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PREPARATORY EXAMINATION

2023

MARKING GUIDELINES

MATHEMATICS (PAPER 1) (10611)

18 pages

INSTRUCTIONS AND INFORMATION:

A – Accuracy

CA – Continued Accuracy

S – Statement

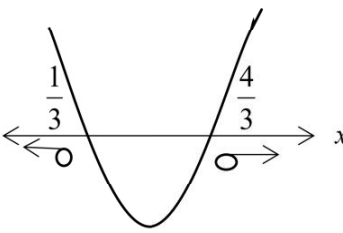
R – Reason

S and R – Statement and Reason

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed OUT an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- It is UNACCEPTABLE to assume values/answers in order to solve a question.

QUESTION 1

1.1	1.1.1	$(2x+1)^2 - 4 = 0$ $4x^2 + 4x + 1 - 4 = 0$ $4x^2 + 4x - 3 = 0$ $(2x+3)(2x-1) = 0$ $x = -\frac{3}{2} \dots or \dots x = \frac{1}{2}$ OR $(2x+1)^2 - 4 = 0$ $2x+1 = \pm\sqrt{4}$ $2x+1 = \pm 2$ $2x = \pm 2 - 1$ $2x = 2 - 1 \dots or \dots 2x = -2 - 1$ $2x = 1 \dots or \dots 2x = -3$ $x = \frac{1}{2} \dots or \dots x = -\frac{3}{2}$ NOTE: In option 2, the simplification mark must only be awarded if the candidate states ± 2 or demonstrates it later in the solution.	✓ standard form ✓ factors ✓ answers ✓ find root ✓ simplification ✓ answers	(3)
	1.1.2	$4x^2 - 11 = -12x$ $4x^2 + 12x - 11 = 0$ $x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(-11)}}{2(4)}$ $x = -3,74 \quad or \quad x = 0,74$ NOTE: Penalise 1 mark for rounding in this question only. Candidate must show the substitution to obtain full marks.	✓ correct substitution ✓✓ answers	(3)
	1.1.3	$15x - 4 < 9x^2$ $\therefore -9x^2 + 15x - 4 < 0$ $\therefore 9x^2 - 15x + 4 > 0$ $(3x-1)(3x-4) > 0$ $\therefore x < \frac{1}{3} \dots or \dots x > \frac{4}{3}$ 	✓ standard form ✓ factors ✓✓ answers	(4)

1.1.4	$\sqrt{2x-2} - \sqrt{7-2x} = 1$ $(\sqrt{2x-2})^2 = (\sqrt{7-2x} + 1)^2$ $2x-2 = 7-2x+2\sqrt{7-2x}+1$ $4x-10 = 2\sqrt{7-2x}$ $(2x-5)^2 = (\sqrt{7-2x})^2$ $4x^2 - 20x + 25 = 7 - 2x$ $4x^2 - 18x + 18 = 0$ $\therefore 2x^2 - 9x + 9 = 0$ $(x-3)(2x-3) = 0$ $\therefore x = 3 \dots \text{or} \dots x = \frac{3}{2}$ <i>reject</i> $\dots x \neq \frac{3}{2}$	✓ isolate surd and square both sides ✓ simplification ✓ standard form ✓ factors ✓ $x = 3$ and rejection	(5)
1.2	$a^2b^2 - 2ab - 8 = 0$ and $\log_2(a+5) = 3$ $\log_2(a+5) = 3$ $2^3 = a+5$ $8 = a+5$ $a = 3$ $\therefore a^2b^2 - 2ab - 8 = 0$ $(3)^2b^2 - 2(3)b - 8 = 0$ $9b^2 - 6b - 8 = 0$ $(3b-4)(3b+2) = 0$ $b = \frac{4}{3}$ or $b = -\frac{2}{3}$ OR $a^2b^2 - 2ab - 8 = 0$ and $\log_2(a+5) = 3$ $\log_2(a+5) = 3$ $2^3 = a+5$ $8 = a+5$ $a = 3$ $a^2b^2 - 2ab - 8 = 0$ $(ab-4)(ab+2) = 0$ $ab = 4$ or $ab = -2$ $\therefore 3b = 4$ $3b = -2$ $b = \frac{4}{3}$ or $b = -\frac{2}{3}$	✓ exponential form ✓ value of a ✓ substitution ✓ factors ✓ both answers for b OR ✓ exponential form ✓ value of a ✓ factors ✓ substitution ✓ both answers for b	(5)

1.3	$p = \frac{\sqrt{x+2}}{\sqrt{16-x^2}}$ $\therefore x+2 \geq 0$ $\therefore x \geq -2 \dots \dots \dots (1)$ $\therefore 16-x^2 > 0$ $(4+x)(4-x) > 0$ $\therefore -4 < x < 4 \dots \dots \dots (2)$ From (1) and (2) $-2 \leq x < 4$	✓ equation (1) ✓ factors ✓ equation (2) ✓ answer	(4)
[24]			

QUESTION 2

2.1	2.1.1	$1-p; 2p-3; p+5$ $d = T_2 - T_1 = T_3 - T_2$ $\therefore (2p-3) - (1-p) = (p+5) - (2p-3)$ $2p-3-1+p = p+5-2p+3$ $3p-4 = -p+8$ $4p = 12$ $\therefore p = 3$	✓ substitution ✓ answer	(2)
	2.1.2	(a) $T_1 = 1-p$ $T_1 = 1-3$ $T_1 = -2$	✓ answer	(1)
		(b) $T_2 = 2p-3$ $T_2 = 2(3)-3$ $\therefore T_2 = 3$ $\therefore d = T_2 - T_1$ $d = 3 - (-2)$ $d = 5$	✓ answer	(1)
	2.1.3	$1-p; \dots 2p-3; \dots p+5; \dots$ $= 1-3; \dots 2(3)-3; \dots 3+5; \dots$ $= -2; \dots \dots \dots 3; \dots \dots \dots 8; \dots 13; \dots 18; \dots$ All the terms except T_1 end in either 3 or 8 while perfect squares end on 1; 4; 9; 6; 5; 0.	✓ correct terms ✓ explanation	(2)

10611/23

2.2	2.2.1	$-3; -2; -3; -6; -11; \dots$ First differences: $+1; -1; -3; -5$ $T_n = a + (n-1)d$ $T_n = 1 + (n-1)(-2)$ $T_n = 1 - 2n + 2$ $T_n = -2n + 3$	✓ first differences ✓ substitution ✓ answer	(3)
	2.2.2	$T_n = -2n + 3$ $T_{35} = -2(35) + 3$ $\therefore T_{35} = -67$	✓ answer	(1)
	2.2.3	$-3 \quad -2 \quad -3 \quad -6 \quad -11$ $\swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow$ $1 \quad -1 \quad -3 \quad -5$ $\swarrow \quad \searrow \quad \swarrow \quad \searrow$ $-2 \quad -2 \quad -2$ 1st differences 2nd differences $T_n = an^2 + bn + c$ but $\dots 2a = -2$ $\therefore a = -1$ $\therefore T_n = -n^2 + bn + c$ $T_1 = -(1)^2 + b(1) + c$ $-3 = -1 + b + c$ $\therefore -2 = b + c \dots \dots (1)$ $T_2 = -(2)^2 + b(2) + c$ $-2 = -4 + 2b + c$ $\therefore 2 = 2b + c \dots \dots (2)$ $(2-1) \dots \therefore b = 4$ sub(1) $\dots -2 = 4 + c$ $\therefore c = -6$ $\therefore T_n = -n^2 + 4n - 6$	✓ second differences ✓ $a = -1$ ✓ $b = 4$ ✓ $c = -6$	(4)
	2.2.4	$T_n = -n^2 + 4n - 6$ $T_n = -(n^2 - 4n + 2^2 - 4 + 6)$ $T_n = -[(n-2)^2 + 2]$ $\therefore T_n = -(n-2)^2 - 2$ $\therefore T_n(\text{max}) = -2$ $\therefore$ NO positive terms.	✓ method ✓ $T_n(\text{max})$	(2)

[16]

QUESTION 3

3.1	$S_n = 4n^2 + 1$ $S_6 = 4(6)^2 + 1$ $\therefore S_6 = 145$ $S_5 = 4(5)^2 + 1$ $\therefore S_5 = 101$ $\therefore T_6 = S_6 - S_5$ $\therefore T_6 = 145 - 101$ $\therefore T_6 = 44$	✓ value of S_6 ✓ value of S_5 ✓ answer	(3)
3.2	$(4x-3) + (4x-3)^2 + (4x-3)^3$ $\therefore r = 4x-3$ $\therefore -1 < r < 1; \dots\dots\dots r \neq 0$ $-1 < 4x-3 < 1; \dots\dots\dots 4x-3 \neq 0$ $\therefore 2 < 4x < 4 \dots\dots\dots x \neq \frac{3}{4}$ $\therefore \frac{1}{2} < x < 1; \dots\dots\dots x \neq \frac{3}{4}$ NOTE: No penalty if candidate does not state: $x \neq \frac{3}{4}$	✓ $r = 4x-3$ ✓ $-1 < r < 1$ ✓ answer	(3)
3.3	$\sum_{k=3}^5 (-1)^k \cdot \frac{2}{k}$ $= (-1)^3 \cdot \frac{2}{3} + (-1)^4 \cdot \frac{2}{4} + (-1)^5 \cdot \frac{2}{5}$ $= -\frac{2}{3} + \frac{1}{2} - \frac{2}{5}$ $= -\frac{17}{30}$ NOTE: Answer only, full marks.	✓ expansion ✓ answer	(2)
[8]			

QUESTION 4

4.1	$x \in \mathbb{R}; \quad x \neq -3$ NOTE: Both conditions must be stated.	✓ answer	(1)
4.2	$y \in \mathbb{R}; \quad y \neq 2$ NOTE: Both conditions must be stated.	✓ answer	(1)
4.3	4.3.1 6 units right. NOTE: Accept 6 units, but not 6 units left.	✓ answer	(1)
	4.3.2 3,5 units upwards. NOTE: Accept 3,5 units, but not 3,5 units downwards.	✓ answer	(1)
4.4	$x = -3$ $y = -\frac{3}{2}$ NOTE: Do not accept any equating in terms of p and q.	✓ answer ✓ answer	(2)
4.5	$0 = \frac{6}{x+3} - 1,5$ $\frac{3}{2} = \frac{6}{x+3}$ $3(x+3) = 6(2)$ $3x+9 = 12$ $3x = 3$ $x = 1$ (1;0) NOTE: The answer does NOT need to be given in coordinate form.	✓ answer	(1)

4.6		✓ asymptotes ✓ intercepts ✓ shape	(3)
4.7	$y = -x + k$ $\therefore -\frac{3}{2} = -(-3) + k$ $-\frac{3}{2} = 3 + k$ $\therefore k = -\frac{9}{2}$	✓ substitute point ✓ answer	(2)
4.8	$x > -3$	✓ answer	(1)
4.9	$y = \frac{-6}{x+3} + \frac{3}{2}$ OR $y = \frac{6}{-x-3} + \frac{3}{2}$ OR $y = -\frac{6}{x+3} + \frac{3}{2}$	✓ answer	(1)
[14]			

QUESTION 5

5.1	$E\left(\frac{7}{2}; \frac{81}{4}\right)$	✓ answer	(1)
5.2	$f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ $f(1) = -\left(1 - \frac{7}{2}\right)^2 + \frac{81}{4} = 14$ $f(5) = -\left(5 - \frac{7}{2}\right)^2 + \frac{81}{4} = 18$ $\therefore AG = \frac{f(5) - f(1)}{5 - 1}$ $= \frac{18 - 14}{4}$ $\therefore AG = 1$	✓ value of $f(1)$ ✓ value of $f(5)$ ✓ substitution ✓ answer	(4)
5.3	$f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ $\therefore f(x) = -x^2 + 7x + 8$ $\because f(x) = g(x)$ $\therefore -x^2 + 7x + 8 = -3x + 24$ $-x^2 + 10x - 16 = 0$ $\therefore x^2 - 10x + 16 = 0$ $(x - 2)(x - 8) = 0$ $\therefore x = 2 \cdots \text{or} \cdots x = 8$ $\therefore x_D = 2$	✓ equating ✓ x-values ✓ answers/selection	(3)
5.4	$ST = f(x) - g(x)$ $ST = -x^2 + 7x + 8 - (-3x + 24)$ $ST = -x^2 + 7x + 8 + 3x - 24$ $ST = -x^2 + 10x - 16$	✓ method ✓ answer	(2)
5.5	For max: $\frac{d(ST)}{dx} = 0$ $\frac{d(ST)}{dx} = -2x + 10 = 0$ $2x = 10$ $\therefore x = 5$ $\text{max: } ST = -(5)^2 + 10(5) - 16$ $\therefore ST = 9$	✓ derivative = 0 ✓ value for x ✓ answer	(3)
			[13]

6.1	$y = 3^x$ Inverse $x = 3^y$ $\therefore y = \log_3 x$	✓ answer	(1)
6.2	$y = f^{-1}(x) = \log_3 x$ x-intercept: $\log_3 x = 0$ $\therefore x = 3^0$ $\therefore x = 1$ y-intercept: None $y = f^{-1}(x-2) = \log_3(x-2)$ x-intercept: $\log_3(x-2) = 0$ $\therefore x-2 = 3^0$ $\therefore x = 3$ y-intercept: None NOTE: If the candidate does not draw the graph but calculates both x-intercepts, award 2 marks.	✓ x-intercept of $f^{-1}(x) = \log_3 x$ ✓ shape $f^{-1}(x) = \log_3 x$ ✓ x-intercept of $f^{-1}(x-2) = \log_3(x-2)$ ✓ shape $f^{-1}(x-2) = \log_3(x-2)$	(4)
6.3	$\log_3(x-2) < 1$ $\therefore 2 < x < 5$	✓ critical values ✓ correct notation	(2)

[7]

QUESTION 7

7.1	$A = P(1+i)^n$ $6\,000\,000 = 5\,700\,000(1+i)^7$ $\frac{6\,000\,000}{5\,700\,000} = (1+i)^7$ $\sqrt[7]{1,0526...} = 1+i$ $\therefore i = 1,00735 - 1$ $\therefore i = 0,00735$ rate: 0,74%	✓ substitution into correct formula ✓ value of i ✓ answer	(3)
7.2	7.2.1 $P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $4\,000\,000 = \frac{30\,000 \left[1 - \left(1 + \frac{0,06}{12} \right)^{-n} \right]}{\frac{0,06}{12}}$ $\frac{4\,000\,000 \times \left(\frac{0,06}{12} \right)}{30\,000} = 1 - \left(1 + \frac{0,06}{12} \right)^{-n}$ $\frac{1}{3} = \left(1 + \frac{0,06}{12} \right)^{-n}$ $\log_{\left(1 + \frac{0,06}{12} \right)} \frac{1}{3} = -n$ $n = 220,27$ $\therefore$ she will make 220 withdrawals.	✓ substitution into correct formula ✓ $i = \frac{0,06}{12}$ ✓ simplification ✓ correct use of logs ✓ answer of 220	(5)

7.2.2	$4\,000\,000 = \frac{20\,000 \left[1 - \left(1 + \frac{0,06}{12} \right)^{-n} \right]}{\frac{0,06}{12}}$ $0 = \left(1 + \frac{0,06}{12} \right)^{-n}$ She can make any number (an infinite number) of withdrawals. Her interest earned equals her withdrawal amount. She will only be drawing the interest amount. OR $A = P(1+i)^n$ $A = 4\,000\,000 \left(1 + \frac{0,06}{12} \right)^1$ $A = R4\,020\,000$ She can make any number (an infinite number) of withdrawals. Her interest earned equals her withdrawal amount. She will only be drawing the interest amount.	✓ valid method ✓ simplification ✓ explanation OR ✓ valid method ✓ answer ✓ explanation	(3)
7.3	$A = P(1+i)^n$ $A = 1\,000 \left(1 + \frac{0,15}{12} \right)^{18}$ $A = R1\,250,58$ $F_v = \frac{x[(1+i)^n - 1]}{i}$ $F_v = \frac{700 \left[\left(1 + \frac{0,15}{12} \right)^{18} - 1 \right]}{\frac{0,15}{12}}$ $F_v = R14\,032,33$ $\text{Amount} = R1\,250,58 + R14\,032,33$ $\therefore \text{Amount} = R15\,282,91$	✓ value of A ✓ substitution in correct formula ✓ value for F_v ✓ answer	(4)
[15]			

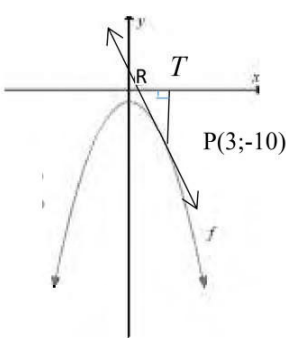
QUESTION 8

8.1	$f(x) = 3x^2 - 6$ $f(x+h) = 3(x+h)^2 - 6$ $f(x+h) = 3(x^2 + 2xh + h^2) - 6$ $f(x+h) = 3x^2 + 6xh + 3h^2 - 6$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 6 - 3x^2 + 6}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$ $f'(x) = \lim_{h \rightarrow 0} 6x + 3h$ $f'(x) = 6x$ NOTE: Penalise 1 mark for notation error in this question only.	✓ $f(x+h)$ ✓ substitution ✓ factorisation ✓ answer	(4)
8.2	$f(x) = (2\sqrt{x} - \frac{1}{x})^2$ $f(x) = 4x - 4 \cdot \frac{1}{\sqrt{x}} + \frac{1}{x^2}$ $f(x) = 4x - 4x^{-\frac{1}{2}} + x^{-2}$ $\frac{d_y}{d_x} = 4 + 2x^{-\frac{3}{2}} - 2x^{-3}$ OR $\frac{d_y}{d_x} = 4 + \frac{2}{\sqrt{x^3}} - \frac{2}{x^3}$	✓ simplification ✓ $f(x) = 4x - 4x^{-\frac{1}{2}} + x^{-2}$ ✓ 4 ✓ $2x^{-\frac{3}{2}}$ or $\frac{2}{\sqrt{x^3}}$ ✓ $-2x^{-3}$ or $\frac{2}{x^3}$	(5)
8.3	$f(x) = 3x^3 - 3x^2 + 6x - 2$ $f'(x) = 9x^2 - 6x + 6$ $f''(x) = 18x - 6$ for concave up $18x - 6 > 0$ $\therefore x > \frac{1}{3}$	✓ $f'(x)$ ✓ $f''(x)$ ✓ correct condition of concavity ✓ answer	(4)
[13]			

QUESTION 9

9.1	9.1.1	$f'(x) = -6x^2 - 6x + 12$ y-intercept $x = 0$ $\therefore f'(0) = 12$ $\therefore A(0 ; 12)$ NOTE: Does not have to be in coordinate form.	✓ answer	(1)
	9.1.2	x-intercepts $y = f'(x) = 0$ $-6x^2 - 6x + 12 = 0$ $\therefore x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $\therefore x = -2 \dots \text{or} \dots x = 1$ $\therefore B(-2 ; 0) \quad ; \quad C(1 ; 0)$ NOTE: Must be in coordinate form.	✓ factors ✓ coordinates of B ✓ coordinates of C	(3)
	9.1.3	Turning points	✓ answer	(1)
	9.1.4	$f(x)$ increases where: $m = f'(x) > 0$ $\therefore -2 < x < 1$ OR $m = f'(x) > 0$ $\therefore x \in (-2 ; 1)$	✓ $m = f'(x) > 0$ ✓✓ answers OR ✓ $m = f'(x) > 0$ ✓✓ answers	(3)
	9.1.5	Point of inflection: $f''(x) = 0$ $\therefore -12x - 6 = 0$ $-12x = 6$ $\therefore x = -\frac{1}{2}$ $f(x) = -2x^3 - 3x^2 + 12x + 20$ $\therefore f(-\frac{1}{2}) = -2(-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 + 12(-\frac{1}{2}) + 20$ $\therefore f(-\frac{1}{2}) = 13,5$	✓ $f''(x) = 0$ ✓ value for x ✓ substitution ✓ value for $f(-\frac{1}{2})$	(4)

10611/23

9.2	$y = -x^2 - 1$ $\frac{dy}{dx} = -2x$ $\frac{dy}{dx} = -2(3)$ $\therefore \frac{dy}{dx} = -6$ $\therefore m_{PR} = -6$ $\therefore \frac{PT}{RT} = -6$ $\therefore \frac{-10}{RT} = -6$ $-6RT = -10$ $\therefore RT = \frac{5}{3}$		✓ derivative ✓ $\frac{dy}{dx} = -6$ ✓ $m_{PR} = -6$ ✓ substitution ✓ answer	(5)
[17]				

QUESTION 10

10.	$W(x) = -\frac{x^3}{150} + 3x^2 - 250x - 2700$ $\therefore \text{max :}$ $W'(x) = -\frac{3x^2}{150} + 6x - 250 = 0$ $-x^2 + 300x - 12500 = 0$ $\therefore x^2 - 300x + 12500 = 0$ $(x - 250)(x - 50) = 0$ $\therefore x = 250 \cdots \text{or} \cdots x = 50$ $W''(x) = -\frac{6x}{150} + 6$ $W''(x) = -\frac{x}{25} + 6$ $\therefore W''(50) = -\frac{50}{25} + 6 > 0$ $\therefore W''(250) = -\frac{250}{25} + 6 < 0$ $\therefore \text{Maximum profit: 250 bicycles weekly.}$	✓ $W'(x)$ ✓ standard form ✓ factors ✓ values of x ✓ $W''(x)$ ✓ $W''(50) > 0$ ✓ $W''(250) < 0$ ✓ conclusion	
[8]			

QUESTION 11

NOTE: Candidates can present solutions either in decimal or fraction form. Do not penalise for rounding-off.

11.1	11.1.1	For independent events: $P(A) \times P(B) = P(A \text{ en } B)$ $\therefore (x + 0,1) \times (0,4) = (0,1)$ $\therefore (x + 0,1) = 0,25$ $\therefore x = 0,15$ $x + 0,1 + 0,3 + y = 1$ $0,15 + 0,1 + 0,3 + y = 1$ $\therefore y = 0,45$	✓ correct substitution ✓ value for x ✓ method ✓ value for y	(4)
	11.1.2	0,15 + 0,3 = 0,45 NOTE: Answer only, full marks.	✓ answer	(1)
11.2	F (unsuccessful)/P (pass/succeed)			
	11.2.1	$\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$ NOTE: Answer only, full marks.	✓ method ✓ answer	(2)
	11.2.2	$\frac{4}{7} \times \frac{2}{5} \times \frac{3}{5} = \frac{24}{175}$ NOTE: Answer only, full marks.	✓ method ✓ answer	(2)
[9]				

QUESTION 12

12.1	12.1.1	1 NOTE: Accept 2.	✓ answer	(1)
	12.1.2	$7!-(2!\times6!)=3\,600$ NOTE: Accept answer as 3 600 or $7!-(2!\times6!)$ No CA if method is meaningless.	✓ 7! ✓ $-(2!\times6!)$ ✓ answer	(3)
12.2	P(win) $=0,7\times0,9+0,3\times0,45$ $=0,765$ $=76,5\%$ NOTE: No CA if method is meaningless.		method answer	(2)
				[6]
				TOTAL: 150