education

Department of Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P1

SEPTEMBER 202

Gr 12 MATHEMATICS P1

12611B

TIME: 3 HOURS

MARKS: 150

This question paper consists of 9 pages and 1 information sheet.

X10

P2

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number your answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answer only will NOT necessarily be awarded full marks.
6. An approved scientific calculator (non-programmable, non-graphic) may be used, unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-1)(2x+1)=0$. (2)

1.1.2 $(x-1)(2x+1)=4$ (correct to two decimal places) (4)

1.1.3 $x+\sqrt{x-2}=4$ (5)

1.1.4 $3x^2+x\leq 0$ (3)

1.2 Solve for x and y in the following simultaneous equations:

$xy=8$ and $2x+y=17$ (6)

1.3 Simplify the following WITHOUT USING A CALCULATOR :

$\sqrt{\sqrt{21x^2}-\sqrt{5x^2}}\times\sqrt{\sqrt{21x^2}+\sqrt{5x^2}}$ (3)

[23]

QUESTION 2

2.1 Consider the series: $a+(a+d)+(a+2d)+...$

2.1.1 Prove that the sum of the first n terms of this arithmetic series will be $S_n=\frac{n}{2}[2a+(n-1)d]$. (3)

2.1.2 Given: $2^x+2.2^x+3.2^x+...$ The sum of the first 20 terms of the series is 1680. Calculate the value of x. (4)

2.2 Given: $S_n=\frac{n^2+n}{4}$, calculate T_8. (3)

2.3 Consider the series: $32+(-16)+8+(-4)+...$
Calculate the sum of the first 10 terms of the series. (3)

[13]

QUESTION 3

3.1 Given the quadratic number pattern $-4; -6; -10; -16; \ldots$

3.1.1 Determine T_n. (4)

3.1.2 Between which consecutive terms of the pattern is the difference -100? (4)

3.2 Given that $\sum_{n=1}^{\infty}\left(k-\frac{3}{2}\right)^n = -\frac{5}{3}$. Calculate the value of k. (4)

[12]

QUESTION 4

Given the function: $h(x) = -\frac{2}{x-2} + 2$.

4.1 Write down the equations of the asymptotes of h. (2)

4.2 Calculate the x-intercept of h. (2)

4.3 Draw the graph of h. Clearly show all asymptotes and intercepts with the axes. (4)

4.4 Determine the equation of the axis of symmetry of h, in the form $y = mx + c$, where $m < 0$. (3)

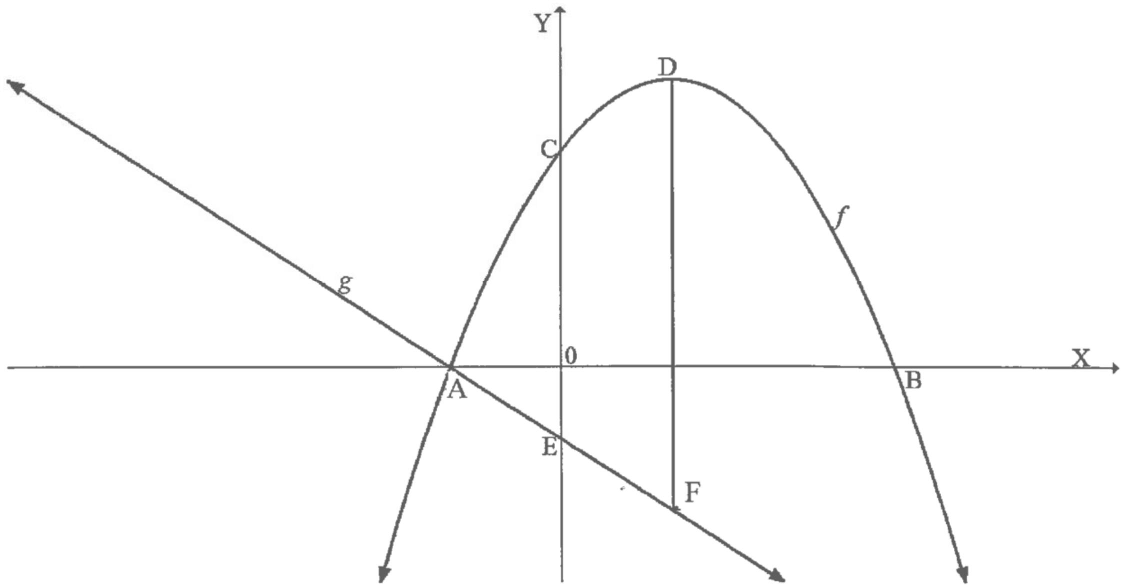
4.5 Determine the range of $-h(x)+1$ (2)

4.6 Determine the values of x where $h(x) \leq 0$. (2)

[15]

QUESTION 5

In the diagram, the graphs of $f(x) = -\frac{1}{2}x^2 + 2x + 6$ and $g(x) = -x - 2$ are drawn. C and E are the y-intercepts of f and g, respectively. The parabola has a turning point at D and cuts the x-axis at A and B. A is also the x-intercept of g. DF is a line parallel to the y-axis with F a point on g.



5.1 Calculate the:

5.1.1 Coordinates of D. (3)

5.1.2 Distance DF. (3)

5.2 For which values of k will $f(x) = k$ have two positive roots? (2)

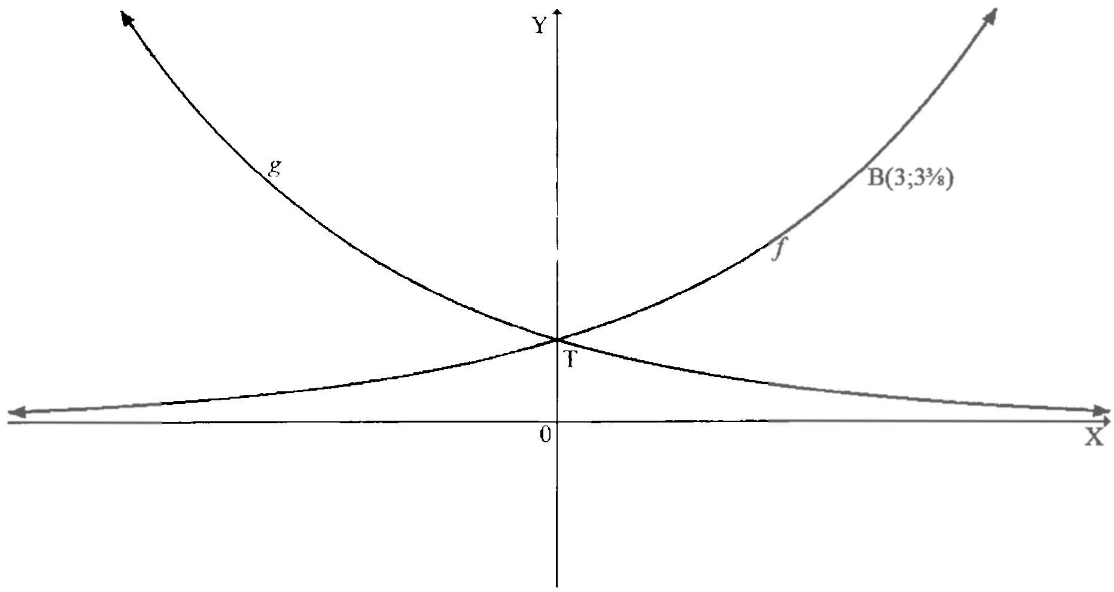
5.3 Given that $f(x) = h'(x)$. Determine the x-coordinates of the turning points of h. (3)

5.4 Determine the value(s) of x where $f'(x) \times g(x) \leq 0$. (2)

[13]

QUESTION 6

The diagram shows the graphs of $f(x) = a^x$ and g, the reflection of f in the y-axis.
$B\left(3;3\frac{3}{8}\right)$ lies on f. The two graphs intersect at T.



6.1 Write down the coordinates of T. (1)

6.2 Calculate the value of a. (2)

6.3 Determine the equation of g. (2)

6.4 Write down the equation of $f^{-1}(x)$, the inverse of f, in the form $y = ...$ (2)

6.5 For which values of x will $f^{-1}(x) \leq 1$? (3)

[10]

QUESTION 7

7.1 How many years will it take to triple an investment if the interest is compounded annually at a rate of 9,8% p.a.? (3)

7.2 Andile needs R64 000 for a holiday. He started to invest a fixed amount of his salary at a rate of 8,5% p.a. compounded monthly, at the end of each month, for ten years.

7.2.1 Calculate the monthly payment he will have to make to achieve this. (3)

7.2.2 If Andile has stopped his payment at the end of eight years, what will the total of his investment be at the end of ten years? (3)

7.3 Madri took out a loan of R400 000 at an interest rate of 10,4% p.a. compounded monthly. She repaid the loan at the end of the first month and every month for 15 years. Her monthly instalment is R4 396,83.

7.3.1 Calculate the outstanding balance after nine years. (3)

7.3.2 How much interest did she pay over the nine years? (3)

[15]

QUESTION 8

8.1 Determine the derivative of $f(x) = 3 - x^2$ using FIRST PRINCIPLES. (5)

8.2 Determine:

8.2.1 $D_x\left[\frac{2}{x} - \sqrt{x}\right]$ (4)

8.2.2 $\frac{dy}{dx}$, if $y = (x^3 - 1)^2$ (3)

8.3 Given: $f(x) = x^3 - 12x - 16$

8.3.1 Calculate the:

(a) Coordinates of the turning points of f. (5)

(b) x-intercepts of f. (3)

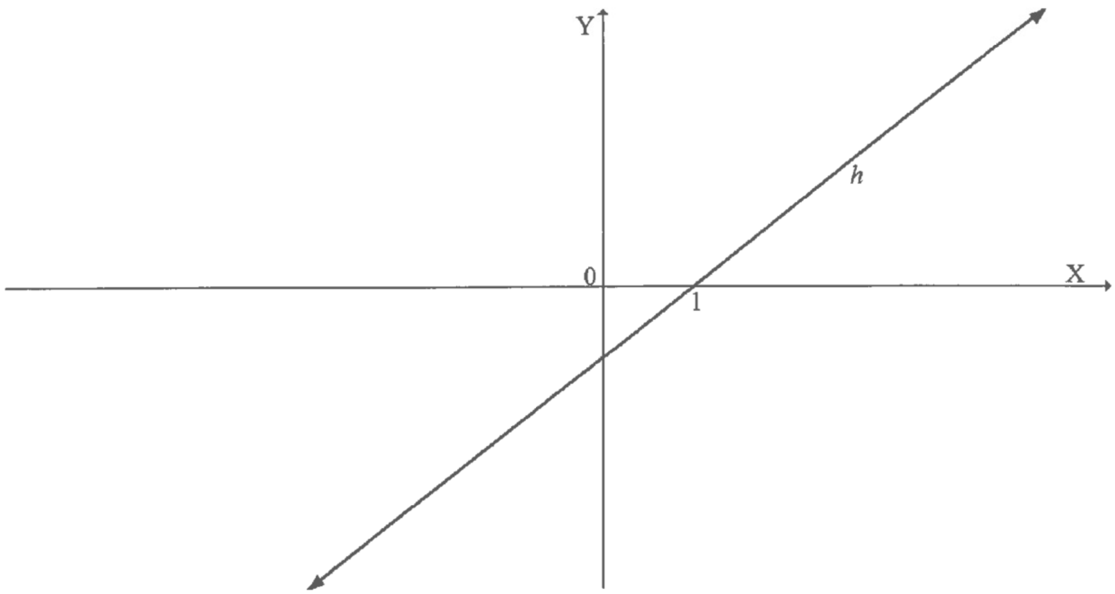
8.3.2 $y = 15x + p$ is a tangent to the graph of f. Calculate the x-coordinates of the point(s) of contact. (4)

8.3.3 For which value(s) of x will the given function be concave up? (3)

[27]

QUESTION 9

9.1 The diagram shows the straight-line h, where $h(x) = f'(x)$.
The x-intercept of h is 1.
The following is true for function f: $f(1) = -3$ and $f(3) = 0$.



Draw a sketch graph of the function f, clearly indicating all x-intercepts and turning point(s). (3)

9.2 During an experiment the temperature, T in $°C$ varies with time t in seconds, to the equation $T(t) = 60 + 27t - t^3$, $t \in [0;6]$.

Calculate:

9.2.1 The average change of temperature between 3 and 6 seconds. (3)

9.2.2 After how many seconds the temperature will be a maximum. (3)
[9]

QUESTION 10

10.1 Given: $P(A) = 0{,}4$ and $P(B) = 0{,}5$

10.1.1 Calculate $P(A \text{ or } B)$ if A and B are mutually exclusive events. (2)

10.1.2 Calculate $P(A \text{ or } B)$ if A and B are independent events. (3)

10.2 A four-digit code must be set using the letters A, E, I, O, U and digits 0 to 9.
The letters may be repeated, but the digits may not be repeated.
The code must consist of two letters and two digits in that order, for example, UO19.

10.2.1 How many different codes are possible with the information given? (4)

10.2.2 What is the probability that a code that is picked randomly will start with an A and be an even number? (4)

[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$