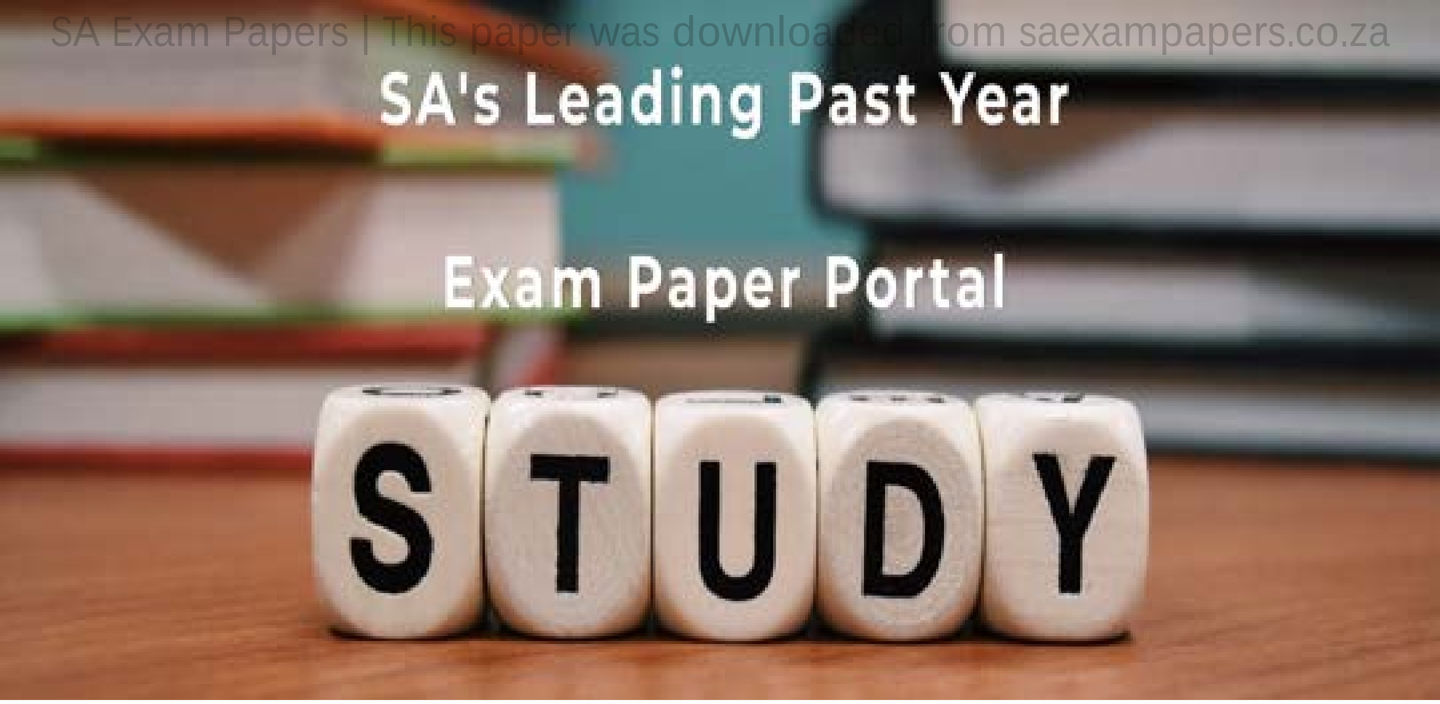
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GRADE 12

MATHEMATICS PAPER 1

2023 SEPTEMBER MOCK EXAMINATION

MARKS: 150

TIME: 3 HOURS

This paper consists of 12 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the question paper.

1. This Question Paper consists of 11 questions. Answer all questions.
2. Answer ALL the questions.
3. Clearly show all the calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
4. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
5. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
6. Number the answers correctly according to the numbering system used in this question paper.
7. Diagrams are not necessarily drawn to scale.
8. Full marks will not necessarily be given to answers only.
9. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

1.1 Given $x^2 + 2x = 0$

1.1.1 Solve for x (2)

1.1.2 Hence, determine the positive values of x for which $x^2 \geq -2x$ (3)

1.2 Solve for x:
$2x^2 - 3x - 7 = 0$ (correct to TWO decimal places) (4)

1.3 Given: $k + 5 = \frac{14}{k}$

1.3.1 Solve for k. (3)

1.3.2 Hence, or otherwise, solve for x if $\sqrt{x+5} + 5 = \frac{14}{\sqrt{x+5}}$. (3)

1.4 Solve for x and y simultaneously if:

$x - 2y - 3 = 0$ and

$4x^2 - 5xy + y^2 = 0$ (7)

1.5 The roots of a quadratic equation is given by $x = \frac{-2 \pm \sqrt{4-20k}}{2}$. Determine the value(s) of k for which the equation will have real roots. (2)

[24]

QUESTION 2

2.1 The first four terms of a quadratic sequence are: $1\ ;\ -5;\ -13;\ -23\ \ldots.$

2.1.1 Calculate the general term of the sequence. (4)

2.1.2 Which term has a value of -643? (3)

2.2 $2k+1\ ;\ 3k\ ;\ 5k-5$ are the first three terms of an arithmetic sequence

2.2.1 Calculate the value of k. (2)

2.2.2 Write down this sequence in sigma notation for the first 20 terms. (3)

2.3 Given the geometric series: $8x^2 + 4x^3 + 2x^4 + \ldots$

2.3.1 For what value(s) of x will the series converge? (3)

2.3.2 Calculate the values of x if $S_\infty = \frac{8}{3}$. (4)

2.4 The first, third and thirteenth terms of arithmetic sequence are the first 3terms of a geometric sequence. If the first term of both sequences is 1, determine:

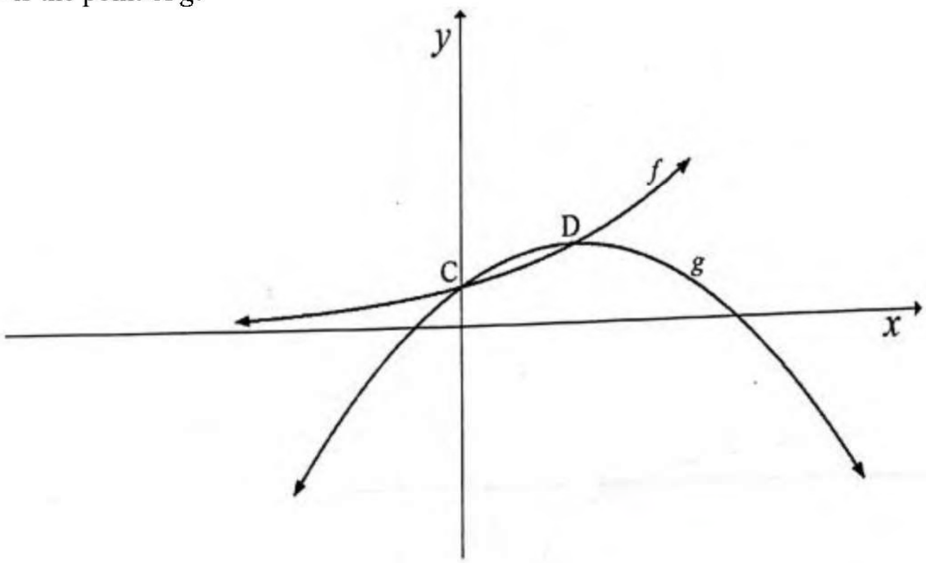
2.4.1 the first three terms of the geometric sequence if $r > 1$ (6)

2.4.2 the sum of 7 terms of the geometric sequence if the sequence is $1\ ;\ 5\ ;\ 25$. (2)

[27]

QUESTION 3

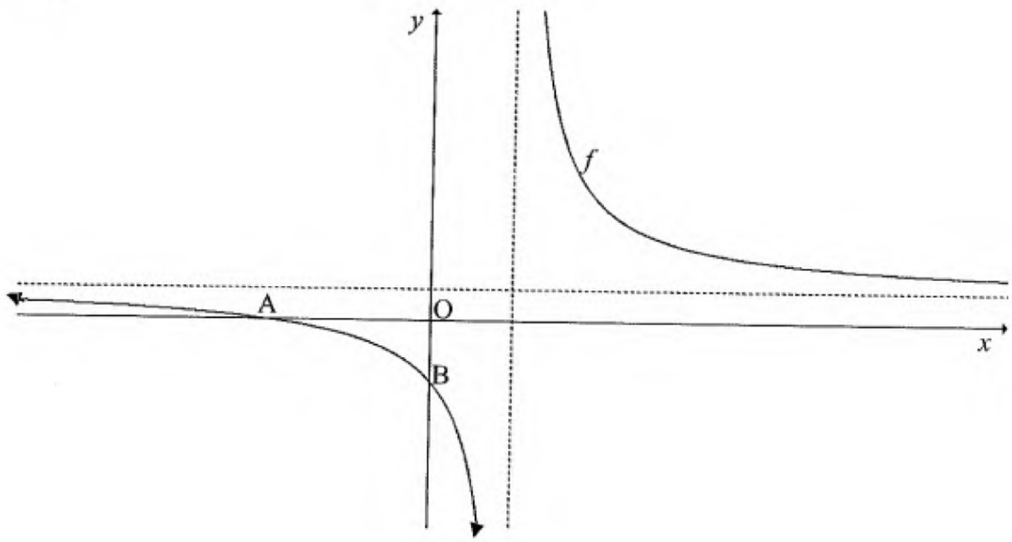
Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + q$, where q is a constant.
The graphs of f and g intersect at C and D.
C is the y-intercept of both f and g.
D is the point of g.



3.1	Show that $q = 2$.	(2)
3.2	Write down the coordinates of the turning point of g.	(2)
3.3	Determine the value(s) of t for $g(x) = t$ if the roots are equal.	(1)
3.4	Write down the equation of $f^{-1}(x)$ in the form $y = \ldots$	(2)
3.5	Sketch the graph of f^{-1} on the same system of axes. Indicate the x-intercept and the coordinates of one other point on your graph.	(3)
3.6	Write down the equation of h if $h(x) = g(x+1) - 2$.	(2)
3.7	How can the domain of h be restricted so that h^{-1} will be a function?	(1)
		[13]

QUESTION 4

The sketch below shows the graph of $f(x)=\frac{2+x}{x-1}$. A and B are the x- and the y-intercepts of f.



4.1	Write the equation of f in the form $f(x)=\frac{a}{x+p}+q$.	(3)
4.2	Determine the equations of the asymptotes of f.	(2)
4.3	Write down the coordinates of A, the x-intercept of f.	(2)
		[7]

QUESTION 5

5.1 Given: $f(x) = 3^{-x}$

5.1.1 Write the equation of $f^{-1}(x)$, the inverse of $f(x)$, in the form $f^{-1}(x) = \cdots$ **(2)**

5.1.2 On the same set of axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$ in your ANSWER BOOK. Clearly indicate the intercepts with the axes and one other point on the graphs and label the TWO graphs. [4]

5.2 A linear function satisfies the following conditions: $p(-3) = 10$ and $p'(x) = -2$. Determine the inverse of p in the form $p^{-1}(x) = \cdots$ (4)

[10]

QUESTION 6

6.1 On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018.

Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (3)

6.2 On 1 February 2018, Genevieve took a loan of R82 000 from the bank to pay for her studies. She will make her first repayment of R3 200 on 1 February 2019 and continue to make payments of R3 200 on the first of each month thereafter until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.

6.2.1 Calculate how much Genevieve will owe the bank on 1 January 2019. (3)

6.2.2 How many instalments of R3 200 must she pay? (5)

6.2.3 Calculate the final payment, to the nearest rand, Genevieve has to pay to settle the loan. (5)

[16]

QUESTION 7

7.1 If $f(x) = -2x^2$, determine $f'(x)$ from first principles. (5)

7.2 Determine:

7.2.1 $\frac{dy}{dx}$ if $y = \frac{2x^2-1}{\sqrt{x}}$ (3)

7.2.2 $D_x[(3x-2)^2$ (3)

7.3 Given: $y = \frac{1}{x^2}$. (3)

Prove that the gradient of the curve is negative at each point on the curve where $x > 0$.

[14]

QUESTION 8

The following information about a cubic polynomial, $y = f(x)$, is given:

- $f(-1) = 0$
- $f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(-1) = f'(1) = 0$
- if $x < -1$ then $f'(x) > 0$
- if $x > 1$ then $f'(x) > 0$

8.1 Use this information to draw a neat sketch graph of f using the grid on the DIAGRAM SHEET (5)

8.2 For which value(s) of x is f decreasing? (2)

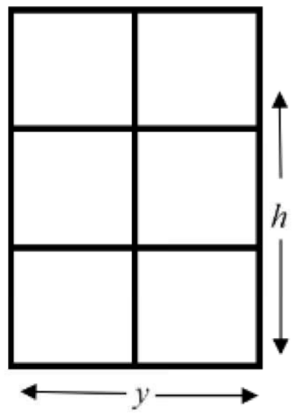
8.3 Use your graph to determine the x-coordinate of the point of inflection of f. (2)

8.4 For which value(s) of x is f concave up? (2)

[11]

QUESTION 9

A window frame with dimensions y by h is illustrated below.
The frame consists of six smaller frames.



9.1 If 12 m of material is used to make the entire frame, show that $y = \frac{1}{4}(12 - 3h)$. (2)

9.2 Show that the area of the window is given by $A = 3h - \frac{3}{4}h^2$ (3)

9.3 Find $\frac{dA}{dh}$ and hence the dimensions, h and y, of the frame so that the area of the window is a maximum. (5)

[10]

QUESTION 10

Consider the word **"CALCULATOR"**

10.1 How many different word arrangements can be made from all the letters of the word CALCULATOR? (2)

10.2 What is the probability of making a word arrangement that will start and end with the letter L? (2)

10.3 In how many ways can all the letters be arranged if no similar letters should be close to each other? (3)

[7]

QUESTION 11

11.1

Given $P(A) = 0,5$; $P(B) = x$ and $P(A \text{ or } B) = 0,88$.

Calculate the value(s) of x, if

11.1.1 Event A and B are mutually exclusive. (2)

11.1.2 Event A and B are dependent. (3)

11.2 South African Women Football Team known as "Banyana Banyana" are to play a friendly football match against their Nigerian counterparts known as "Super Falcons". There should be a winner on the day of the match. Penalties would decide the winner in case the match is drawn after regulation time. On the day the match takes place, there is a 30% chance that it could rain (R), 45% chance that it could be sunny(S) or it could be cloudy(C). Banyana Banyana has a 24% of winning on a rainy day, a 65% of winning on a sunny day or a 33% chance of wining on a cloudy day.

11.2.1 Draw a tree diagram to represent all outcomes of the above scenario. (3)

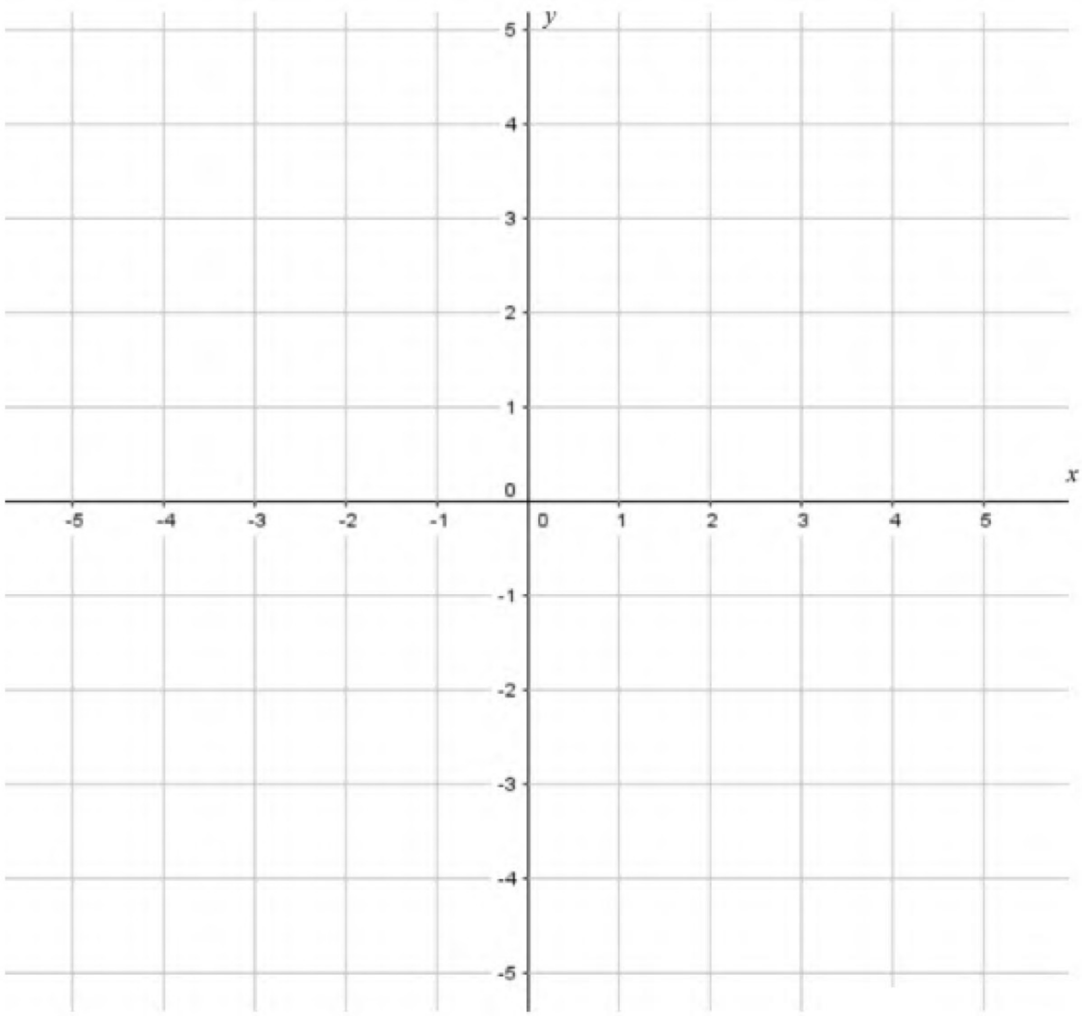
11.2.2 What is the probability of Banyana Banyana winning the match? (3)

[11]

Name and Surname: …………………………………………………………………
Class: …………………………………………

DIAGRAM SHEET

QUESTION 8.1



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}\ ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$