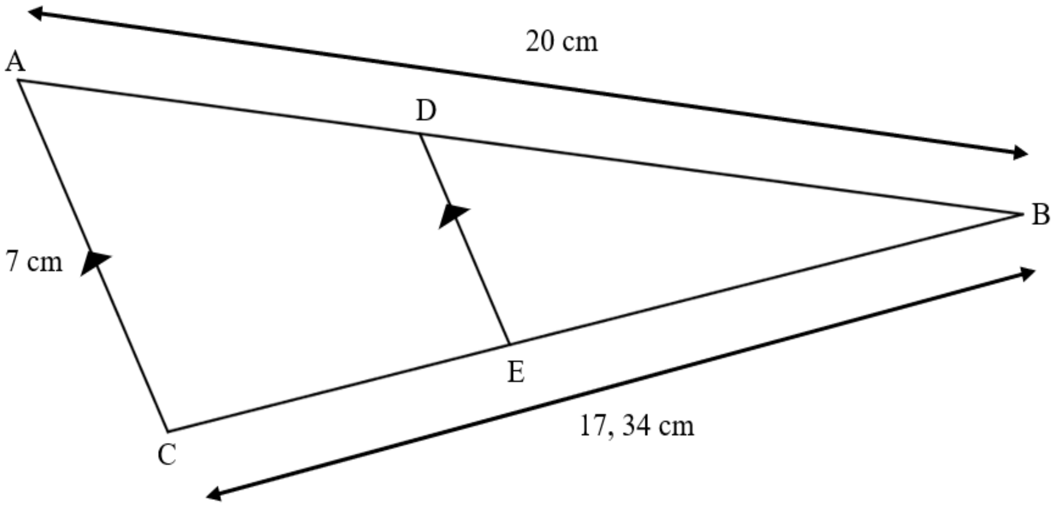
8.2.5 $F\hat{D}K$ (2)

8.3 Show, stating reasons, whether EHDF is cyclic. (3)

[14]

QUESTION 9

In ΔABC below, AB = 20 cm, BC = 17,34 cm and AC = 7 cm.
D and E are points on the sides of the triangle such that DE ∥ AC.
AD : DB = 2 : 3



9.1 Determine the lengths of AD and DB. (3)

9.2 Calculate, stating reasons, the length of BE. (3)

9.3 Prove, stating reasons, that ΔBDE ||| ΔBAC. (3)

9.4 Hence, determine the length of DE. (3)

[12]

QUESTION 10

A wheel with a diameter 250 mm, has a circumferential velocity of 108 kilometres per hour.

10.1 Convert 108 km/h to m/s. (2)

10.2 Determine the rotational frequency of the wheel in seconds. (5)

10.3 Determine the angular velocity of the wheel in seconds. (3)

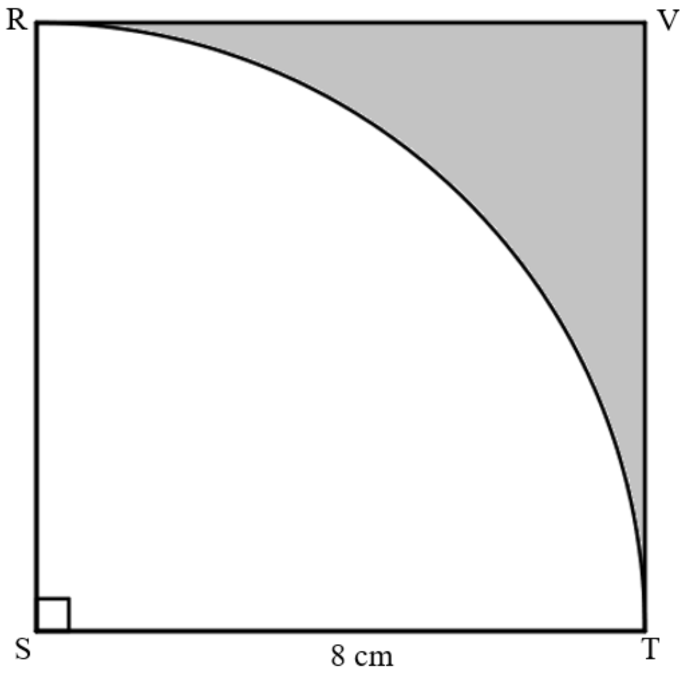
10.4 Determine the distance, in km, a point on the wheel will cover in 10 min. (3)

10.5 Determine how long it will take the wheel to make 20 revolutions. (2)

[15]

QUESTION 11

11.1 In diagram below, RSTV is a square with sides 8 cm.
RT is an arc of the sector RST. (2)

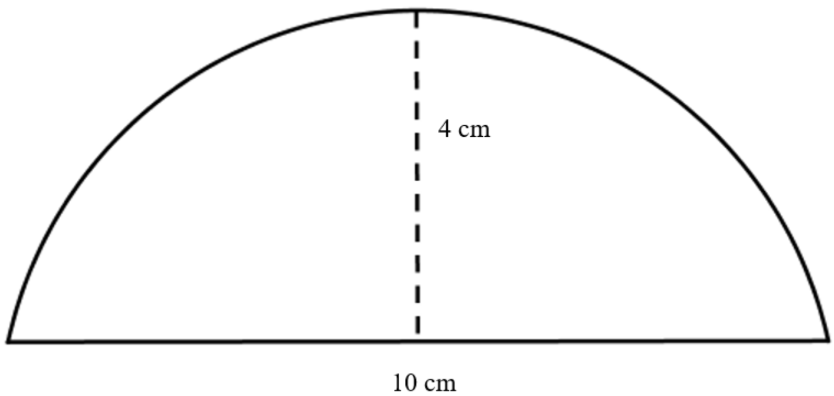


11.1.1 Determine the length of arc RT. (3)

11.1.2 Determine the area of sector RST. (3)

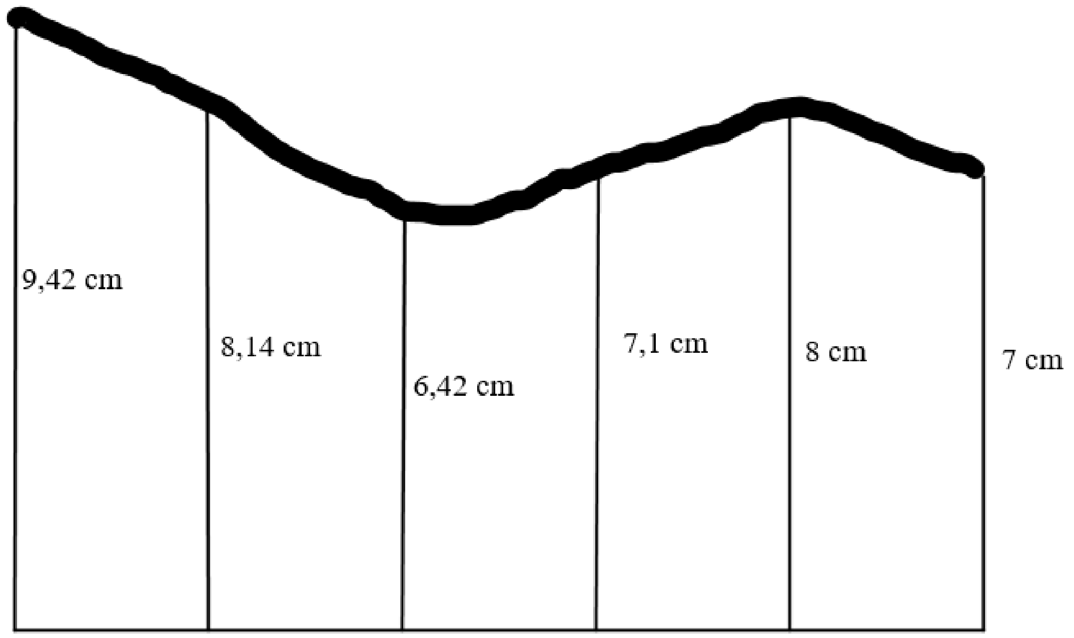
11.1.3 Hence, calculate the area of the shaded area. (3)

11.2 The diagram below is a minor segment of a circle with height 4 cm and chord 10 cm.



Determine the length of the radius of the circle. (5)

11.3 The ordinates in the irregular figure are 9,42; 8,14; 6,42; 7,1; 8 and 7 cm as indicated in the diagram below. The area of the irregular figure is 113,61 cm^2.



Determine the width of the equal parts on the horizontal axis. (4)

[18]

TOTAL: 150

INFORMATION SHEET

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $x = -\frac{b}{2a}$ $\quad$ $y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b$ $\quad$ $a > 0$, $a \neq 1$ en $b > 0$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$

$A = P(1 + i)^n$ $\quad$ $A = P(1 - i)^n$ $\quad$ $i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$ $\quad$ $\int kx^n dx = k \cdot \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$

$\int \frac{1}{x} dx = \ln(x) + C, \quad x > 0$ $\quad$ $\int \frac{k}{x} dx = k \cdot \ln(x) + C, \quad x > 0$

$\int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$ $\quad$ $\int ka^{nx} dx = k \cdot \frac{a^{nx}}{n \ln a} + C \quad , \quad a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In ΔABC:

$\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$ $\quad$ $a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$ $\quad$ $\text{Area} = \frac{1}{2} ab.\sin \mathrm{C}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360°n$ where n = rotation frequency

Circumferential velocity = $v = \pi Dn$ where D = diameter and n = rotation frequency

Circumferential velocity = $v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius and s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of the circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates