FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

MARKING GUIDELINE

COMMON TEST

JUNE 2023

MARKS: 150

QUESTION 1

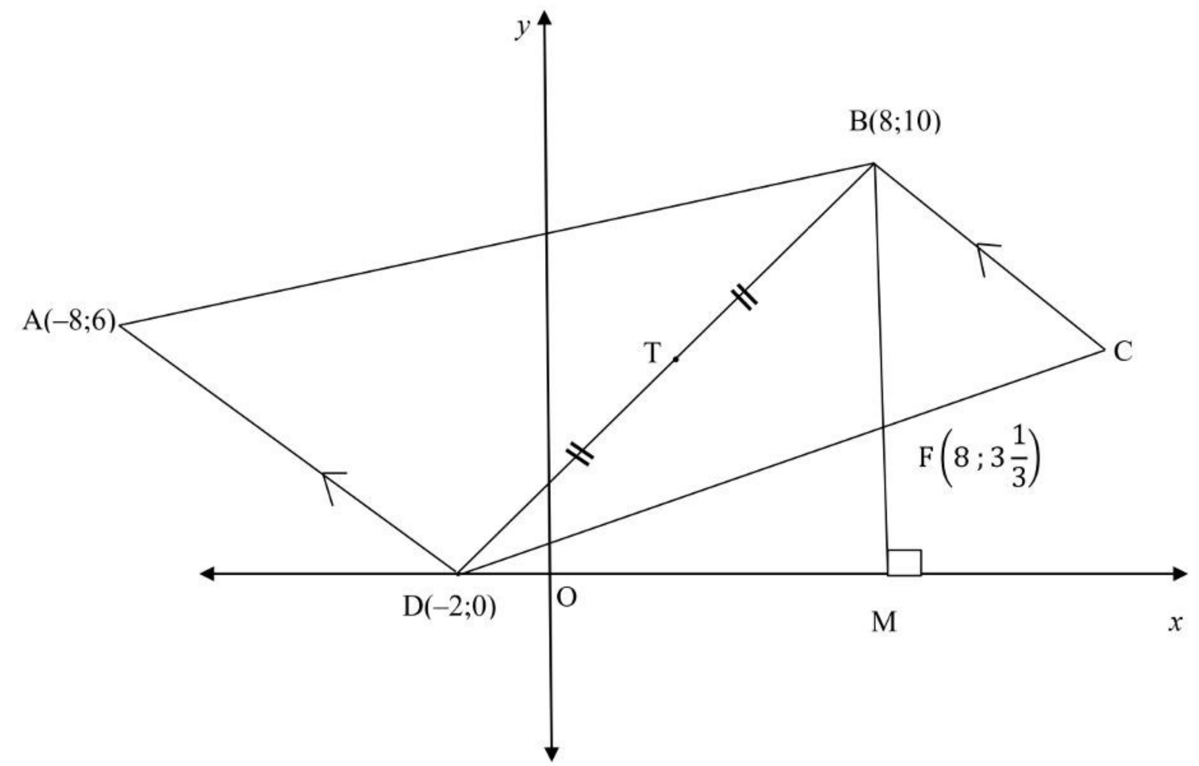
1.1	$g = 42$ Minimum : $a = 42 - 35 = 7$ $d = 23$ $f - 23 = 14$ ..Therefore $f = 37$ $f - b = 22$ …Therefore $b = 15$ $\frac{7 + 15 + c + 23 + 2c + 37 + 42}{7} = 25$ $\frac{3c + 124}{7} = 25$ $3c + 124 = 175$ $3c = 51$ $c = 17$ Therefore $e = 34$	A✓ Value of g A✓ Value of a A✓ Value of f A✓ Value of d CA✓ Value of b CA✓ Setting up equation CA✓ Simplifying numerator CA✓ equation in c CA✓ Value of c CA✓ Value of e	(10)
			[10]

QUESTION 2

Time (in minutes)	Number of Learners	Midpoint of Interval(x)	f.x	Cf
$0 < t \leq 10$	5	**5**	**25**	**5**
$10 < t \leq 20$	8	**15**	**120**	**13**
$20 < t \leq 30$	18	**25**	**450**	**31**
$30 < t \leq 40$	7	**35**	**245**	**38**
$40 < t \leq 50$	2	**45**	**90**	**40**
Total	**40**		**930**	

2.1	Estimated Mean = $\frac{930}{40}$ = 23,25	A✓930 A✓40 CA✓Answer (only if denominator is 40) **(Answer only full marks)**	(3)
2.2	DO NOT MARK THIS QUESTION		
2.3	60 % of 50 minutes = 30 minutes 9 learners	A✓Calculation A✓30 minutes A✓Answer **(AO full marks)**	(3)

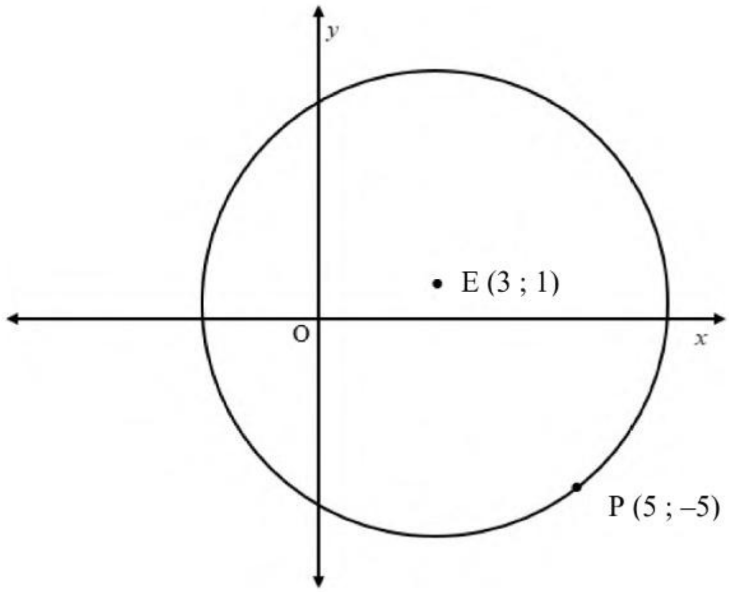
QUESTION 3



3.1	$mAD = \frac{6-0}{-8+2} = \frac{6}{-6} = -1$	A✓subst. of points A and D CA✓Answer	(2)
3.2	$mAD = mBC = -1$ …..(AD ∥ BC) $y = mx + c$ $10 = -1(8) + c$ $18 = c$ $y = -x + 18$	CA✓gradient of BC CA✓subst. of point C and gradient CA✓Answer	(3)
3.3	$mBD = \frac{10-0}{8+2} = \frac{10}{10} = 1$ Since the product of the gradients of lines AD and BD = –1 Therefore BD ⊥ AD	A✓subst. of points A and D A✓Gradient of BD A✓Justification	(3)
3.4	$\tan\theta = 1$ $B\hat{D}M = 45^\circ$	CA✓$\tan\theta = 1$ CA✓Answer	(2)
3.5	T(3 ; 5) … midpoint C(13 ; 5)	A ✓coordinates CA CA✓✓coordinates of C	(3)

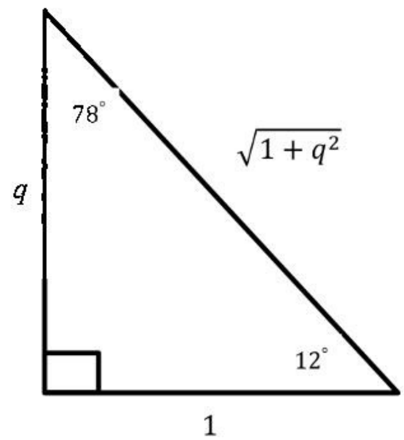
3.6	$BF = \sqrt{(8-8)^2 + \left(10 - 3\frac{1}{3}\right)^2} = 6\frac{2}{3} = \frac{20}{3}$ Area of $\Delta BDF = \frac{1}{2}(10)\left(\frac{20}{3}\right) = \frac{100}{3} = 33\frac{1}{3}$ square units. **OR** Area of Δ BDF = Area of Δ DBM – area of Δ DFM $= \frac{1}{2}(10)(10) - \frac{1}{2}(10)\left(\frac{10}{3}\right)$ $= \frac{100}{33} = 33{,}3$ units ^{2} **OR** $m_{DF} = \frac{1}{3}$ $\tan\ F\hat{D}M = \frac{1}{3}$ $F\hat{D}M = 18.43^\circ$ $BD = 10\sqrt{2}$ $DF = \frac{10\sqrt{10}}{3}$ $\therefore$ Area of Δ BDF $= \frac{1}{2}\ 10\sqrt{2}\ \text{x}\ \frac{10\sqrt{10}}{3}\ \sin 26.57^\circ$ $= 33{,}3$ units^2	A✓Substitution A✓ BF value CA✓Subst. into formula CA✓Simplification CA✓Answer	(5)
			[18]

QUESTION 4

4.1			
4.1.1	$EP = \sqrt{(3-5)^2 + (1+5)^2}$ $EP = \sqrt{(-2)^2 + (6)^2}$ $EP = \sqrt{40}$ $(x-3)^2 + (y-1)^2 = 40$ $x^2 - 6x + 9 + y^2 - 2y + 1 = 40$ $x^2 + y^2 - 6x - 2y - 30 = 0$	A✓Substitution CA✓EP value CA✓Substitution CA✓Answer	(4)
4.1.2	Radius: $m = \dfrac{1+5}{3-5} = -3$ Tangent: $m = \dfrac{1}{3}$ $y = mx + c$ $-5 = \dfrac{1}{3}(5) + c$ $-15 = 5 + 3c$ $-20 = 3c$ $c = -6\dfrac{2}{3}$ $y = \dfrac{1}{3}x - 6\dfrac{2}{3}$	A✓Gradient of radius CA✓Gradient of tangent (provided it is positive) CA✓Subst. Of point and gradient. CA✓c – value CA✓Answer	(5)

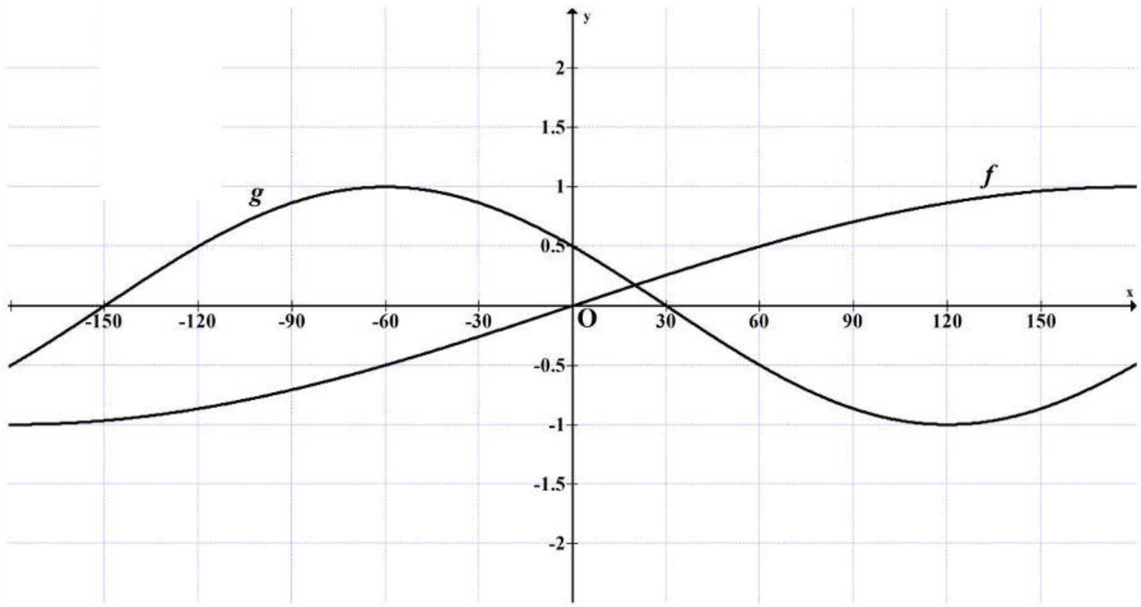
4.2.1	Coordinates of Centre of smaller circle: $\left(\frac{3+5}{2}\,;\frac{1-5}{2}\right) = (4;-2)$	A✓ midpoint formula A✓Substitution CA CA✓✓Answer **(AO full marks)**	(4)
4.2.2	Radius of larger circle: $2\sqrt{10}$ Radius of smaller circle: $r = \frac{1}{2}(2\sqrt{10}) = \sqrt{10}$ units **OR** Radius = $\sqrt{(4-3)^2 + (-2-1)^2}$ $= \sqrt{10}$ **OR** Raduis = $\sqrt{(4-5)^2 + (-2-5)^2}$ $= \sqrt{10}$	A ✓$\frac{1}{2}(2\sqrt{10})$ CA ✓ Answer	(2)
4.3	EC $= \sqrt{(9-3)^2 + (3-1)^2}$ $= \sqrt{6^2 + 2^2}$ $= \sqrt{36+4}$ $= \sqrt{40}$ $\therefore$ EC = radius $\therefore$ C lies on the circle, centre E **OR** $x^2 - 6x + 9 + y^2 - 2y + 1 = 40$ $x^2 + y^2 - 6x - 2y - 30 = 0$ **OR** $(x-3)^2 + (y-1)^2 = 40$ LHS = $(9-3)^2 + (3-1)^2 = 40$ RHS = 40 $\therefore$ LHS = RHS $\therefore$ It lies on the circle	A✓ Substitution CA ✓ EC = $\sqrt{40}$ CA ✓ equating to radius CA ✓ Conclusion	(4)
			[19]

QUESTION 5

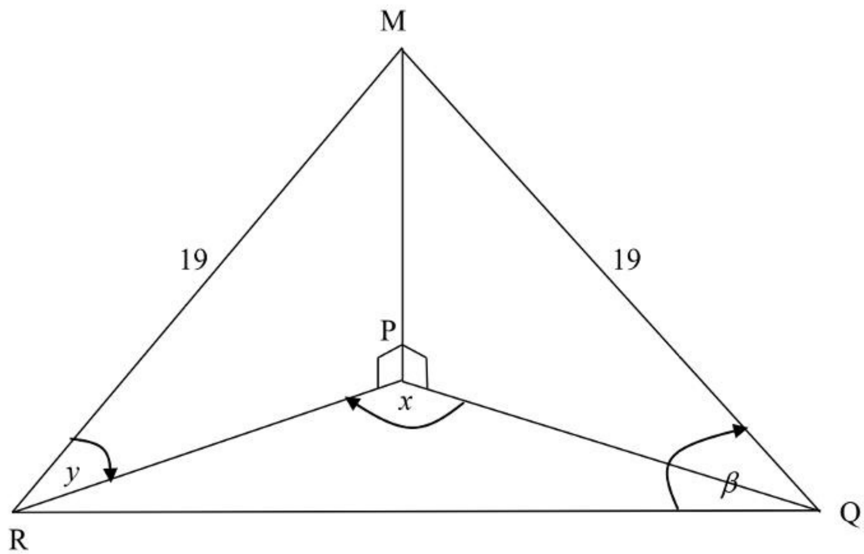
5.1			
5.1.1	$\cos 192^\circ$ $= -\cos 12^\circ$ $= -\dfrac{1}{\sqrt{1+q^2}}$	A✓ $\sqrt{1+q^2}$ A✓ $-\cos 12^\circ$ CA✓Answer	(3)
5.1.2	$\cos 24^\circ$ $= 2\cos^2 12^\circ - 1$ $= 2\left(\dfrac{1}{\sqrt{1+q^2}}\right)^2 - 1$ $= \dfrac{2}{1+q^2} - 1$ **OR** $\text{Cos } 24^\circ = 1 - 2\sin^2 12^\circ$ $= 1 - 2\left(\dfrac{q}{\sqrt{1+q^2}}\right)^2$ $= 1 - \dfrac{2q}{1+q^2}$ **OR** $\text{Cos } 24^\circ = \left(\dfrac{1}{\sqrt{1+q^2}}\right)^2 - \left(\dfrac{q}{\sqrt{1+q^2}}\right)^2$ $= \dfrac{1}{1+q^2} - \dfrac{q^2}{1+q^2}$ $= \dfrac{1-q^2}{1+q^2}$	A✓Double angle Expansion CA✓Substitution CA✓Answer	(3)

5.1.3	$1-2\sin^2 6^\circ$ $=\cos 12^\circ$ $=\dfrac{1}{\sqrt{1+q^2}}$	A✓Double angle CA✓Answer	(2)
5.2	$\dfrac{2\sin^2(x-180^\circ)\cos(180^\circ-x)}{\cos(90^\circ+x)\sin x-\cos(x-90)\sin(720^\circ-x)}$ $=\dfrac{2(-sinx)^2.(-\cos x)}{-\sin x.\sin x-(\sin x.-\sin x)}$ $=\dfrac{-2\sin^2 x\ cos\ x}{0}$ $=undefined$	A✓$(-sinx)^2$ A✓$-\cos x$ A✓$-\sin x$ A✓$\sin x$ A✓- $\sin x$ CA✓$\dfrac{-2\sin^2 x\ cos\ x}{0}$ CA✓Answer	(7)
5.3.1	LHS: $(1-\tan A)\left(\dfrac{\cos A}{\cos 2A}\right)$ $=\left(1-\dfrac{\sin A}{\cos A}\right)\left(\dfrac{\cos A}{\cos^2 A-\sin^2 A}\right)$ $=\left(\dfrac{\cos A-\sin A}{\cos A}\right)\dfrac{\cos A}{(\cos A-\sin A)(\cos A+\sin A)}$ $=\dfrac{1}{\cos A+\sin A}$ = RHS	A ✓ $\dfrac{\sin A}{\cos A}$ A ✓ $\cos^2 A-\sin^2 A$ A ✓ simplification	(3)
5.3.2	DO NOT MARK THIS QUESTION		
5.4	DO NOT MARK THIS QUESTION		
			[18]

QUESTION 6

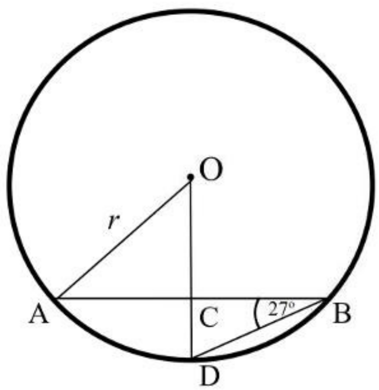
6.1			
	Graph of f: A 1 mark for x – intercepts A 1 marks for minimum and maximum points A 1 mark for shape Graph of g: A 1 mark for end points A 1 mark for x – intercepts A 1 mark for y – intercept		(6)
6.2.1	$y \in [-1\,;1]$	CA✓ $[-1\,;1]$ A ✓notation	(2)
6.2.2	360°	A✓Answer	(1)
6.2.3	$\sin\frac{1}{2}x = \cos(x + 60^\circ)$ $\sin\frac{1}{2}x = \sin[90^\circ - (x + 60^\circ)]$ $\sin\frac{1}{2}x = \sin[30^\circ - x]$ $\frac{1}{2}x = 30^\circ - x$ $\frac{3}{2}x = 30^\circ$ $x = 20^\circ$	A✓Co - Ratio A✓Equation in sine only A✓$\frac{1}{2}x = 30^\circ - x$ A✓Answer CA(**AO full marks)**	(4)
6.2.4	$h(x) = g(x + 30)$ $h(x) = cos(x + 30 + 60)$ $h(x) = cos(x + 90)$ $h(x) = -\sin x$	A ✓substitution A✓ $cos(x + 90)$ A✓Answer **(AO full marks)**	(3)
			[14]

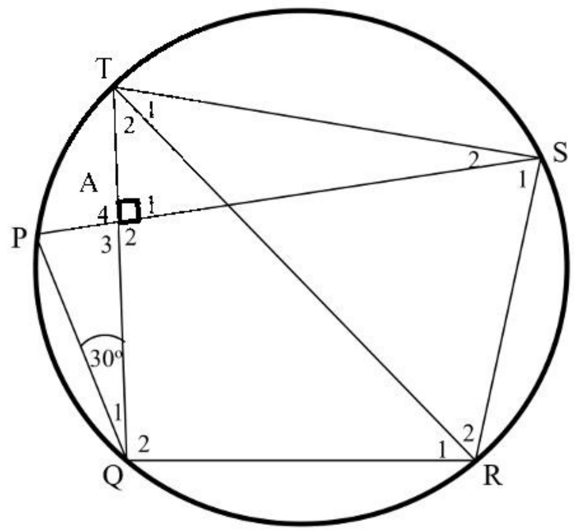
QUESTION 7



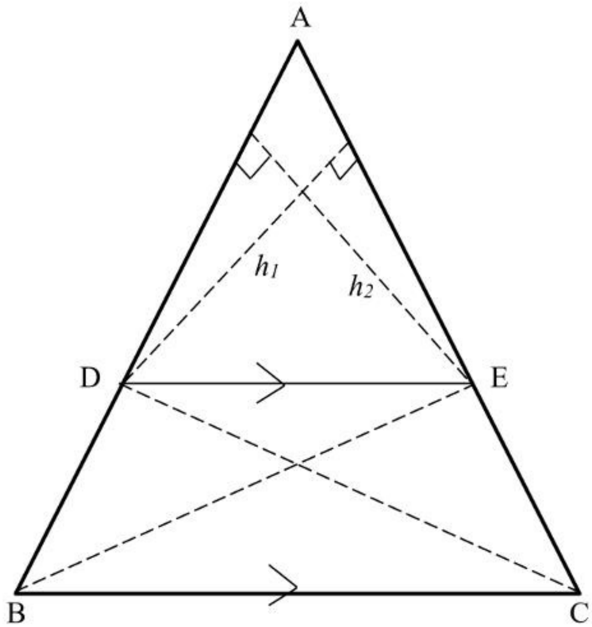
7.1	$\frac{PR}{19} = \cos y$ $PR = 19\cos y$ Now PR = PQ …..(ΔMPR ≡ ΔMPQ …90deg HS) Area of Δ PQR $= \frac{1}{2}(19\cos y)(19\cos y)\sin x$ $= \frac{361\sin x\cos^2 y}{2}$	A✓Trig ratio A✓Length of PR A✓Congruent triangles A✓Reason SAS A✓Substitution into Area Formula	(5)
7.2	$RQ^2 = 19^2 + 19^2 - 2(19)(19)\cos(180^\circ - 2\beta)$ $RQ^2 = 361 + 361 - 722(-\cos 2\beta)$ $RQ^2 = 722 + 722\cos 2\beta$ $RQ^2 = 722 + 722(2\cos^2\beta - 1)$ $RQ^2 = 1444\cos^2\beta$ $RQ = 38\cos\beta$ **OR** $MR^2 = RQ^2 + MQ^2 - 2RQ\,.\,MQ\cos\beta$ $19^2 = RQ^2 + 19^2 - 2(RQ)(19)\cos\beta$ $0 = RQ^2 - 38\,RQ\,.\cos\beta$ $\frac{38\,RQ\cos\beta}{RQ} = \frac{RQ^2}{RQ}$ $RQ = 38\cos\beta$	A✓Size of angle RMQ A✓Subst. Cosine rule A✓Reduction A✓Simplifying A✓Double angle expansion A✓$1444\cos^2\beta$	(6)
			[11]

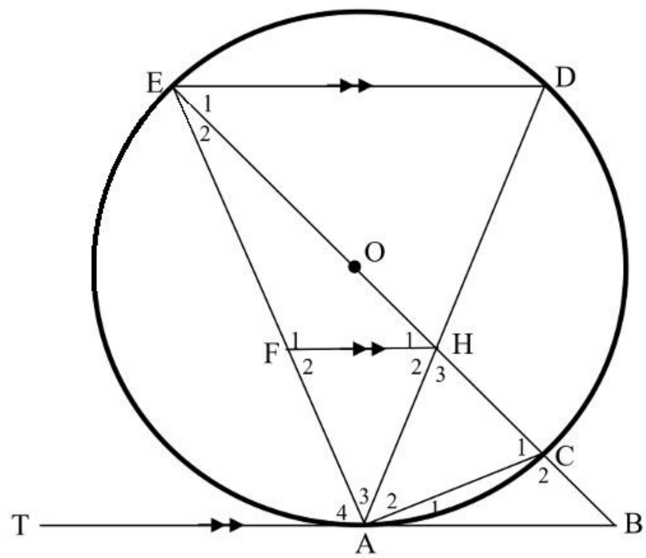
QUESTION 8

8.1	O is the centre of the circle, radius r, and chord AB = $\sqrt{128}$ cm. OCD ⊥ AB and OC : CD = 3 : 2. $A\hat{B}D = 27°$ 		
8.1.1	$\frac{OC}{CD} = \frac{3}{2} = \frac{3r}{2r}$ AC = CB = $4\sqrt{2}$ cm … (line from centre ⊥ chord) In ΔAOC: $r^2 = \left(\frac{3}{5}r\right)^2 + \left(4\sqrt{2}\right)^2$ … Pythagoras $\frac{16}{25}r^2 = 32$ $r^2 = \frac{32}{1} \times \frac{25}{16} = 50\text{ cm}^2$ $r = 5\sqrt{2}$ cm	A✓ OC and CD in terms of r. A✓ S/R A✓ Pythagoras (S/R) CA✓ Simplifying CA✓ Answer	(5)
8.1.2	$A\hat{O}D = 54°$ (angle at the centre 2 x angle at the circum)	A ✓ S A✓ R	(2)

8.2.1	(Exterior angle of cyclic quad = int. opp. A)	S ✓✓	(2)
8.2.2			
a)	$\hat{P}$ $Q\hat{T}S$ $\hat{S}_1$	S✓ S✓ S✓	(3)
b)	$Q\hat{R}S = 120^\circ$…[Opposite angles of cyclic quad.]	S✓ R✓	(2)
c)	$Q\hat{R}S + \hat{S}_1 = 120° + 60° = 180^\circ$ PS ∥ QR (Co-Interior angles are supplementary)	S✓ R✓	(2)
d)	$\hat{Q}_2 = 90^\circ$ …..(Corresponding Angles ; PS ∥ QR) TR is a diameter of the circle. (Conv. Angle in the semi -circle) **OR** $\hat{S}_1 = 60^\circ$ ... given $\hat{Q}_1 = 30^o$ ... given $\hat{Q}_1 + \hat{Q}_2 + \hat{S}_1 = 180^\circ$ opp ∠'s of a cyclic quad $\therefore \hat{Q}_2 = 90^\circ$ ∴ TR is a diameter (∠'s subt by 90°) Conv ∠'s on semi circle)	S/R✓ R✓	(2)
			[18]

QUESTION 9

9.1		S✓ Construction	
	Proof: $\dfrac{\text{Area } \Delta \text{ ADE}}{\text{Area } \Delta \text{ DBE}} = \dfrac{\frac{1}{2} AD \times h_2}{\frac{1}{2} DB \times h_2} = \dfrac{AD}{DB}$ (Common vertex E, same height h_2)	✓S✓R	
	$\dfrac{\text{Area } \Delta \text{ AED}}{\text{Area } \Delta \text{ DBE}} = \dfrac{\frac{1}{2} AE \times h_1}{\frac{1}{2} EC \times h_1} = \dfrac{AE}{EC}$ (Common vertex D, same height h_1)	S/R✓	
	Area Δ DBE = Area Δ ECD (Common base DE, same height, DE // BC)	✓S✓R	
	$\dfrac{\text{Area } \Delta \text{ ADE}}{\text{Area } \Delta \text{ DBE}} = \dfrac{\text{Area } \Delta \text{ AED}}{\text{Area } \Delta \text{ ECD}}$ $\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC}$ Area of Δ ADE is common and Area Δ DBE = Area Δ ECD	**(No Marks if no construction indicated in drawing or words)**	(6)

9.2			
9.2.1	In Δ's ABC and EBA 1) $\widehat{B} = \widehat{B}$ ….. [Common] 2) $\widehat{A}_1 = \widehat{E}_2$ … [Tangent – Chord Theorem] 3) $\hat{C}_2 = E\hat{A}B$ …. [Remaining Angles of Δ's] ΔABC ||| ΔEBA…[AAA]	S/R✓ S✓R✓ R✓	(4)
9.2.2	ΔABC ||| ΔEBA $\frac{AB}{EB} = \frac{BC}{AB} = \frac{AC}{EA}$ similar triangles $\frac{5}{2r + \frac{2r}{3}} = \frac{\frac{2r}{3}}{5}$ $\frac{2r}{3}\left(2r + \frac{2r}{3}\right) = 25$ $\frac{4r^2}{3} + \frac{4r^2}{9} = 25$ $\frac{16r^2}{9} = 25$ $r^2 = 25 \times \frac{9}{16}$ $r = \sqrt{25 \times \frac{9}{16}} = \frac{15}{4}$ metres	S/R✓ S✓Substitution S✓Multiplication S✓Answer	(4)
9.2.3	Let AH = 5*a* and HD = 7*a* $\frac{AF}{FE} = \frac{AH}{HD} = \frac{5a}{7a} = \frac{5}{7}$ ….. [Prop. Theorem; FH || ED]	✓S ✓R	(2)

9.2.4	$\Delta AFH = \frac{5}{12}\Delta AHE$ …. [Common Vertex; Equal Heights] $\Delta AFH = \frac{5}{12}\left(\frac{5}{12}\Delta AED\right)$ …. [Common Vertex; Equal Heights] $\frac{\Delta AFH}{\Delta AED} = \frac{25}{144}$ **OR** $\frac{\text{Area } \Delta \text{ AFH}}{\text{Area } \Delta \text{ AED}} = \frac{\frac{1}{2} AF . AH \text{ Sin } \hat{A}_3}{\frac{1}{2} AE . AD \text{ Sin } A_3}$ $\therefore \left(\frac{AF}{AE}\right)^2$ since $\frac{AF}{FE} = \frac{AH}{AD}$ $= \left(\frac{5a}{12a}\right)^2$ $= \frac{25}{144}$	✓S ✓R ✓S/R ✓S	(4)
9.2.5	Let OH = x Then HC = $r - x = \frac{15}{4} - x$ metres $\frac{AF}{FE} = \frac{BH}{HE}$ …..[Prop. Theorem; FH || AB $\frac{5}{7} = \frac{\frac{15}{4} - x}{\frac{15}{4} + x}$ $5\left(\frac{15}{4} + x\right) = 7\left(\frac{15}{4} - x\right)$ $\left(\frac{75}{4} + 5x\right) = \left(\frac{105}{4} - 7x\right)$ $75 + 20x = 105 - 28x$ $48x = 30$ $x = \frac{30}{48} = \frac{5}{8}$ metres	✓S/R ✓S ✓S	(3)
			[22]

TOTAL MARKS: 150

THIS PAPER SHOULD BE MARKED OUT OF 139