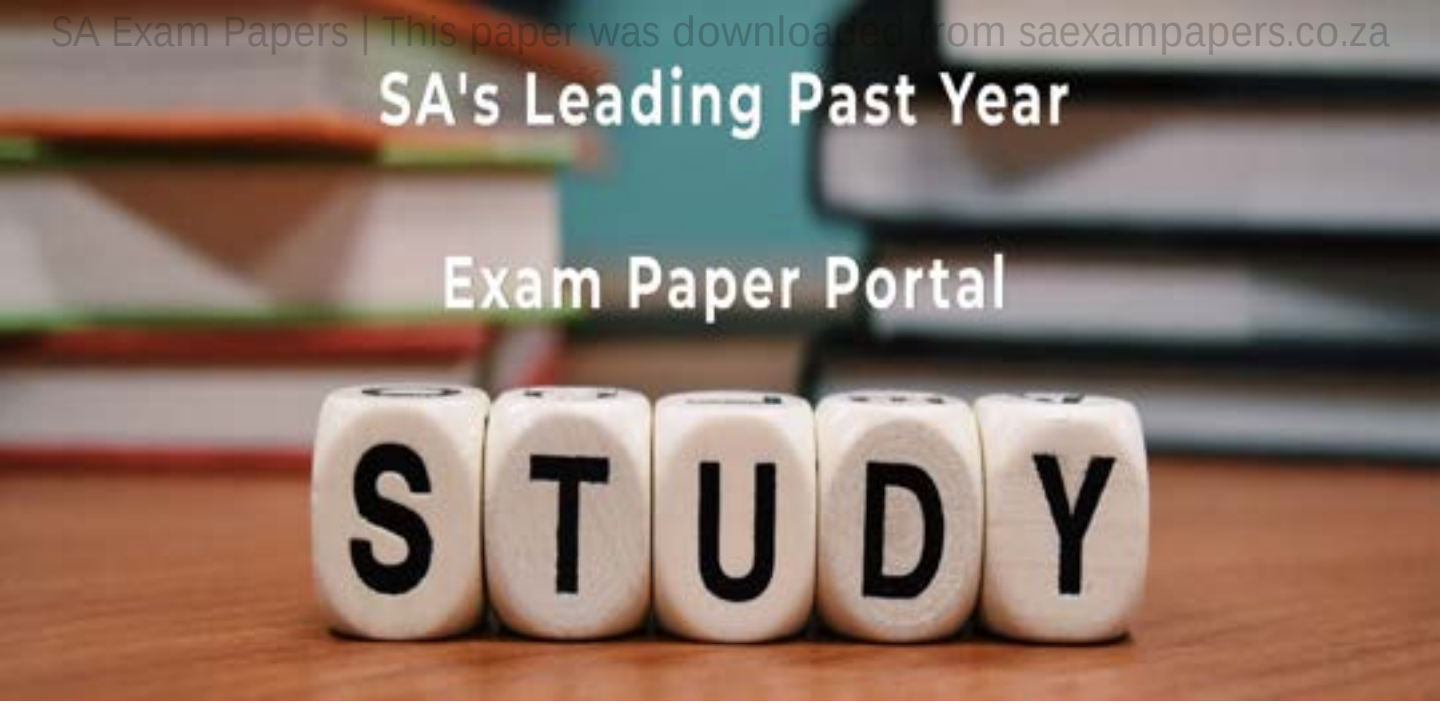


education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2023

MARKING GUIDELINES

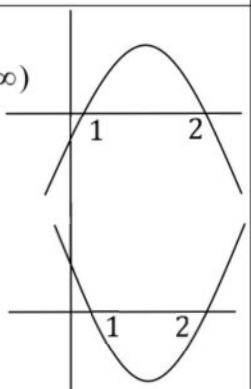
MARKS: 150

These marking guidelines consists of 16 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.

QUESTION 1

1.1.1	$x^2 = 5x$ $x^2 - 5x = 0$ $x(x-5) = 0$ $x = 0$ or $x = 5$	✓ standard form ✓ factors ✓ x-values (3)
1.1.2	$x^2 - 2x - 13 = 0$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-13)}}{2(1)}$ $x = 4,74$ or $x = -2,74$	✓ substitution into the correct formula ✓ $x = 4,74$ ✓ $x = -2,74$ (3)
1.1.3	$(x-2)(1-x) \leq 0$ $x \leq 1$ or $x \geq 2$ / $x \varepsilon (-\infty;1]$ or $[2;\infty)$  **OR** $(x-2)(x-1) \geq 0$ $x \leq 1$ or $x \geq 2$ / $x \varepsilon (-\infty;1]$ or $[2;\infty)$	✓ critical values ✓ $x \leq 1$ ✓ $x \geq 2$ (3)
1.1.4	$2\sqrt{2x-1} = x - 11$ $4(2x-1) = x^2 - 22x + 121$ $8x - 4 = x^2 - 22x + 121$ $x^2 - 30x + 125 = 0$ $(x-25)(x-5) = 0$ $x = 25$ or $x \neq 5$ **OR** $2\sqrt{2x-1} = x - 11$ $\sqrt{2x-1} = \frac{x-11}{2}$ $2x - 1 = \frac{x^2 - 22x + 121}{4}$ $8x - 4 = x^2 - 22x + 121$ $x^2 - 30x + 125 = 0$ $(x-25)(x-5) = 0$ $x = 25$ or $x \neq 5$	✓ square on both sides ✓ standard form ✓ factors/formula ✓ both answers ✓ reject $x = 5$ (5) ✓ square on both sides ✓ standard form ✓ factors/formula ✓ both answers ✓ reject $x = 5$ (5)

1.2	$3x-y=4$ $y=3x-4$	✓ $y=3x-4$
	$x^2+xy=24$ $x^2+x(3x-4)=24$ $x^2+3x^2-4x=24$ $4x^2-4x-24=0$ $x^2-x-6=0$ $(x-3)(x+2)=0$ $x=3$ or $x=-2$ $y=3(3)-4$ or $y=3(-2)-4$ $y=5$ $y=-10$	✓substitution ✓ standard form ✓ factors/formula ✓ both x-values ✓ both y-values (6)
	OR	
	$3x=y+4$ $x=\frac{y+4}{3}$	✓ $x=\frac{y+4}{3}$
	$x^2+xy=24$ $\left(\frac{y+4}{3}\right)^2+y\left(\frac{y+4}{3}\right)=24$ $\frac{y^2+8y+16}{9}+\frac{y^2+4y}{3}=24$ $y^2+8y+16+3y^2+12y=216$ $4y^2+20y-200=0$ $y^2+5y-50=0$ $(y+10)(y-5)=0$ $y=-10$ or $y=5$ $x=\frac{(-10)+4}{3}$ or $x=\frac{(5)+4}{3}$ $x=-2$ $x=3$	✓substitution ✓ standard form ✓ factors/formula ✓ both y-values ✓ both x-values (6)
1.3	$S=\left(1+\frac{1}{7}\right)\left(1+\frac{1}{8}\right)\left(1+\frac{1}{9}\right)....\left(1+\frac{1}{m}\right)$ $S=\left(\frac{8}{7}\right)\left(\frac{9}{8}\right)\left(\frac{10}{9}\right)...\left(\frac{m+1}{m}\right)$ $S=\frac{m+1}{7}$ For S to be a natural number, $m+1$ must be a multiple of 7 $m+1=14$ or $m+1=21$ or $m+1=28$ $m=13$ $m=20$ $m=27$	✓ $\left(\frac{8}{7}\right)\left(\frac{9}{8}\right)\left(\frac{10}{9}\right)...$ ✓ $\frac{m+1}{m}$ ✓ $S=\frac{m+1}{7}$ ✓ multiples 14,21,28 ✓ answer (5)
		[25]

QUESTION 2

2.1.1	$-120 \quad -99 \quad -80 \quad -63$ $\quad 21 \quad 19 \quad 17$ $\quad -2 \quad -2 \quad -2$ The next TWO terms: -48 ; -35	✓ -48 ✓ -35 (2)
2.1.2	$2a=-2 \quad 3a+b=21 \quad a+b+c=-120$ $a=-1 \quad 3(-1)+b=21 \quad (-1)+(24)+c=-120$ $b=24 \quad c=-143$ $T_n=-n^2+24n-143$	✓ 2^{nd} diff $=-2$ ✓ $a=-1$ ✓ $b=24$ ✓ $c=-143$ (4)
2.1.3	$T'n=-2n+24=0$ $n=12$ $T_n=-(12)^2+24(12)-143$ $T_n=1$ A maximum of 1 Add -1 to Tn **OR** $n=\frac{-(24)}{2(-1)}=12$ $T_n=-(12)^2+24(12)-143$ $T_n=1$ A maximum of 1 Add -1 to Tn **OR** $T_n=-n^2+24n-143+k$ $\Delta=(24)^2-4(-1)(k-143)$ $=576+4k-572$ $=4k+4$ but $\Delta=0$ $4k+4=0$ $k=-1$	✓ method ✓ $n=12$ ✓ maximum 1 ✓ -1 (4) ✓ method ✓ $n=12$ ✓ maximum 1 ✓ -1 (4) ✓ method ✓ $\Delta=4k+4$ ✓ $\Delta=0$ ✓ -1 (4)
2.2.1	$9+14+19+...+124$ $T_n=(9)+(n-1)(5)$ $T_n=5n+4$ Answer only: full marks	✓ substitution into the correct formula ✓ $T_n=5n+4$ (2)

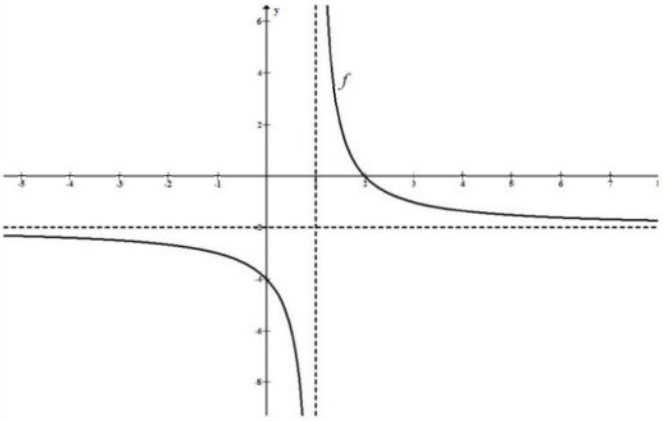
2.2.2	$T_n = 5n+4 = 124$ $5n = 120$ $n = 24$ $\sum_{n=1}^{24}(5n+4)$	✓ =124 ✓ $n=24$ ✓ answer (3)
2.3	$2^x + 2.2^x + 3.2^x + 4.2^x ....$ $a = 2^x$ $d = 2^x$ $S_{30} = \frac{30}{2}\left[2(2^x)+29(2^x)\right]$ $3720 = 15(31.2^x)$ $248 = 31.2^x$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$	✓ $a=2^x$ and $d=2^x$ ✓ substitution into the correct formula ✓ $2^x = 8$ ✓ $x=3$ (4)
		[19]

QUESTION 3

3.1.1	$5;10;20;....$ $T_n = a.r^{n-1}$ $T_n = (5)(2)^{n-1}$	✓ answer (1)
3.1.2	$S_n = \frac{a\left(r^n-1\right)}{r-1}$ $S_{18} = \frac{5\left[(2)^{18}-1\right]}{2-1}$ $S_{18} = 1310715$	✓ substitution into the correct formula ✓ answer (2)
3.2	$r = \frac{(2x+4)(2x-4)}{2x-4} = 2x+4$ Converge: $-1 < r < 1$ $-1 < 2x+4 < 1$ $-5 < 2x < -3$ $-\frac{5}{2} < x < -\frac{3}{2}$	✓ $r=2x+4$ ✓ $-1<r<1$ ✓ substitution ✓ answer (4)

3.3	$\frac{S_\infty}{S_2} = \frac{\frac{2}{1-\frac{1}{\sqrt{2}}}}{\frac{2\left(1-\left(\frac{1}{\sqrt{2}}\right)^2\right)}{1-\frac{1}{\sqrt{2}}}}$ $= \frac{1}{1-\frac{1}{2}}$ $= 2$	✓ S_∞ ✓ S_2 ✓ dividing ✓ answer (3)
		[10]

QUESTION 4

4.1	$x = 1$ $y = -2$	✓ $x = 1$ ✓ $y = -2$ (2)
4.2		✓ x-asymptote and y-asymptote ✓ x-intercept ✓ y-intercept ✓ form-decreasing (4)
4.3	$y = -x + c$ $-2 = -(1) + c$ $-1 = c$ $y = -x - 1$ OR $y - y_1 = -1(x - x_1)$ $y = -x + 1 - 2$ $y = -x - 1$ Answer only: full marks	✓ method ✓ answer (2)
4.4	$0 \leq x < 1$ **OR** $x \varepsilon [0\,;1)$	✓ 0 and 1 ✓ inequalities (2) ✓ 0 and 1 ✓ brackets (2)
		[10]

QUESTION 5

5.1	$y = a\left(\frac{1}{5}\right)^x - 5$ $-4 = a\left(\frac{1}{5}\right)^{-2} - 5$ $1 = a(25)$ $\frac{1}{25} = a$	✓ substitution ✓ simplification (2)
5.2	$0 = \frac{1}{25}\left(\frac{1}{5}\right)^x - 5$ $5 = \frac{1}{25}\left(\frac{1}{5}\right)^x$ $125 = \left(\frac{1}{5}\right)^x$ $5^3 = 5^{-x}$ $x = -3 \ \ (-3;0)$	✓ $y = 0$ ✓ simplifying ✓ answer (3)
5.3.1	$h: \ y = \left(\frac{1}{5}\right)^x$ $x = \left(\frac{1}{5}\right)^y$ $y = \log_{\frac{1}{5}} x$ or $y = -\log_5 x$ Answer only: full marks	✓ swop x and y ✓ answer (2)
5.3.2	$g: \ y = \left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^x - 5$ $g(x) = \left(\frac{1}{5}\right)^{x+2} - 5$ 2 units right and 5 units up	✓ rewriting g ✓ 2 units right and 5 units up (2)
		[9]

QUESTION 6

6.1	$y=\frac{1}{2}x+2$ $0=\frac{1}{2}x+2$ $x=-4$ $B(-4;0)$	✓ $y=0$ ✓ answer	(2)
6.2	$y=a(x-x_1)(x-x_2)$ $y=a(x+2)(x+6)$ $-12=a(0+2)(0+6)$ $-12=12a$ $a=-1$ $y=-(x^2+8x+12)$ $y=-x^2-8x-12$ **OR** $(-2;0):\ 0=a(-2)^2+b(-2)-12$ $12=4a-2b$ $6=2a-b$.....(1) and $\frac{-b}{2a}=-4$ $-b=-8a$ $b=8a$.....(2) $6=2a-8a$ $6=-6a$ $a=-1$ $b=8(-1)=-8$ $y=-x^2-8x-12$	✓ coordinates of A$(-6;0)$ ✓ substitution ✓ $a=-1$ ✓ answer ✓ substitution ✓ substitution ✓ $a=-1$ ✓ answer	(4) (4)

6.3	$FH = -x^2 - 8x - 12 - \left(\frac{1}{2}x + 2\right)$ $= -x^2 - 8x - 12 - \frac{1}{2}x - 2$ $= -x^2 - \frac{17}{2}x - 14$ $\frac{dFH}{dx} = -2x - \frac{17}{2} = 0$ or $x = -\frac{b}{2a}$ $-2x = \frac{17}{2}$ $x = -\frac{(-8,5)}{2(-1)}$ $x = -\frac{17}{4}$ $x = -\frac{17}{4}$ $G\left(-\frac{17}{4};0\right)$	✓ $f(x) - g(x)$ ✓ FH ito x ✓ method ✓ answer (4)
6.4	$-4 < x < 0$ **OR** $x\varepsilon(-4;0)$	✓ answer ✓ inequility (2) ✓ -4 and 0 ✓ inequility (2)
		[12]

QUESTION 7

7.1	$1 + i_{eff} = \left(1 + \frac{i^m}{m}\right)^m$ $1 + 0,113 = \left(1 + \frac{i^4}{4}\right)^4$ $1,027... = 1 + \frac{i^4}{4}$ $0,027... = \frac{i^4}{4}$ $0,1085.. = i^4$ $r = 10,85\%$	✓ substitution ✓ 4^{th} root ✓ answer (3)

7.2	$A = 10000\left(1+\frac{0,053}{12}\right)^{60}$ $A = 13026,71$ $F = \frac{500\left[\left(1+\frac{0,053}{12}\right)^{58}-1\right]}{\frac{0,053}{12}}$ $F = R32970,51$ $Total = 13026,71+32970,51$ $= R45997,22$ **OR** $= 10000\left(1+\frac{0,053}{12}\right)^{60}+\frac{500\left[\left(1+\frac{0,053}{12}\right)^{58}-1\right]}{\frac{0,053}{12}}$ $= R45997,22$	✓ substitution into the correct formula ✓ $n = 60$ and $i = \frac{0,053}{12}$ ✓ substitution into the F formula ✓ $n = 58$ ✓ answer (5) ✓ substitution into the correct formula ✓ $n = 60$ and $i = \frac{0,053}{12}$ ✓ substitution into the F formula ✓ $n = 58$ ✓ answer (5)
7.3.1	$860000 = \frac{7200\left[1-\left(1+\frac{0,095}{12}\right)^{-n}\right]}{\frac{0,095}{12}}$ $0,945... = 1-\left(1+\frac{0,095}{12}\right)^{-n}$ $-0,054... = -\left(1+\frac{0,095}{12}\right)^{-n}$ $0,054... = \left(1+\frac{0,095}{12}\right)^{-n}$ $-n = \log_{(1,007..)} 0,054...$ $-n = -369,212...$ Sam will have 370 installments	✓ substitution into the correct formula ✓ $i = \frac{0,095}{12}$ ✓ correct use of log ✓ answer (4)

7.3.2	$A = 860\,000\left(1+\frac{0,095}{12}\right)^{369}$ $A = R15\,782\,859,31$	✓ loan and interest ✓ $n = 369$
	$F = \frac{7\,200\left[\left(1+\frac{0,095}{12}\right)^{369}-1\right]}{\frac{0,095}{12}}$ $F = R15\,781\,334,69$	✓ payment and interest
	Balance $= R1524,62$ (after 369 installments)	✓ method
	Last installment $= 1524,62\left(1+\frac{0,095}{12}\right)^{1}$ $= R1536,69$	✓ answer (5)
	OR	
	$P = \frac{7\,200\left(1-\left(1+\frac{0,095}{12}\right)^{-0,2127679...}\right)}{\frac{0,095}{12}}$	✓ method ✓ $n = -0,2127679...$
	$P = R1524,62$ (after 369 installments)	✓ balance
	Last installment $= 1524,62\left(1+\frac{0,095}{12}\right)^{1}$ $= R1536,69$	✓ method ✓ answer (5)
		[17]

QUESTION 8

8.1	$f(x)=1-x^2$ $f(x+h)=1-(x+h)^2$ $=1-(x^2+2xh+h^2)$ $=1-x^2-2xh-h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{1-x^2-2xh-h^2-(1-x^2)}{h}$ $=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $f'(x)=-2x$	✓ $=1-x^2-2xh-h^2$ ✓ substitution ✓ $\frac{-2xh-h^2}{h}$ ✓ $-2x-h$ ✓ $-2x$ (5)
8.2.1	$D_x\left[3x^2-\frac{2}{x}\right]$ $D_x\left[3x^2-2x^{-1}\right]$ $=6x+2x^{-2}$	✓ $-2x^{-1}$ ✓ $6x$ ✓ $2x^{-2}$ (3)
8.2.2	$y=\sqrt{x}\left(\sqrt[3]{x}-5x\right)$ $y=x^{\frac{1}{2}}\left(x^{\frac{1}{3}}-5x\right)$ $y=x^{\frac{5}{6}}-5x^{\frac{3}{2}}$ $\frac{dy}{dx}=\frac{5}{6}x^{-\frac{1}{6}}-\frac{15}{2}x^{\frac{1}{2}}$	✓ change from surd to exponential form ✓ $x^{\frac{5}{6}}$ and $-5x^{\frac{3}{2}}$ ✓ $\frac{5}{6}x^{-\frac{1}{6}}$ ✓ $-\frac{15}{2}x^{\frac{1}{2}}$ (4)
		[12]

QUESTION 9

9.1	$f(x)=ax^3+bx^2$ $-\frac{1}{3}=a(2)^3+b(2)^2$ $-\frac{1}{3}=8a+4b.....(1)$ $f'(x)=3ax^2+2bx$ $f'(2)=3a(2)^2+2b(2)=0$ $12a+4b=0$ $4b=-12a$ $b=-3a.....(2)$ $-\frac{1}{3}=8a+4(-3a)$ $-\frac{1}{3}=8a-12a$ $-\frac{1}{3}=-4a$ $a=\frac{1}{12}$ $b=-\frac{1}{4}$	✓ substitution $\left(2\,;-\frac{1}{3}\right)$ ✓ $f'(x)=0$ ✓ substitution $x=2$ ✓ solve simultaneously (4)
9.2	$f''(x)=\frac{1}{2}x-\frac{1}{2}$ $\frac{1}{2}x-\frac{1}{2}<0$ $x<1 \quad / \quad x\varepsilon(-\infty\,;1)$	✓ $f''(x)$ ✓ $f''(x)<0$ ✓ answer (3)

9.3	$f(x)=\frac{1}{12}x^3-\frac{1}{4}x^2$ $f(-2)=\frac{1}{12}(-2)^3-\frac{1}{4}(-2)^2=-\frac{5}{3}$ $\left(-2;-\frac{5}{3}\right)$ $m=f'(x)=\frac{1}{4}x^2-\frac{1}{2}x$ $f'(-2)=\frac{1}{4}(-2)^2-\frac{1}{2}(-2)$ $m=2$ $y=2x+c$ $-\frac{5}{3}=2(-2)+c$ $c=\frac{7}{3}$ $y=2x+\frac{7}{3}$	✓ $y=-\frac{5}{3}$ ✓ gradient ✓ substitution of x,y and m ✓ answer (4)
9.4	$-\frac{1}{3}<k<0 \quad / \quad k\,\varepsilon\left(-\frac{1}{3};0\right)$	✓✓ answer (2)
		[13]

QUESTION 10

10.1	$\tan 60°=\frac{DF}{x}=\sqrt{3}$ $DF=\sqrt{3}x$ $\therefore$ Area rectangle $=DF\times DE$ $=\sqrt{3}x(m-2x)$	✓ $\tan 60°=\sqrt{3}$ ✓ $DF=\sqrt{3}x$ ✓ DE$=(m-2x)$ (3)

10.2	$\text{Area} = \sqrt{3}mx - 2\sqrt{3}x^2$	
	$\frac{dA}{dx} = \sqrt{3}m - 4\sqrt{3}x = 0$	✓ $f'(x) = 0$
	$x = \frac{m}{4}$	✓ $x = \frac{m}{4}$
	$\text{Max Area} = \sqrt{3}x(m - 2x)$	
	$= \sqrt{3}\left(\frac{m}{4}\right)\left(m - 2\left(\frac{m}{4}\right)\right)$	✓ substitution
	$= \frac{\sqrt{3}}{8}m^2$	✓ $\frac{\sqrt{3}}{8}m^2$ (4)
	OR	
	$\frac{dA}{dx} = \sqrt{3}m - 4\sqrt{3}x = 0$	✓ $f'(x) = 0$
	$x = \frac{m}{4}$	✓ $x = \frac{m}{4}$
	$\sqrt{3}mx - 2\sqrt{3}(x^2)$	
	$= \sqrt{3}m\left(\frac{m}{4}\right) - 2\sqrt{3}\left(\frac{m}{4}\right)^2$	✓ substitution
	$= \frac{\sqrt{3}m^2}{4} - \frac{\sqrt{3}m^2}{8}$	
	$= \frac{2\sqrt{3}m^2 - 8\sqrt{3}m^2}{8}$	
	$= \frac{\sqrt{3}m^2}{8}$	✓ $\frac{\sqrt{3}}{8}m^2$ (4)
		[7]

QUESTION 11

11.1.1	$P(A \text{ and } C) = 0$	✓ answer	(1)
11.1.2	$P(A \text{ and } B) = P(A) \times P(B)$ $P(A \text{ and } B) = 0,3 \times 0,43$ $P(A \text{ and } B) = 0,129$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = 0,3 + 0,43 - 0,129$ $P(A \text{ or } B) = 0,6$	✓ $P(A \text{ and } B) = 0,129$ ✓ substitution ✓ answer	(3)
11.2.1	$P(G \text{ and } T) = \frac{105}{250} = \frac{21}{50} = 42\%$	✓ answer	(1)
11.2.2	Independent: $P(T \text{ and } G) = P(T) \times P(G)$ $P(T) \times P(G)$ $\quad$ $P(G \text{ and } T)$ $= \frac{173}{250} \times \frac{130}{250}$ $\quad$ $= \frac{105}{250}$ $= 0.36$ $\quad$ $= 0.42$ $P(G \text{ and } T) \neq P(G) \times P(T)$ Events are not independent.	✓ $P(T) = \frac{173}{250}$ ✓ $P(G) = \frac{130}{250}$ ✓ $P(T) \times P(G) = 0,36$ ✓ answer	(4)
11.3.1	12!	✓ answer	(1)
11.3.2	Pieter and John next to one another = 10!2! 11!–10!2! = 32 659 200	✓ 10!2! ✓ 11!–10!2! ✓ answer	(3)
11.3.3	$\frac{11!2!}{12!}$ $= \frac{1}{6}$	✓ 11! 2! ✓ 12! ✓ answer	(3)
			[16]
		TOTAL:	**150**