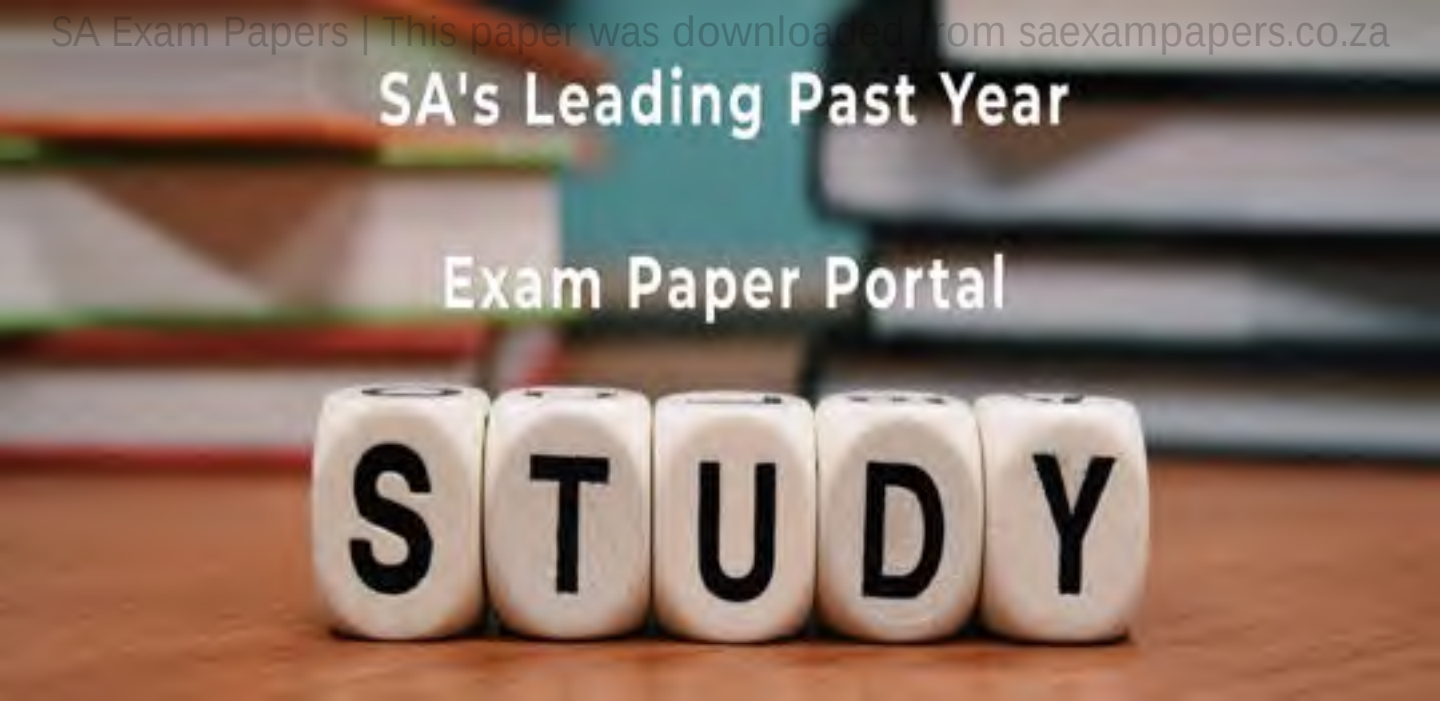


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12/*GRAAD 12*

MATHEMATICS/*WISKUNDE*

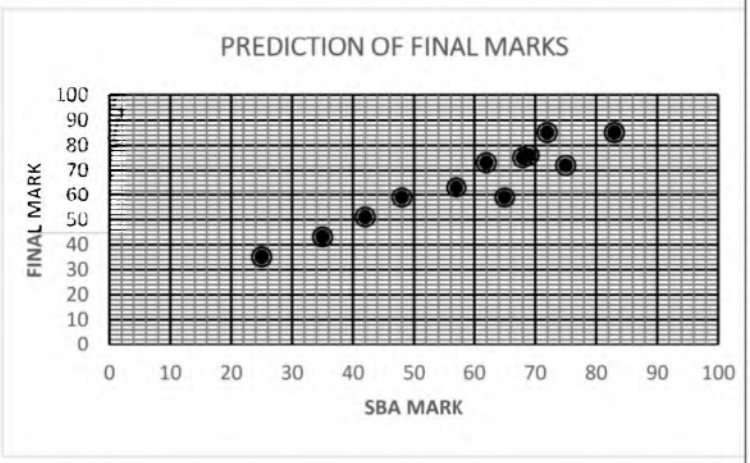
MEMORANDUM P2/*V2*

SEPTEMBER 2023

MARKS/*PUNTE*: 150

This memorandum consists of 19 pages/*Hierdie nasienriglyne bestaan uit 19 bladsye*.

QUESTION/*VRAAG* 1

1.1		✓✓✓✓all dots correct/*alle punte korrek* ✓✓✓9 – 11 dots correct/ *punte korrek* ✓✓6 – 8 dots correct/ *punte korrek* ✓1 – 6 dots correct/ *punte korrek* (4)
1.2	$r = 0,94$	✓✓answer/*antw* (2)
1.3	The strong correlation between the SBA mark and the final mark implies that the points lie close to the least squares regression line. Hence the prediction is reliable.	✓reason/*rede* ✓conclusion/*konklusie* (2)
1.4	$a = 14,49$ $b = 0,86$ $y = 14,49 + 0,86x$	✓value/*waarde a* ✓value/*waarde b* ✓equation/*vgl* (3)
1.5	$y = 14,49 + 0,86(66)$ $y = 71,25\%$ **OR/*OF*** $y = 71,18\%$	✓subst/*vervang* ✓answer/*antw* (2) ✓✓answer/*antw* (2)

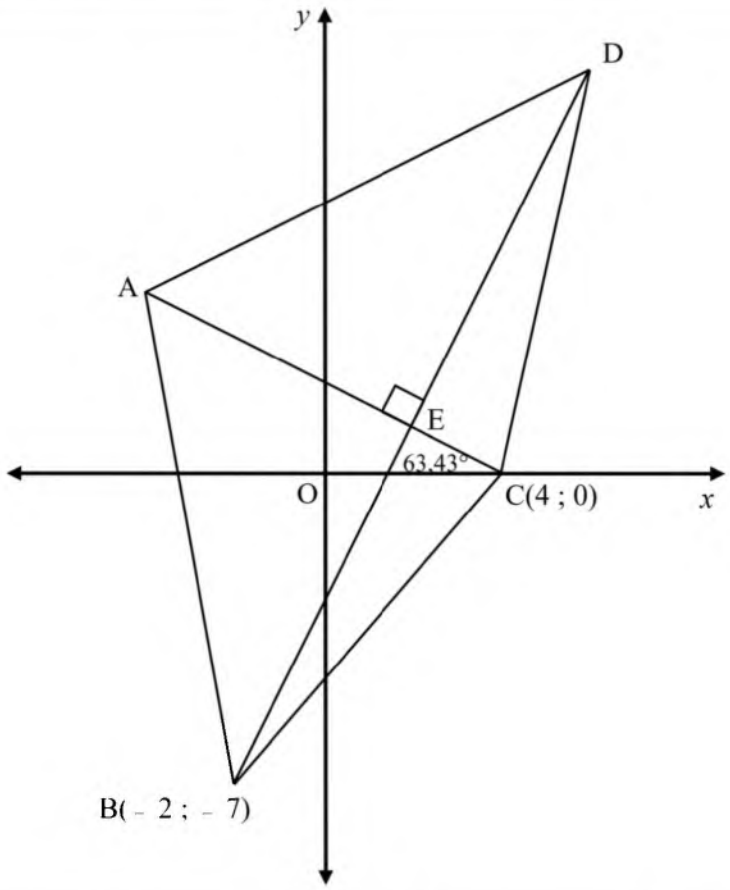
[13]

QUESTION/*VRAAG* 2

2.1.1	$\frac{3+4+4+6+10+12+12+4+y}{9}=7$ $\frac{55+y}{9}=7$ $55+y=63$ $y=8$	✓$\frac{55+y}{9}=7$ ✓value of/*waarde y* (2)
2.1.2	Median = 6	✓6 (1)
2.2.1	$\bar{x}=\frac{3+4+4+4+6+8+10+12+12+7-n+7+n}{11}$ $\bar{x}=\frac{77}{11}$ $\bar{x}=7$ **OR/*OF*** $\bar{x}=\frac{63+7-n+7+n}{11}$ $\bar{x}=\frac{77}{11}$ $\bar{x}=7$	✓77 ✓7 (2) ✓77 ✓7 (2)
2.2.2	$\bar{x}-\sigma_x=3$ $7-\sigma_x=3$ $\sigma_x=4$ **OR/*OF*** $\bar{x}+\sigma_x=11$ $7+\sigma_x=11$ $\sigma_x=4$	✓equation/*vgl* ✓answer/*antw* (2) ✓equation/*vgl* ✓answer/*antw* (2)

[7]

QUESTION/*VRAAG* 3



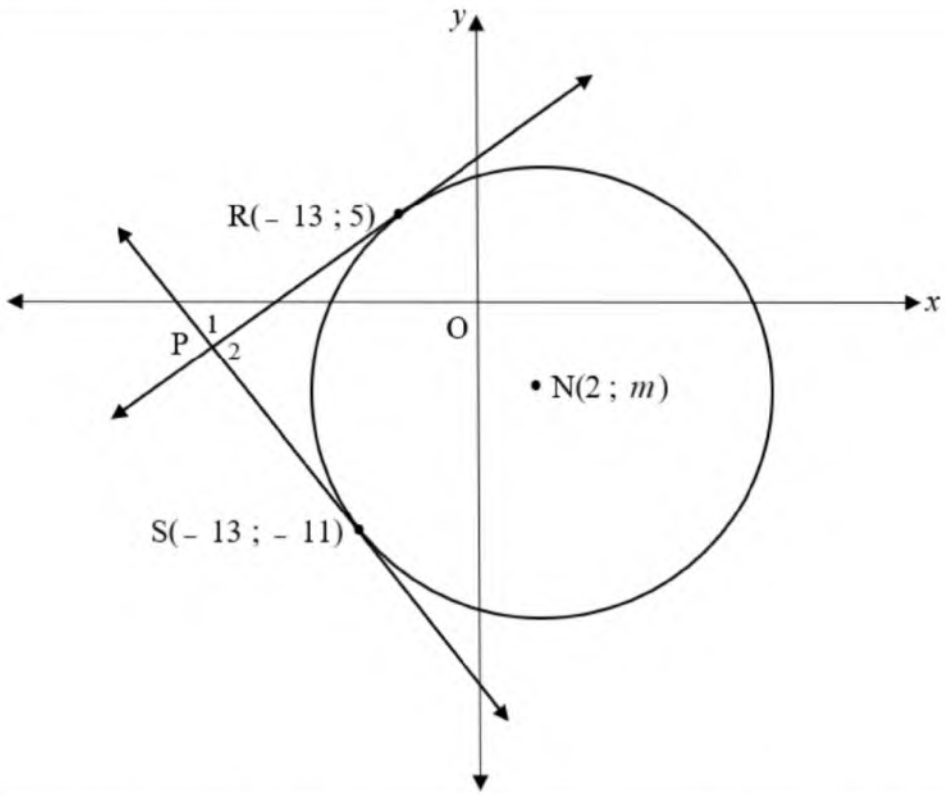
3.1	$y-y_1=m(x-x_1)$ $y-(-7)=2(x-(-2))$ $y+7=2(x+2)$ BD : $y=2x-3$	$m(\text{AC})=-\frac{1}{2}$ $m(\text{BD})=2$	✓ $m(\text{AC})=-\frac{1}{2}$ ✓ $m(\text{BD})=2$ ✓subt *m* and point B /*verv m en punt B* ✓answer/*antw* (4)
	OF/*OR*		
	$y=mx+c$ $-7=2(-2)+c$ $-7=-4+c$ $c=-3$ BD : $y=2x-3$	$m(\text{AC})=-\frac{1}{2}$ $m(\text{BD})=2$	✓ $m(\text{AC})=-\frac{1}{2}$ ✓ $m(\text{BD})=2$ ✓subt *m* and point B /*verv m en punt B* ✓answer/*antw* (4)

3.2	$-\frac{1}{2}x+2=2x-3$ $-x+4=4x-6$ $-5x=-10$ $x=2$ subst into BD: $y=2(2)-3$ $y=1$ E (2 ; 1)	✓equating/*vgl* ✓value of/*waarde x* ✓value of/*waarde y* (3)
3.3	A$(-4\ ;\ 4)$ through translation **OR/*OF*** Find a point P so that CE = EP P(0 ; 2) Then CP = PA A$(-4\ ;\ 4)$	✓value of/*waarde x* ✓value of/*waarde y* (2) ✓P(0 ; 2) ✓A(–4 ; 4) (2)
3.4	$AC=\sqrt{(-4-4)^2+(4-0)^2}=4\sqrt{5}$ $BE=\sqrt{(-2-2)^2+(-7-1)^2}=4\sqrt{5}$ Area of kite = 2 × area of ΔABC $=2\times\left(\frac{1}{2}\times4\sqrt{5}\times4\sqrt{5}\right)$ $=80$ Enlarged by scale factor 2: $80\times4=320$ sq unit **OR/OF** $AC=\sqrt{(-4-4)^2+(4-0)^2}=4\sqrt{5}$ $\therefore PR=8\sqrt{5}$ $BE=\sqrt{(-2-2)^2+(-7-1)^2}=4\sqrt{5}$ $\therefore QS=8\sqrt{5}$ Area of kite = 2 × area of ΔPQR $=2\times\left(\frac{1}{2}\times8\sqrt{5}\times8\sqrt{5}\right)$ $=320$	✓length/*lengte* AC ✓length/*lengte* BE ✓method/*metode* ✓80 ✓answer/*antw* (5) ✓length/*lengte* AC ✓length/*lengte* PR ✓length/*lengte* QS ✓method/*metode* ✓answer/*antw* (5)

[14]

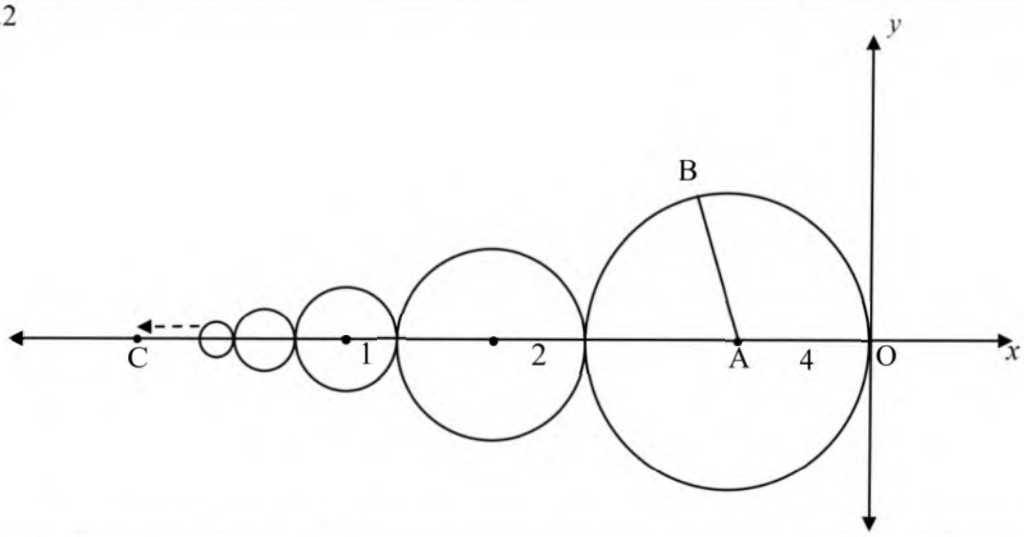
QUESTION/*VRAAG* 4

4.1



4.1.1 (a)	$(-13-2)^2+(5-m)^2=17^2$ $225+25-10m+m^2=289$ $m^2-10m-39=0$ $(m-13)(m+3)=0$ $m=13$ or $m=-3$ NA **OR/*OF*** $(-13-2)^2+(-11-m)^2=17^2$ $225+(-11-m)^2=289$ $(-11-m)^2=64$ $-11-m=\pm 8$ $m=-19$ or $m=-3$ NA	✓subst R and N/*verv R en N* ✓simplify/*vereenv* ✓std form/*std vorm* ✓value/*value m* (4) ✓subst S and N/*verv S en N* ✓simplify/*vereenv* ✓std form/*std vorm* ✓value/*waarde m* (4)

	OR/*OF*	
	$(-13-2)^2+(-11-m)^2=(-13-2)^2+(5-m)^2$ $225+m^2+22m+121=225+m^2-10m+25$ $32m=-96$ $m=-3$	✓✓equating ✓simplify/*vereenv* ✓value/*waarde m* (4)
	OR/*OF*	
	RS ∥ $y-$axis ∴ $x=-13$ same $x-$values Draw line NK ⊥ RS ∴ NK bisect RS at $(-13\ ;\ -3)$ line from centre of circle ⊥ to chord ∴ N(2 ; −3)	✓use line $x=-13$ ✓Use NK ✓midpt of NK ✓$y=m=-3$ (4)
4.1.1 (b)	$(x-2)^2+(y+3)^2=289$	✓answer/*antw* (1)
4.1.2 (a)	$m(\text{NR})=\frac{-3-5}{2-(-13)}$ $m(\text{NR})=\frac{-8}{15}$	✓subst in gradient formula/*verv in gradiënt formule* ✓gradient of NR (2)
4.1.2 (b)	$m(\text{NS})=\frac{-3-(-11)}{2-(-13)}$ $m(\text{NS})=\frac{8}{15}$	✓gradient of NS/ *gradiënt van NS* (1)
4.1.3	NR ⊥ PR NS ⊥ PS $\therefore m(\text{PR})=\frac{15}{8}$ $\therefore m(\text{PS})=-\frac{15}{8}$ $\tan\alpha=\frac{15}{8}$ $\tan\beta=-\frac{15}{8}$ $\alpha=61{,}93°$ $\beta=180°-61{,}93°$ $\beta=118{,}07°$ $\hat{P}_1=118{,}07°-61{,}93°=56{,}14°$ $\therefore \hat{P}_2=180°-56{,}14°=123{,}86°$	✓$\tan\alpha$ def ✓value/*waarde* α ✓$\tan\beta$ def ✓value/*waarde* β ✓method/*metode* ✓answer/*antw* (6)

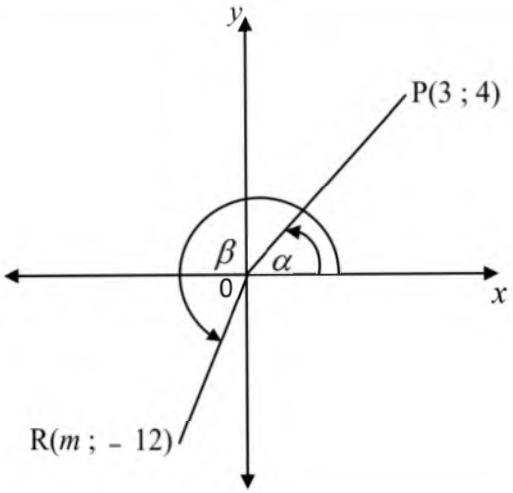
	OF/*OR* Inclination of NS = 28,07° Inclination of NR = 151,93° $R\hat{N}S = 28,07° + 28,07°$ $R\hat{N}S = 56,14°$ NRPS is cyclic quadrilateral Opp $\angle's$ suppl $\therefore \hat{P}_2 = 123,86°$	✓28,07° ✓151,93° ✓ $R\hat{N}S = 2 \times 28,07°$ ✓56,14° ✓Opp $\angle's$ cyclic quad/*teenoorst* $\angle'e$ *kvh* ✓answer/*antw* (6)
4.1.4	Reflection about x – axis: $(2\ ;\ -3) \rightarrow (2\ ;\ 3)$ Shift 2 units up: $(2\ ;\ 3) \rightarrow (2\ ;\ 5)$ Circle M: $(x-2)^2 + (y-5)^2 = 289$	✓✓equation/*vgl* (2)
4.2		
4.2.1	Diameter = 8 + 4 + 2 + 1 ….= $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8}{1-\frac{1}{2}}$ $S_\infty = 16$ $\therefore OC = 16$	✓sum of diameters/ *som middellyne* ✓subst in sum formula/*verv in som formule* (2)
4.2.2	90°, radius ⊥ tangent	✓answer/*antw* ✓reason/*rede* (2)

4.2.3	$AC = 16 - 4 = 12$ $AB = 4$ $BC^2 = AC^2 - AB^2$ (Pyth) $BC^2 = 12^2 - 4^2$ $BC^2 = 128$ $BC = 8\sqrt{2}$ $\therefore \tan C = \frac{4}{8\sqrt{2}} = \frac{\sqrt{2}}{4}$	✓AC ✓AB ✓BC ✓tan ratio (4)
4.2.4	$y - y_1 = m(x - x_1)$ $y - 0 = \frac{\sqrt{2}}{4}(x - (-16))$ $y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$ **OR/*OF*** $y = \frac{\sqrt{2}}{4}x + c$ $0 = \frac{\sqrt{2}}{4}(-16) + c$ $c = 4\sqrt{2}$ $\therefore y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$	✓subst/*verv* m ✓subst point/*verv punt* ✓equation/*vgl* (3) ✓subst/*verv* m ✓subst point/*verv punt* ✓equation/*vgl* (3)

[27]

QUESTION/*VRAAG* 5

5.1



5.1.1	$\tan\alpha = \frac{4}{3}$	✓answer/*antw* (1)
5.1.2	$\sin(90° + \alpha)$ $= \cos\alpha$ $= \frac{3}{5}$	✓reduction/*reduksie* ✓$r = 5$ ✓answer/*antw* (3)
5.1.3	$12 + 13\sin\beta = 0$ $\sin\beta = -\frac{12}{13}$ $m^2 = 13^2 - (-12^2)$ (Pyth) $m^2 = 25$ $m = \pm 5$ $\therefore m = -5$	✓std form/*std vorm* ✓subst into Pyth ✓simpl/*vereenv* ✓answer/*antw* (4)
5.1.4	$\cos(\alpha + \beta)$ $= \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $= \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$ $= -\frac{3}{13} + \frac{48}{65}$ $= \frac{33}{65}$	✓expansion/*uitbrei* ✓subst/*vervang* ✓answer/*antw* (3)

5.2.1	$\sqrt{4^{\sin 150^\circ}.2^{3\tan 225^\circ}}$ $=\sqrt{\left(2^2\right)^{\sin 30^\circ}.2^{3\tan 45}}$ $=\sqrt{2^{2\left(\frac{1}{2}\right)}.2^{3(1)}}$ $=\sqrt{2.2^3}$ $=\sqrt{16}$ $=4$	✓sin 30° ✓tan 45° ✓special values/ *spesiale waardes* ✓simpl/*vereenv* ✓answer/*antw* (5)
5.2.2	$\frac{\tan(180^\circ+x)\cos x}{\sin(180^\circ+x)\cos x-\cos(540^\circ+x)\cos(90^\circ+x)}$ $=\frac{(\tan x)(\cos x)}{(-\sin x)(\cos x)-(-\cos x)(-\sin x)}$ $=\frac{\frac{\sin x}{\cos x}.\cos x}{-\sin x\cos x-\cos x\sin x}$ $=\frac{\sin x}{-2\sin x\cos x}$ $=-\frac{1}{2\cos x}$	✓$\tan x$ ✓$-\sin x$ ✓$-\cos x$ ✓$-\sin x$ ✓$\frac{\sin x}{\cos x}$ ✓answer/*antw* (6)
5.3	$\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\frac{1-(1-2\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{1-1+2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\frac{\sin x}{\cos x}$ $=\tan x$	✓cos $2x$ expansion/ *uitbrei* ✓sin $2x$ expansion/ *uitbrei* ✓simpl/*vereenv* ✓factors/*faktore* (4)

5.4	$\sin P\sin Q-\cos P\cos Q=\frac{1}{2}$ $\therefore \cos P\cos Q-\sin P\sin Q=-\frac{1}{2}$ $\cos(P+Q)=-\frac{1}{2}$ $P+Q=180°-60°$ $P+Q=120°$ .....1 $\sin(P-Q)=\frac{1}{2}$ $P-Q=30°$ .....2 $\therefore 2P=150°$ $P=75°$ $Q=45°$	✓rearrange terms/ *herrangskik terme* ✓cos identity/ *identiteit* ✓2nd quadrant/ *kwadrant* ✓equation/*vgl* ✓1st quadrant/ *kwadrant* ✓75° ✓45° (7)

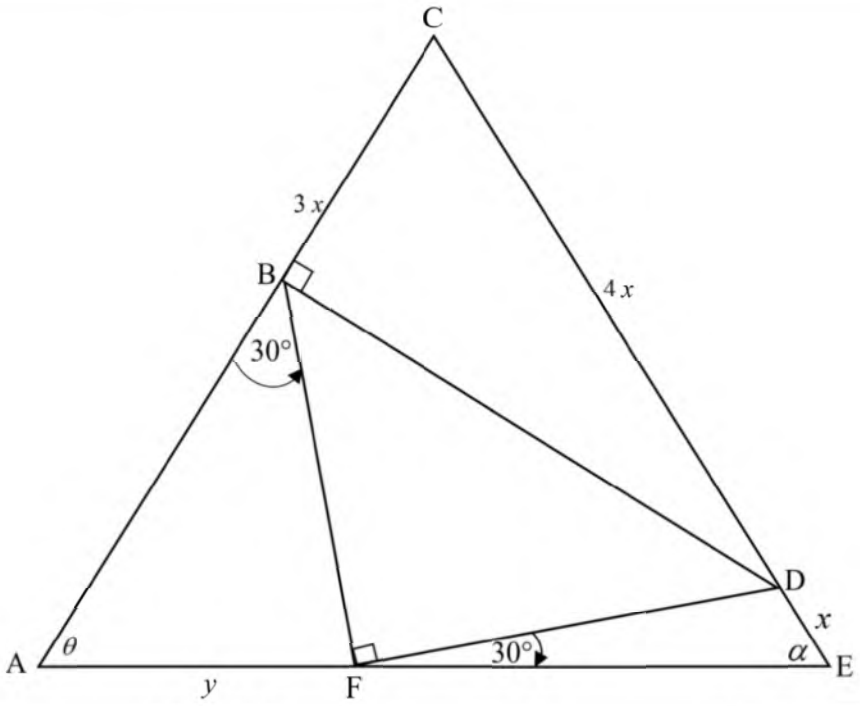
[33]

QUESTION/*VRAAG* 6

6.1	$a=2$ and $b=1$	✓value/*waarde a* ✓value/*waarde b* (2)
6.2.1	$2\sin x-\cos x=0$ $2\sin x=\cos x$ $\frac{\sin x}{\cos x}=\frac{1}{2}$ $\tan x=\frac{1}{2}$ $x=26,57°$	✓ $\tan x=\frac{1}{2}$ ✓answer/*antw* (2)
6.2.2	$x\in[0°\,;\,180°]$	✓interval ✓notation/*notasie* (2)
6.2.3	$y=2^{2f(x)-1}$ $y\in\left[-\frac{1}{2}\,;\,7\right]$	✓✓answer/*antw* (2)

[8]

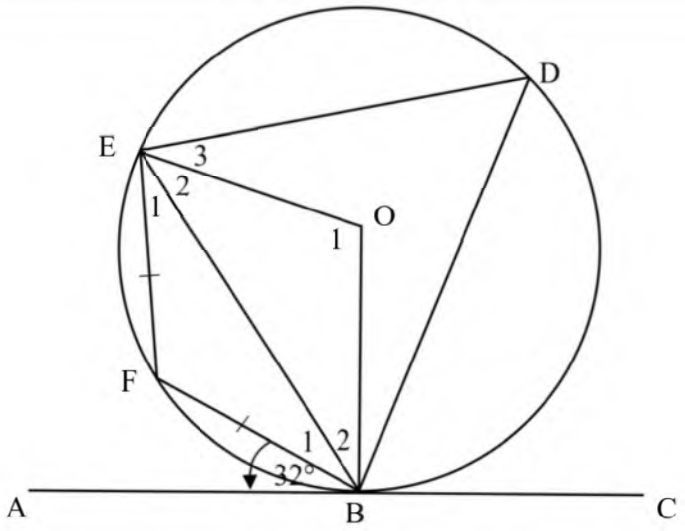
QUESTION/*VRAAG* 7



7.1	In ΔABF: $\frac{BF}{\sin A} = \frac{AF}{\sin B}$ $\frac{BF}{\sin\theta} = \frac{y}{\sin 30°}$ $BF = \frac{y\sin\theta}{\frac{1}{2}}$ $BF = 2y\sin\theta$	✓correct subst in sine rule/*korrekte verv in sinreel* ✓subst special value/ *verv spasiale waarde* ✓answer/*antw* (3)
7.2	In ΔEDF: $\frac{DF}{\sin E} = \frac{DE}{\sin E}$ $\frac{DF}{\sin\alpha} = \frac{x}{\sin 30°}$ $DF = \frac{x\sin\alpha}{\frac{1}{2}}$ $DF = 2x\sin\alpha$	✓correct subst in sine rule/*korrekte verv in sinreel* ✓answer/*antw* (2)
7.3	$BD^2 = BF^2 + FD^2$ (Pyth) $BD^2 = (2y\sin\theta)^2 + (2x\sin\alpha)^2$ $BD^2 = 4y^2\sin^2\theta + 4x^2\sin^2\alpha$	✓ subst/*verv* in Pyth (1)

7.4	In $\Delta BDC: BD^2 = (4x)^2 - (3x)^2$ $BD^2 = 16x^2 - 9x^2$ $BD^2 = 7x^2$	✓ $BD^2 = 7x^2$
	$7x^2 = 4y^2 \sin^2\theta + 4x^2 \sin^2\alpha$ $7x^2 - 4x^2 \sin^2\alpha = 4y^2 \sin^2\theta$ $x^2(7 - 4\sin^2\alpha) = 4y^2 \sin^2\theta$ $x^2 = \dfrac{4y^2 \sin^2\theta}{(7 - 4\sin^2\alpha)}$ $x = \sqrt{\dfrac{4y^2 \sin^2\theta}{7 - 4\sin^2\alpha}}$	✓equating ✓simpl/*vereenv* (3)
	OF/*OR*	
	$CD^2 = BD^2 + BC^2$ $16x^2 = 4y^2\sin^2\theta + 4x^2\sin^2\alpha + 9x^2$ $16x^2 - 9x^2 - 4x^2\sin^2\alpha = 4y^2\sin^2\theta$ $7x^2 - 4x^2\sin^2\alpha = 4y^2\sin^2\theta$ $x^2(7 - 4\sin^2\alpha) = 4y^2\sin^2\theta$ $x^2 = \dfrac{4y^2\sin^2\theta}{7 - 4\sin^2\alpha}$ $x = \sqrt{\dfrac{4y^2\sin^2\theta}{7 - 4\sin^2\alpha}}$	✓subst/*verv* in Pyth ✓simpl/*vereenv* ✓ i.t.o x^2 (3)
		[9]

QUESTION/*VRAAG* 8

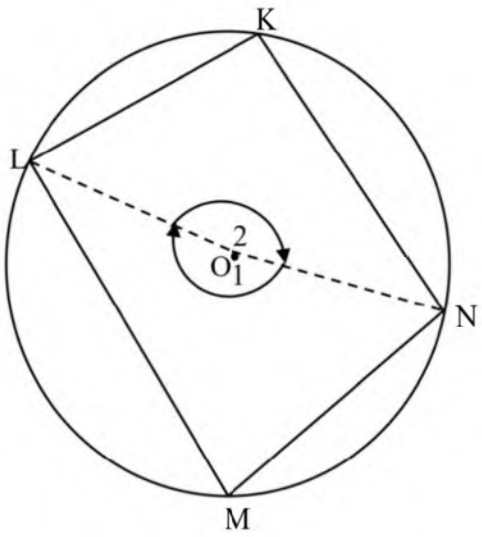


8.1	$A\hat{B}F = \hat{E}_1 = 32°$	tan-chord theorem	✓S ✓R (2)
8.2	$\hat{E}_1 = \hat{B}_1 = 32°$ $\hat{F} = 116°$	∠'s opp equal sides sum ∠'s of Δ	✓S/R ✓S/R (2)
8.3	$\hat{D} + \hat{F} = 180°$ $\hat{D} = 64°$	opp ∠'s of cyclic quad	✓S ✓R (2)
8.4	$\hat{O}_1 = 2 \times \hat{D}$ $\hat{O}_1 = 128°$	midpt ∠ = 2 × circumf ∠	✓S ✓R (2)
8.5	$\hat{E}_2 + \hat{O}_1 + \hat{B}_2 = 180°$ $\hat{E}_2 = \hat{B}_2$ $2\hat{E}_2 + 128° = 180°$ $2\hat{E}_2 = 52°$ $\hat{E}_2 = 26°$ **OR/*OF*** $A\hat{B}F + \hat{B}_1 + \hat{B}_2 = 90°$ $32° + 32° + \hat{B}_2 = 90°$ $\hat{B}_2 = 26°$ $\hat{B}_2 = \hat{E}_2 = 26°$	sum ∠'s of Δ ∠'s opp equal sides rad ⊥ tangent ∠'s opp equal sides	✓S/R ✓S/R (2) ✓S/R ✓S/R (2)

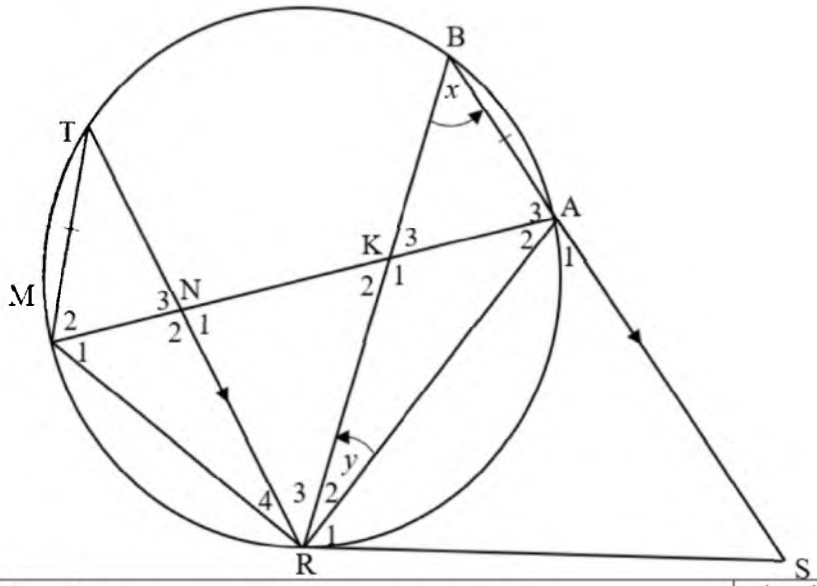
[10]

QUESTION/*VRAAG* 9

9.1

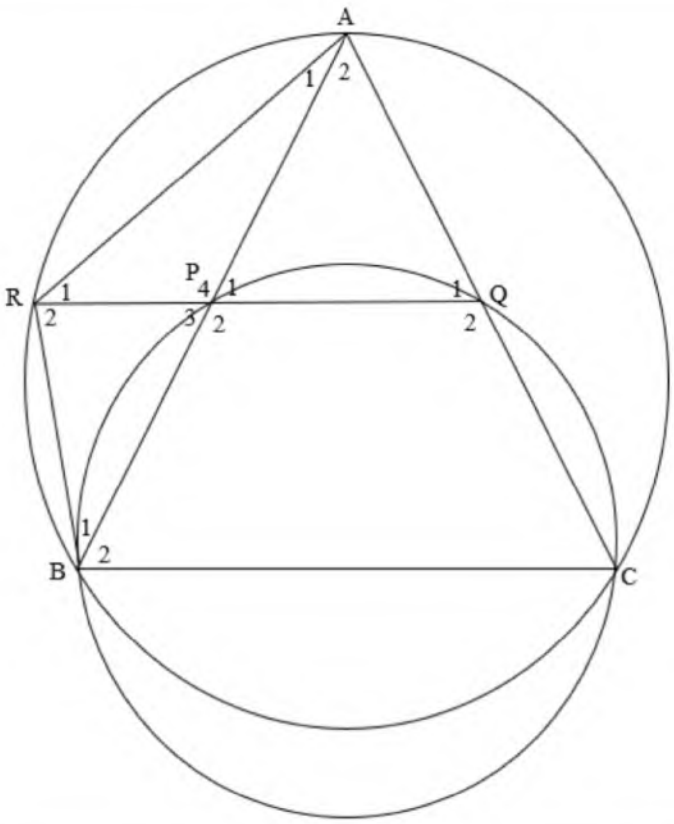


	Construction: Join LO and ON	✓constr/*konstr*
	$\hat{O}_1 = 2 \times \hat{K}$ ∠at centre = 2 × ∠ at circumf	✓S ✓R
	$\hat{O}_2 = 2 \times \hat{M}$ ∠at centre = 2 × ∠ at circumf	✓S/R
	$\hat{O}_1 + \hat{O}_2 = 360°$ revolution	✓R
	$2\hat{M} + 2\hat{K} = 360°$ equating	
	$\hat{M} + \hat{K} = 180°$	
		(5)

9.2		
9.2.1	$\hat{B} = \hat{M}_1 = x$ ∠'s in the same segment $\hat{B} = \hat{R}_3 = x$ alt ∠'s, TR ∥ BS $\hat{B} = \hat{R}_1 = x$ tan-chord theorem	✓S ✓R ✓S ✓R ✓S ✓R (6)
9.2.2	Equal chords equal circumf ∠'s	✓R (1)
9.2.3(a)	$\hat{A}_1 = x + y$ ext ∠ of Δ	✓S/R (1)
9.2.3(b)	$\hat{N}_1 = x + y$ ext ∠ of Δ	✓S/R (1)
9.2.4	In Δ SAR and Δ KNR (i) $\hat{R}_1 = \hat{R}_3$ both = x (ii) $\hat{A}_1 = \hat{N}_1$ both = $x + y$ (iii) $\hat{K}_2 = \hat{S}$ sum ∠'s of Δ $\therefore$ Δ SAR ||| Δ KNR ∠∠∠ **OR/*OF*** In Δ SAR and Δ KNR (i) $\hat{R}_1 = \hat{R}_3$ both = x (ii) $\hat{A}_1 = \hat{N}_1$ both = $x + y$ $\therefore$ Δ SAR ||| Δ KNR ∠∠∠	✓S ✓S ✓S (3) ✓S ✓S ✓R (3)
9.2.5	$\hat{K}_2 = \hat{S}$ Δ SAR ||| Δ KNR SAKR is a cyclic quad ext ∠ of quadrilateral **OR** converse ext ∠ of cyclic quad	✓S ✓R (2)

[19]

QUESTION/*VRAAG* 10



10.1	$\hat{P}_1 = C$	ext $\angle$ of cyclic quad	✓S ✓R
	$\hat{C} = 180° - (\hat{R}_1 + \hat{R}_2)$	opp $\angle$'s of cyclic quad	✓S ✓R
	$\therefore \hat{P}_1 = 180° - (\hat{R}_1 + \hat{R}_2)$		✓method/*metode*
	$\hat{R}_1 + \hat{R}_2 = 180° - (\hat{A}_1 + \hat{B}_1)$	sum $\angle$'s of Δ	
	$\therefore \hat{P}_1 = 180° - [180° - (\hat{A}_1 + \hat{B}_1)]$		
	$\therefore \hat{P}_1 = 180° - 180 + (\hat{A}_1 + \hat{B}_1)$		
	$\therefore \hat{P}_1 = \hat{A}_1 + \hat{B}_1$		(5)

	OR/*OF*	✓S ✓R
	$A\hat{R}B = 180° - (\hat{A}_1 + \hat{B}_1)$	✓S ✓R
	$A\hat{R}B = 180° - \hat{C}$	✓method/*metode*
	$\therefore 180° - (\hat{A}_1 + \hat{B}_1) = 180° - \hat{C}$	
	$\hat{C} = \hat{A}_1 + \hat{B}_1$	
	$\hat{C} = \hat{P}_1$	(5)
	$\therefore \hat{P}_1 = \hat{A}_1 + \hat{B}_1$	
10.2	In ΔARP and ΔABR	✓ identify/ *identifiseer* Δ's
	(i) $\hat{A} = \hat{A}$ common $\angle$	✓S
	(ii) $\hat{P}_1 = \hat{A}_1 + \hat{B}_1$ proven	✓method/*metode*
	$180° - \hat{P}_1 = 180° - (\hat{A}_1 + \hat{B}_1)$	
	$\therefore \hat{P}_4 = \hat{R}_1 + \hat{R}_2$	
	(iii) $\hat{R}_1 = \hat{B}_1$ sum $\angle$'s of Δ	✓R
	$\therefore \Delta ARP \text{ III } \Delta ABR$ $\angle\angle\angle$	
	$\frac{AR}{AB} = \frac{AP}{AR}$ III Δ's	✓S
	$AR^2 = AB.AP$	
	OR/*OF*	(5)
	In ΔARP and ΔABR	
	(i) $\hat{A} = \hat{A}$ common $\angle$	✓Identify/ *identifiseer* Δ's
	(ii) $\hat{P}_1 = \hat{A}_1 + \hat{B}_1$ proven	✓S
	$180° - \hat{P}_1 = 180° - (\hat{A}_1 + \hat{B}_1)$	✓method/*metode*
	$\therefore \hat{P}_4 = \hat{R}_1 + \hat{R}_2$	
	$\therefore \Delta ARP \text{ III } \Delta ABR$ $\angle\angle\angle$	
	$\frac{AR}{AB} = \frac{AP}{AR}$ III Δ's	✓R
	$AR^2 = AB.AP$	✓S
		(5)
		[10]