LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

MOGALAKWENA DISTRICT

GRADE 12

MATHEMATICS
TERM 2

PRE JUNE EXAM PAPER 1

MARKING GUIDELINE

MAY/JUNE 2023

MARKS: 150

INSTRUCTIONS:

NOTE:

- This memorandum consists of 19 pages including the cover page.
- If the candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking memorandum.

QUESTION 1

1.1.1	$x = 3 ; x = -\frac{2}{5}$	✓✓each root	(2)
1.1.2	$3x^2 \quad 10x \quad 1 = 0$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{-(-10) \pm \sqrt{(-10)^2-4(3)(-1)}}{2(3)}$ $= \frac{10 \pm \sqrt{112}}{6}$ $x = 3{,}43$ of $x = -0{,}1$	✓subst ✓✓each root (-1, if incorrect rounding)	(3)
1.1.3	$x^2 - 4x \geq 21$ $x^2 - 4x - 21 \geq 0$ $(x-7)(x+3) \geq 0$ $\therefore \quad x \leq -3$ and $x \geq 7, x \in R$	✓std form ✓factors ✓critical pts ✓✓conclusion	(5)
1.1.4	$2.3^x + \frac{3^x}{2} = 7\frac{1}{2} (\times 2)$ $2.2.3^x + 3^x = \frac{15}{2} \times \frac{2}{1}$ $4.3^x + 3^x = 15$ $3^x(4+1) = 15$ $3^x = 3$ $x = 1$	✓simplify ✓factorisation ✓simplify ✓$x = 1$	(4)

1.2			
	$x + 2y = 2$ ....(1) $x^2 + 8y = 8$ ....(2) $x = -2y+2$...(3) Subst 3 in 2. $(-2y + 2)(-2y + 2) + 8y = 8$ $4y^2 - 8y + 4 + 8y - 8 = 0$ $4y^2 - 4 = 0$ $y^2 - 1 = 0$ $(y - 1)(y + 1) = 0$ $y = 1$ or $y = -1$ Subst $y = 1$ in 3 Subst $y = -1$ in 3 $x = -2(1) + 2$ $x = -2(-1) + 2$ $x = 0$ $x = 4$ Solution $(0 ; 1)$ $(4 ; -1)$	✓substitution ✓std form ✓factorisation ✓y-value ✓✓x –values.	(6)

1.2.1			
	$2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x(2^1 + 1) = 3^y(3^2 - 1)$ $2^x(3) = 3^y(8)$ $\therefore \frac{2^x}{8} = \frac{3^y}{3}$ $\therefore \frac{2^x}{2^3} = \frac{3^y}{3}$ $\therefore 2^{x-3} = 3^{y-1}$ $\therefore x - 3 = 0$ and $y - 1 = 0$ $\therefore x = 3$ and $y = 1$ **OR** $2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x(2^1 + 1) = 3^y(3^2 - 1)$ $2^x(3) = 3^y(8)$ $2^x(3) = 3^y(2^3)$ $\therefore x = 3$ and $y = 1$	✓ factorise ✓simplify equated bases ✓answers **OR** ✓factorise ✓simplified equated bases ✓answers	(3)

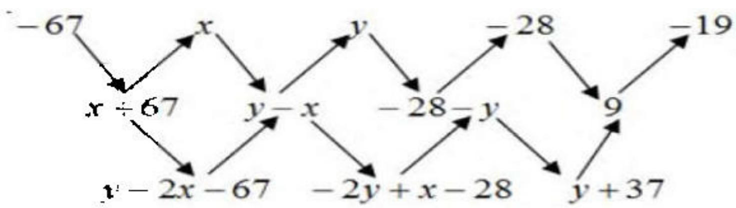
1.2.2	$\sqrt{x-2}+3=\frac{10}{\sqrt{x-2}}$ Let $k=\sqrt{x-2}$ $k+3=\frac{10}{k}$ $k^2+3k-10=0$ $(k+5)(k-2)=0$ $k=-5$ **or** $k=2$ $\sqrt{x-2}\neq-5$ **or** $\sqrt{x-2}=2$ $\left(\sqrt{x-2}\right)^2=(2)^2$ $\therefore x=6$	✓ standard form ✓ factors ✓ both answers for k ✓ selection ✓ answer	
	OR		
	$\sqrt{x-2}+3=\frac{10}{\sqrt{x-2}}$ $\left(\sqrt{x-2}\right)\left(\sqrt{x-2}\right)+3\sqrt{x-2}=10$ $x-2+3\sqrt{x-2}=10$ $3\sqrt{x-2}=12-x$ $\left(3\sqrt{x-2}\right)^2=(12-x)^2$ $9x-18=144-24x+x^2$ $x^2-33x+162=0$ $(x-6)(x-27)=0$ $x=6$ **or** $x\neq27$	✓ simplified both sid ✓ squaring both sides ✓ standard form ✓ factors ✓ selection	(5)

1.3.1	$\Delta = 16-4p = 16-4(4) = 0$ $\therefore$ Roots equal rational	✓equal & rational	(1)
1.3.2	$\Delta = 16-4p < 0$ non-real $-4p < -16$ $p > 4$	✓$\Delta<0$ ✓$p>4$	(2)
1.4	$x^2+rx+m=0$ and $x^2+mx+r=0$ $x^2+rx+m=0$ For real and equal roots, $\Delta=0$ $b^2-4ac=0$ $(r)^2-4(1)(m)=0$ $r^2-4m=0$ $r^2=4m$ $m=\frac{r^2}{4}$ ... (1) $x^2+mx+r=0$ $b^2-4ac=0$ $m^2-4(1)(r)=0$ $m^2-4r=0$ ... (2) Substitute (1) in (2) $\left(\frac{r^2}{4}\right)^2-4r=0$ $\frac{r^4}{16}-4r=0$ $r^4-64r=0$ $r(r^3-64)=0$	✓ substitute into $\Delta=0$ ✓ equation for m ✓ equation 2 ✓ substitute for m	

	$r(r-4)(r^2+4r+16)=0$ $\therefore r=4$ $m=\frac{r^2}{4}$ $m=\frac{4^2}{4}$ $\therefore m=4$ **OR** $x^2+rx+m=0$ and $x^2+mx+r=0$ For real and equal roots quadratic must be a perfect square. $x^2+rx+m=0$ $\left(x+\sqrt{m}\right)^2=0$ $x^2+2\sqrt{m}x+m=0$ $r=2\sqrt{m}$ $r^2=4m$ $\frac{r^2}{4}=m$ ... (1) $x^2+mx+r=0$ $\left(x+\sqrt{r}\right)^2=0$ $x^2+2\sqrt{r}+r=0$ $m=2\sqrt{r}$ $m^2=4r$ ….(2)	✓value of r ✓value of m ✓ $\left(x+\sqrt{m}\right)^2=0$ ✓ equation for m ✓ equation 2 ✓ substitute for m	

	Substitute (1) in (2) $\left(\frac{r^2}{4}\right)^2 - 4r = 0$ $\frac{r^4}{16} - 4r = 0$ $r^4 - 64r = 0$ $r(r^3 - 64) = 0$ $r(r-4)(r^2+4r+16) = 0$ $\therefore r = 4$ $m = \frac{r^2}{4}$ $m = \frac{4^2}{4}$ $\therefore m = 4$	✓value of r ✓value of m	
			(6)
			[37]

QUESTION 2

2.1	 $y-2x-67=y+37$ $-2x=104$ $x=-52$ $y-2x-67=-2y+x-28$ $3y-3x=39$ $y-x=13$ $y-(-52)=13$ $y=-39$	✓first differences ✓second differences ✓equating second differences / ✓x-value ✓ equating second differences/ ✓y-value	(6)
2.2	$-67 \quad -52 \quad -39 \quad -28$ $15 \quad 13 \quad 11$ $-2 \quad -2$ $2a=-2$ $a=-1$ $3a+b=15$ $3(-1)+b=15$ $b=18$ $a+b+c=-67$ $-1+18+c=-67$ $c=-84$ $T_n=-n^2+18n-84$	✓value of a ✓value of b ✓value of c ✓answer	(4)

2.3	$T_n > 0$ $-n^2+18n-84>0$ $n^2-18n+84<0$ no Solution $\therefore$ the sequence will never contain a positive term	✓ $-n^2+18n-84>0$ ✓ $n^2-18n+84<0$ ✓conclusion	(3)
			[13]

QUESTION 3

3.1	$T_4 = 24$ $\quad\quad T_9 = 768$ $\frac{T_9}{T_4} = r^{9-4}$ $\frac{768}{24} = r^5$ $32 = r^5$ $2^5 = r^5$ $r = 2$ $\frac{T_4}{T_1} = r^3$ $\quad$ *or* $\quad$ $\frac{T_9}{T_1} = r^8$ $\frac{24}{a} = 2^3$ $\quad\quad$ $\frac{768}{a} = 2^8$ $a = \frac{24}{8}$ $\quad\quad$ $a = \frac{768}{256}$ $a = 3$ $\quad\quad$ $a = 3$ $T_n = ar^{n-1}$ $T_n = 3 \cdot 2^{n-1}$ 3; 6; 12; ..... **OR**	✓ $\frac{768}{24} = r^5$ ✓simplification ✓ r ✓ a ✓ T_2 & T_3	(5)

	$T_4 = 24$ $\quad T_9 = 768$ $T_n = ar^{n-1}$ $T_4 = ar^3 = 24$..................equation1 $T_9 = ar^8 = 768$..................equation2 $\frac{\text{equation2}}{\text{equation1}}$ $ar^8 = 768$ $ar^3 = 24$ $r^5 = 32$ $r^5 = 2^5$ $r = 2$ $ar^3 = 24$ or $ar^8 = 768$ $a \cdot 2^3 = 24$ $\quad a \cdot 2^8 = 768$ $a = 3$ $\quad a = 3$ $T_n = ar^{n-1}$ $T_n = 3 \cdot 2^{n-1}$ 3; 6; 12;	✓equations ✓ $r^5 = 32$ ✓r ✓a ✓ T_2& T_3	
3.2.1	$T_{12} = S_{12} - S_{11}$ $= 324 - 275$ $= 49$	✓ $T_{12} = S_{12} - S_{11}$ ✓substitution ✓answer	(3)
3.2.2	$T_n = S_n - S_{n-1}$ $= 2n^2 + 3n - [2(n-1)^2 + 3(n-1)]$ $= 2n^2 + 3n - [2n^2 - 4n + 2 + 3n - 3]$ $= 2n^2 + 3n - 2n^2 + n + 1$ $= 4n + 1$	✓ $T_n = S_n - S_{n-1}$ ✓substitution ✓simplification ✓answer	(4)

3.3	$\sum_{n=2}^{18}(2n-1)$ $3+5+7+...$ $a=3 \qquad S_n=\frac{n}{2}[2a+(n-1)d]$ $d=2 \qquad S_{17}=\frac{17}{2}[2\cdot3+(17-1)2]$ $n=17 \qquad =323$	✓a & d ✓$n = 17$ ✓answer	(3)

3.4	$S_\infty=\frac{40}{3}$ $\qquad T_2=\frac{5}{2}$ $S_\infty=\frac{a}{1-r}=\frac{40}{3}$ $\qquad T_n=ar^{n-1}$ $T_2=ar=\frac{5}{2}$ $a=\frac{40}{3}(1-r)$ $\qquad r=\frac{5}{2a}$ $a=\frac{40}{3}\left(1-\frac{5}{2a}\right)$ $a=\frac{40}{3}-\frac{100}{3a}$ $3a^2=40a-100$ $3a^2-40a+100=0$ $(3a-10)(a-10)=0$ $a=\frac{10}{3}$ or $a=10$	✓substitution ✓standard form ✓factors ✓answer(both values of a)	(3)

3.5.1	$a = \frac{24}{x}$ $r = \frac{6x}{12}$ *or* $\frac{3x^2}{6x}$ *or* $\frac{x}{2}$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{\frac{24}{x}}{1-\frac{x}{2}}$ $S_\infty = \frac{48}{2x-x^2}$	✓ value of r ✓ substitution in correct formula ✓ correct numerator ✓ correct denominator	(4)

3.5.2	It exists when: $-1 < \frac{x}{2} < 1$ i.e.: $-1 < r < 1$ $\therefore -2 < x < 2$ **ANSWER ONLY: Award full marks.**	✓ $-1 < r < 1$ ✓ answer	(2)
			[24]

QUESTION 4

4.1	$f(x) = a(x-2)^2 + 9$ $a(3-2)^2 + 9 = 8$ $a + 9 = 8$ $a = -1$ $\therefore f(x) = -1(x-2)^2 + 9$ $f(x) = -1(x^2 - 4x + 4) + 9$ $f(x) = -x^2 + 4x - 4 + 9$ $f(x) = -x^2 + 4x + 5$	✓$T_n = 2^n$ ✓$a = 1$ ✓$b = -1$ ✓$c = 2$ ✓Answer	

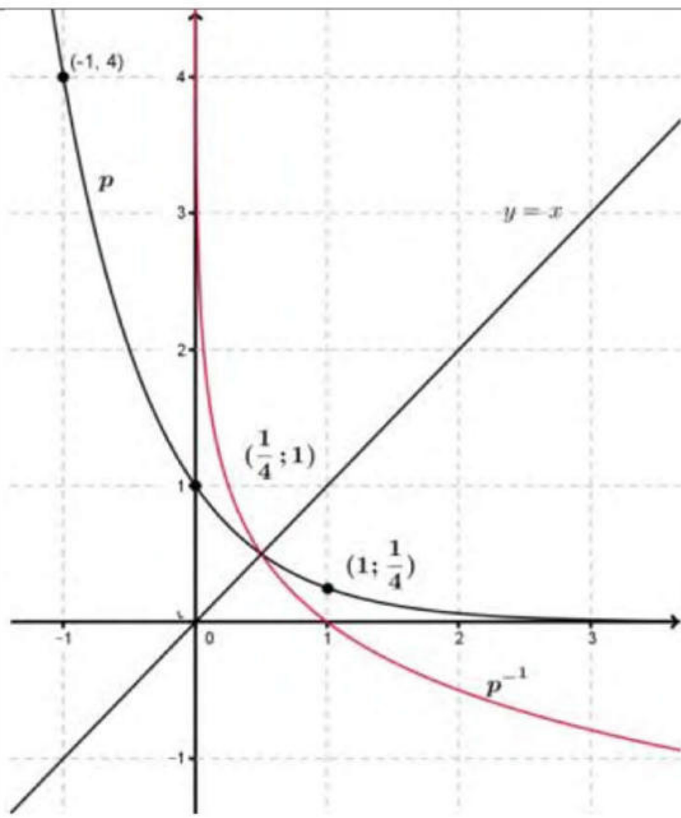
4.2	$x = 2$	✓Answer	(1)
4.3	$g(x) = a^x$ $a^3 = 8$ $a^3 = 2^3$ $a = 2$	✓Substitution ✓Answer	(2)
4.4	(5; 0)	✓Answer	(1)
4.5	$x < -1$ *or* $x > 5$	Mark per answer ✓$x < -1$ ✓$x > 5$	(2)
4.6	$g(x) = 2^x$ $g(2) = 2^2 = 4$ $AC = 9 - 4$ $AC = 5$ *units*	✓$g(2) = 4$ ✓Answer	(2)
4.7	Roots will be real and equal	✓Real ✓Equal	(2)
			[14]

QUESTION 5

5.1	x − intercept of the line: $-2x + 2 = 0 \quad \therefore \quad x = 1$ $p = -1$ y-intercept of line: $y = 2$ $q = 2$	✓x int ✓p ✓q	(3)
5.2	$g(x) = \frac{a}{x-1} + 2$ $\frac{a}{0-1} + 2 = 4$ $\therefore \quad a = -2$	✓Substitute point (0;4) ✓Answer	(2)

5.3	$g(x) = -\frac{2}{x-1} + 2$ $-\frac{2}{x-1} + 2 = 0$ $-\frac{2}{x-1} = -2$ $-2x + 2 = -2$ $\therefore x = 2$ $\therefore A = (2;0)$	✓Subst $y = 0$ ✓Answer	(2)
5.4	The graph moves up 1 unit	✓Answer	(1)
			[8]

QUESTION 6

6.1	$x = \left(\frac{1}{4}\right)^y$ $y = \log_{\frac{1}{4}} x$	✓Interchange ✓Answer	(2)
6.2		Intercept of p ✓ Intercept of p^{-1} ✓ Line: $y = x$ ✓ Shape of p and p^{-1} ✓✓	(5)

6.3	$0 < x \leq \frac{1}{4}$	✓ critical points ✓ answer	(2)
6.4	$h(x) = \left(\frac{1}{4}\right)^{x-2} + 3$	✓ -2 ✓ +3	(2)
			[11]

QUESTION 7

7.1	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} \frac{5-2(x+h)^2-(5-2x^2)}{h}$ $= \lim_{h\to 0} \frac{5-2x^2-4xh-2h^2-5+2x^2}{h}$ $= \lim_{h\to 0} \frac{-4xh-2h^2}{h}$ $= \lim_{h\to 0} \frac{h(-4x-2h)}{h}$ $= \lim_{h\to 0}(-4x-2h)$ $= -4x$	✓ substitution ✓ expansion ✓ simplification ✓ notation and $\lim_{h\to 0}(-4x-2h)$ ✓ answer	(5)
7.2.1	$y = 7x^4 + \frac{2x^2}{\sqrt{x}}$ $= 7x^4 + 2x^{\frac{3}{2}}$ $\therefore \frac{dy}{dx} = 28x^3 + 3x^{\frac{1}{2}}$	✓ $2x^{\frac{3}{2}}$ ✓ $28x^3$ ✓ $3x^{\frac{1}{2}}$	(3)

7.2.2	$xy = 5$ $y = \frac{5}{x}$ $y = 5x^{-1}$ $\frac{dy}{dx} = -5x^{-2}$	✓ $y = 5x^{-1}$ ✓answer	(2)
7.2.3	$= D_x\left[\frac{3x^2 - 7x - 6}{x}\right]$ $= D_x\left[3x - 7 - 6x^{-1}\right]$ $= 3 + 6x^{-2}$	✓ $3x - 7$ ✓ $-6x^{-1}$ ✓ 3 and differentiating constant ✓ $+6x^{-2}$	(4)
			[14]

QUESTION 8

8.1	$x^3 - 6x^2 + 9x - 4 = 0$ $(x-1)(x^2 - 5x + 4) = 0$ $(x-1)(x-4)(x-1) = 0$ $x = 1 ; x = 4$ B(4 ; 0) $\therefore f(3) = -4$ D : $(3 ; -4)$	✓$(x-1)$ ✓$(x^2 - 5x + 4)$ ✓$(x-4)(x-1)$ ✓coordinates	(4)
8.2	$(x) = x^3 - 6x^2 + 9x - 4$ $f'(x) = 3x^2 - 12x + 9 = 0$ $x^2 - 4x + 3 = 0$ $x = \frac{-(-4) \mp \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$ $x = 3 ; x = 1$ $\therefore f(3) = -4$ ∴ D $(3 ; -4)$	✓$3x^2 - 12x + 9$ ✓=0 ✓subs. into formula ✓x=3 ✓y = - 4	(5)
8.3	$f'(x) = 3x^2 - 12x + 9$ $f''(x) = 6x - 12 = 0$ $x = 2$	✓$6x - 12 = 0$ ✓x=2	(2)
8.4	$k < -4$ or/of $k > 0$	✓$k < -4$ ✓$k > 0$	**(2)**

8.5	$x < 1 \quad or/of \ x > 3$	✓$x < 1$ ✓or ✓$x > 3$	(3)
8.6	**0**	✓	(1)
			[17]

QUESTION 9

9.1	After t hours: $BF = 30t$ km and $CD = 40t$ km $\therefore BC = 100 - 40t$ $FC = \sqrt{(30t)^2 + (100 - 40t)^2}$ $= \sqrt{900t^2 + 10000 - 8000t + 1600t^2}$ $= \sqrt{2500t^2 - 8000t + 10000}$	✓ $BF = 30t$ ✓ $BC = 100 - 40t$ ✓Pythagoras ✓answer	**(4)**
9.2	FC is a minimum when FC^2 is a minimum. $FC^2 = 2500t^2 - 8000t + 10000$ $\frac{dFC^2}{dt} = 5000t - 8000 = 0$ $t = \frac{8000}{5000} = 1{,}6\text{hrs} \quad (96 \text{ minutes})$	✓ $BF = 30t$ ✓ $BC = 100 - 40t$ ✓Pythagoras ✓answer	**(4)**
9.3	$FC = \sqrt{2500t^2 - 8000t + 10000}$ $= \sqrt{2500(1.6)^2 - 8000(1.6) + 10000}$ $= 60$ They will be 60km apart.	✓subs into equation ✓answer	**(2)**
			[10]