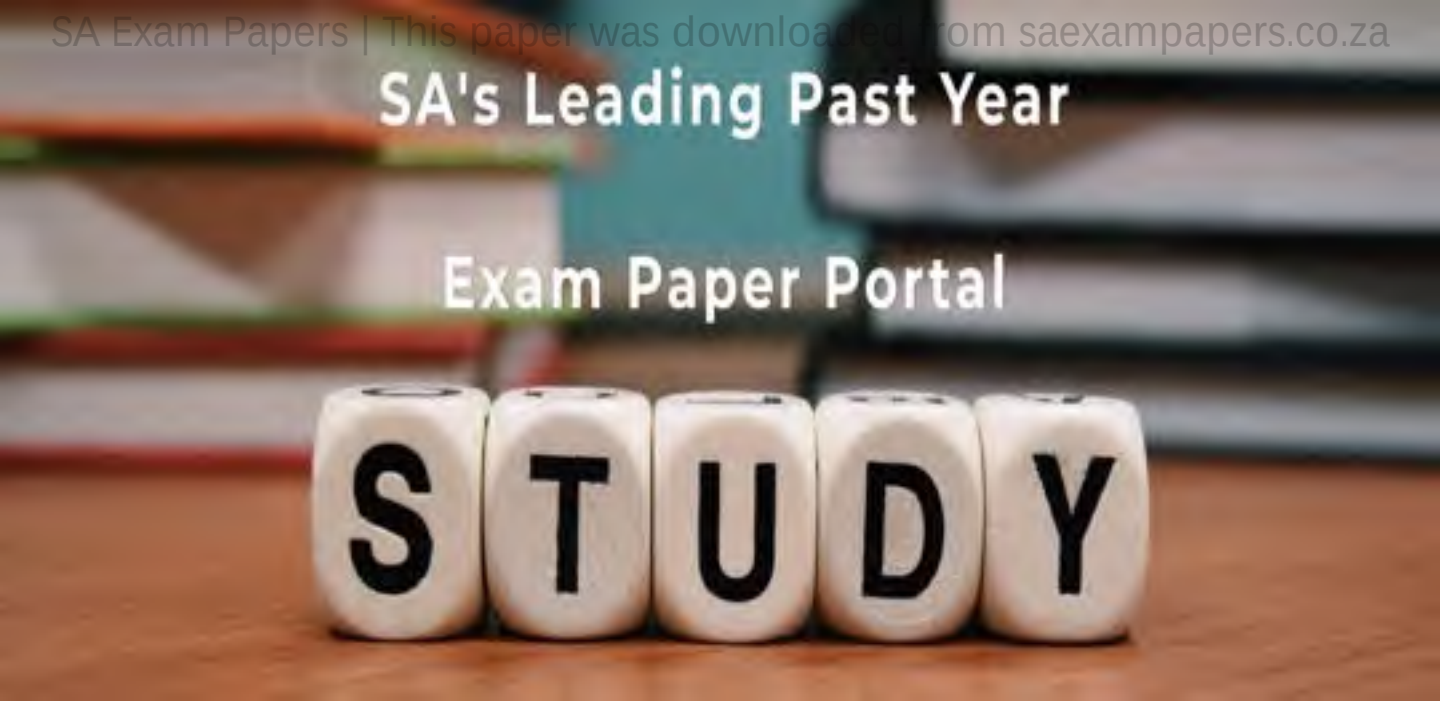


KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2023

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 12 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-4)=0$ (2)

1.1.2 $2x^2+3x=7$ (write your answer to two decimal places). (4)

1.1.3 $x^2-5x+4>0$ (3)

1.1.4 $3^{2x}-10.3^x+21=0$ (5)

1.2 Solve simultaneously for x and y: (6)

$x+y=2$; $x^2+y^2+6x=4y-4$

1.3 The roots of a quadratic equation are $x=\dfrac{-4\pm\sqrt{25-n^2}}{6}$.

For which values of n will the roots be equal? (3)

[23]

QUESTION 2

2.1 Consider the arithmetic sequence:

3 ; 7 ; 11 ; ...; 399.

2.1.1 Determine the twentieth term of the sequence. (2)

2.1.2 How many terms are in this sequence? (2)

2.2 The first term of an arithmetic sequence is a and the thirteenth term $a+24$.

2.2.1 Determine the common difference of the sequence. (2)

2.2.2 Hence determine the sum of the first 200 terms in terms of a. (2)

[8]

QUESTION 3

The values below are the consecutive terms of a quadratic sequence.
The fourth term is 49.

$-\ ;\ -\ ;\ -\ ;\ 49\ ;\ 77\ ;\ 111\ ;\ 151;$ .....

3.1 Determine the third term of the quadratic sequence. (1)

3.2 Determine the general term, T_n of the quadratic sequence. (4)

3.3 Between which two consecutive terms of the quadratic sequence is the first difference 418? (3)

[8]

QUESTION 4

4.1 The first two terms of a geometric sequence are x and $x+1$.

4.1.1 Write down the common ratio. (1)

4.1.2 Write down the third term. (2)

4.1.3 If $x=2$, will the sequence converge? Motivate your answer. (2)

4.2 The given sequence below has **four terms only**, such that the first three terms, $T_1\ ;\ T_2$ and T_3 form an arithmetic sequence and the last three terms $T_2\ ;\ T_3$ and T_4 form a geometric sequence:

$$6\ ;\ a\ ;\ b\ ;\ 16.$$

Calculate the values of a and b. (5)

[10]

QUESTION 5

Given: $g(x)=\dfrac{-3}{3-x}-1$

5.1 Write down the equations of the asymptotes of g. (2)

5.2 Calculate the x and y intercepts of g. (3)

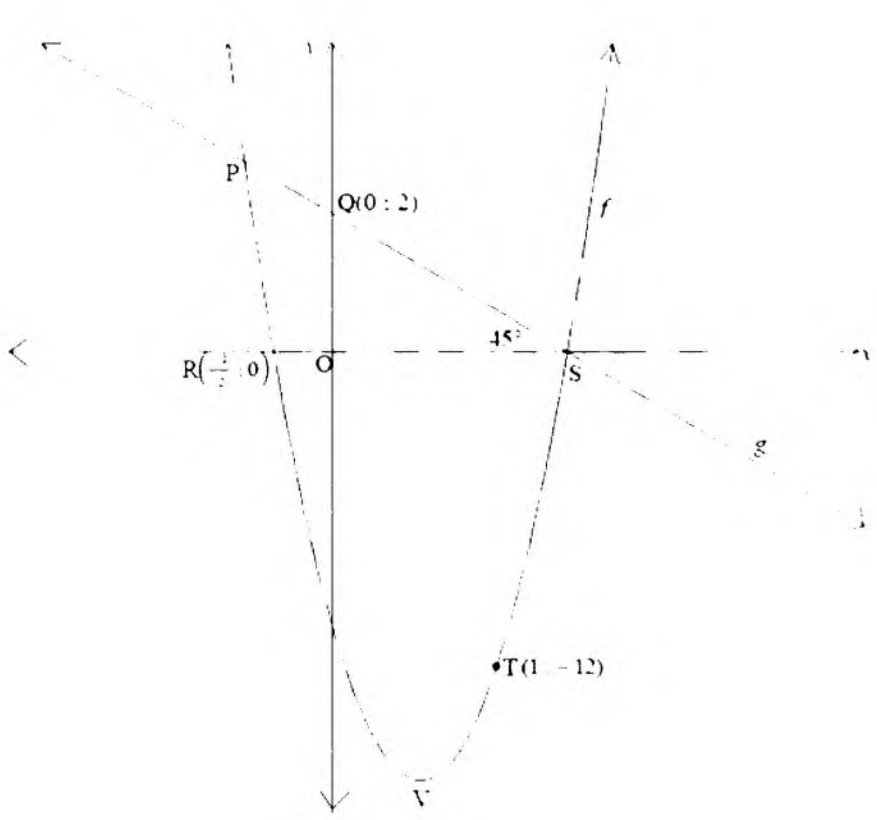
5.3 Draw the graph of g showing all asymptotes and intercepts with the axes (3)

5.4 Determine the equation of the axis of symmetry of g with a negative gradient. (2)

[10]

QUESTION 6

The graphs of a parabola f and a straight line g intersect at P and S as shown below.
Points R $\left(-\frac{1}{2};0\right)$ and T$(1;-12)$ are on the parabola, and Q$(0;2)$ is a point on g.
V is the turning point of the parabola. The straight line makes an angle of 45^0 with the x- axis.



6.1 Show that the equation of g is $g(x)=-x+2$. (2)

6.2 Write down the coordinates of S. (2)

6.3 Show that the equation of f is $f(x)=8x^2-12x-8$. (4)

6.4 Determine the coordinates of V the turning point of f. (3)

6.5 Use the graph to determine the values of k, if $f(x)=k$ has two distinct unequal roots. (1)

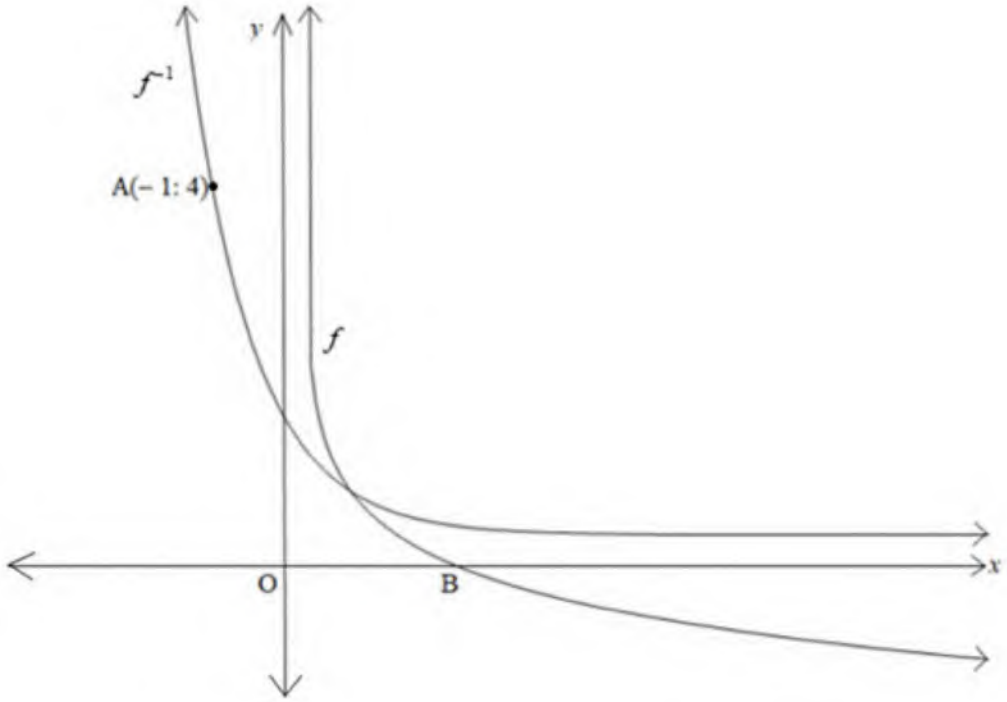
6.6 For which values of x is $f'(x).g'(x)<0$? (1)

6.7 A new graph h is obtained by first reflecting the graph of f about the y - axis and then shifting it in the upward direction so that the x - axis is the tangent of the new graph h. Write down the coordinates of point T′ (the image of T) which lies on the graph of h. (2)

[15]

QUESTION 7

The diagram shows the graphs of $f(x) = \log_a x$ and f^{-1}. Point $A(-1;4)$ lies on the graph of the inverse of f.



7.1 Write down the coordinates of A', the image of A after reflection in the $y = x$. (1)

7.2 Show that the value of a is $\frac{1}{4}$. (2)

7.3 Determine the equation of f^{-1} in the form $y = ...$ (1)

7.4 Calculate the length of AB. (3)

7.5 Write down the value(s) of x for which $f(x) < -1$. (1)

[8]

QUESTION 8

8.1 Minenhle bought a laptop for R15 800. The laptop depreciates at 12% p.a. on a reducing balance. After how many years will the value of this laptop be R10 767,26? (3)

8.2 Calculate the effective yearly interest rate if an investment offers a nominal interest rate of 7,64% p.a compounded half yearly. (2)

8.3 Mandla is planning to buy a car in 2 years' time. He invests R500 in a savings account at the end of each month for 2 years, starting in one month's time so that he can use the money as a deposit for his car. Interest is calculated at 5,8% p.a. compounded monthly.

8.3.1 How much money will be in his savings account at the end of 2 years? (3)

8.3.2 The cost of the car is R368 400. Mandla uses all the money from his savings account as the deposit for his new car. He takes out a loan to pay off the balance after paying the deposit. Interest is calculated at 10,4% p.a. compounded monthly. Calculate Mandla's monthly instalments if the car must be paid for in 6 years. (4)

8.3.3 Calculate the outstanding balance immediately after paying the 56^{th} instalment. (3)

[15]

QUESTION 9

9.1 From first principles, determine the derivative of $f(x)=2x^2+9$. (5)

9.2 Determine the following:

9.2.1 $\frac{dy}{dx}$ if $y=x(2x+1)$. (2)

9.2.2 $\frac{dy}{dx}$ if $\sqrt{y+x}=x+3$ (3)

9.2.3 $\frac{d}{dx}\left[\frac{4+\sqrt{3x}}{x}\right]$. (3)

9.3 The derivative of a function f at point P is given below:

$$\lim_{h\to 0}\frac{\left[(2+h)^3+1\right]-\left[2^3+1\right]}{h}=12$$

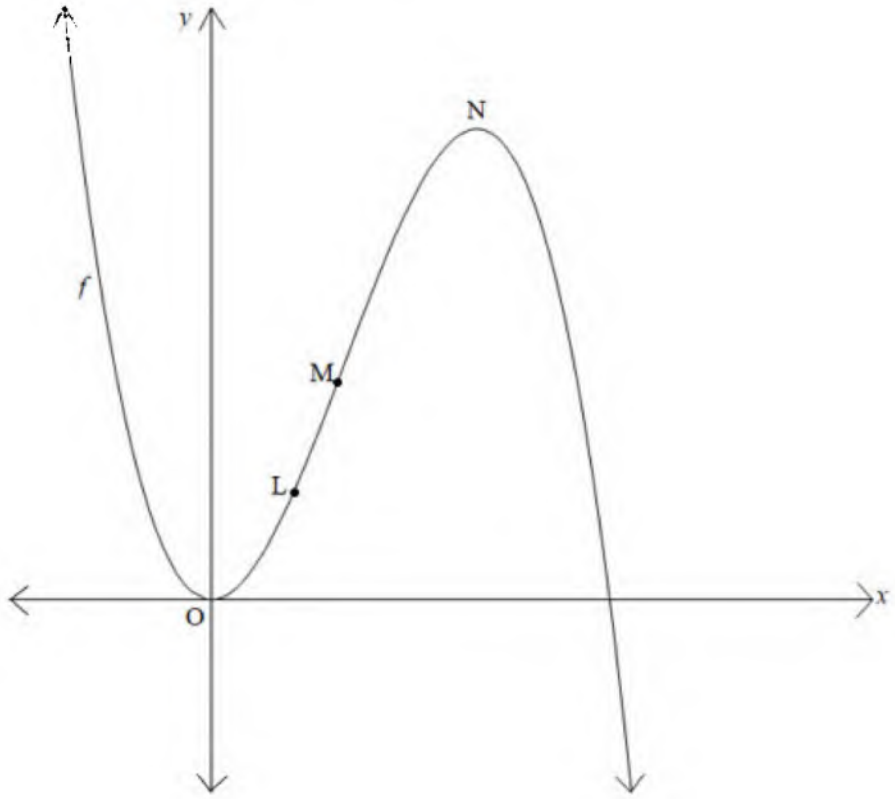
9.3.1 Write down the equation of f. (2)

9.3.2 A tangent is drawn at P. Determine the equation of the tangent to f at point P. (3)

[18]

QUESTION 10

The diagram below shows the graph of $f(x)=ax^3+bx^2$. The x-coordinates of L and M, are 1 and 2 respectively. The average gradient of f between L and M is 5,5. The equation of the tangent to the curve of f at $x=6$ is $y=-18x+c$.



10.1 Show that $a=-\frac{1}{2}$ and $b=3$ (5)

10.2 The equation $f(x)=-\frac{1}{2}x^3+3x^2$ is given. Determine the coordinates of N, the turning point of f. (4)

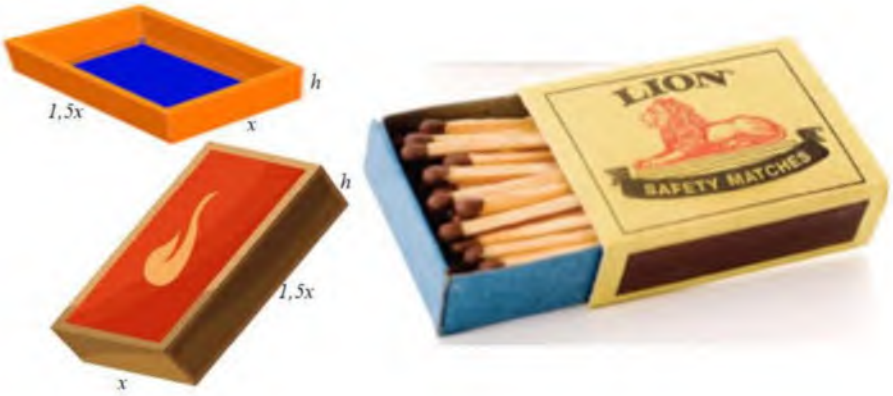
10.3 The graph of f is concave up for $x<k$. Calculate the value of k. (3)

[12]

QUESTION 11

A match box consists of an outer cover, open at both ends, into which slides a rectangular box without a top.

- The length of the box is one and a half times its breadth.
- The thickness of the material is negligible.
- The volume of the box is 25cm^3.



If the breadth of the box is x cm:

11.1 Write down the height of the box in terms of x. (2)

11.2 Show that the amount of the material used for the box is given by: (2)

$$A = \frac{100}{x} + 4,5x^2 + \frac{50}{1,5x}$$

11.3 Hence determine the breadth of the box such that the material used is a minimum. (3)

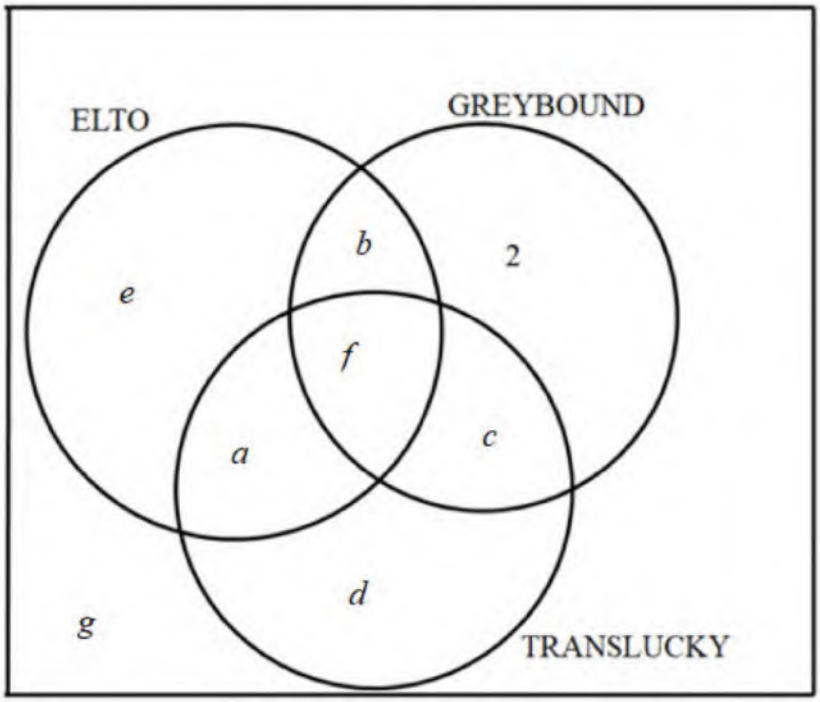
[7]

QUESTION 12

A group of 40 people was asked which bus company they liked to travel by: Elto, Greybound or Translucky bus company.

- 3 people liked travelling by all 3 bus companies.
- 7 people liked to travel by Translucky and Elto.
- 3 people did not like using any of the bus companies.
- The probability of a randomly selected person liking to travel by Greybound and Elto is $\frac{2}{5}$.
- The probability of a randomly selected person liking to travel by Greybound and Translucky is $\frac{1}{5}$.
- The probability of a randomly selected person liking to travel by Translucky only is $\frac{1}{10}$.
- The probability of a randomly selected person liking to travel by Elto is $\frac{13}{20}$.

The partially completed Venn diagram drawn below represents the given information.



12.1 Use the given information to determine the values of *a, b, c, d* and *e*. (5)

12.2 How many people liked to travel by Greybound bus company? (1)

12.3 Calculate the probability of a randomly selected person liking to travel by only one bus company. (2)

[8]

QUESTION 13

Nonhle who is a Grade 12 learner has 8 textbooks from eight different subjects: Mathematics, English, Accounting, History, Tourism, Afrikaans, Geography and Drama which she wants to arrange in a line on a shelf.

13.1 In how many ways can the textbooks be arranged? (1)

13.2 In how many ways can the textbooks be arranged if the Mathematics textbook and the Accounting textbook must be on each end of the shelf? (3)

13.3 If the Mathematics textbook and the Accounting textbook must be on each end of the shelf, what is the probability that the History textbook and the Tourism textbook are not next to each other? (4)

[8]

TOTAL MARKS: 150

INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum f.x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A \text{ or } B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \text{ and } B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$