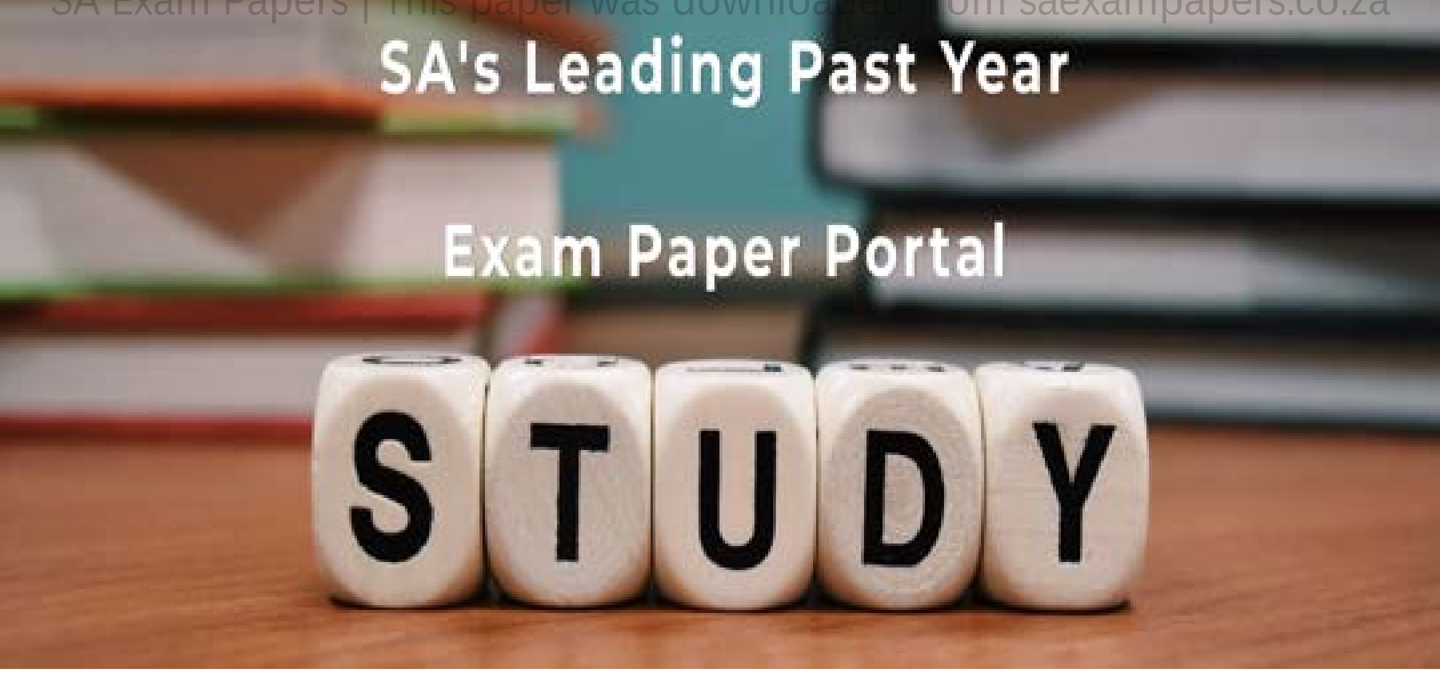
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REPUBLIC OF SOUTH AFRICA

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GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2023

MARKING GUIDELINES

MARKS: 150

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

QUESTION/*VRAAG* 1

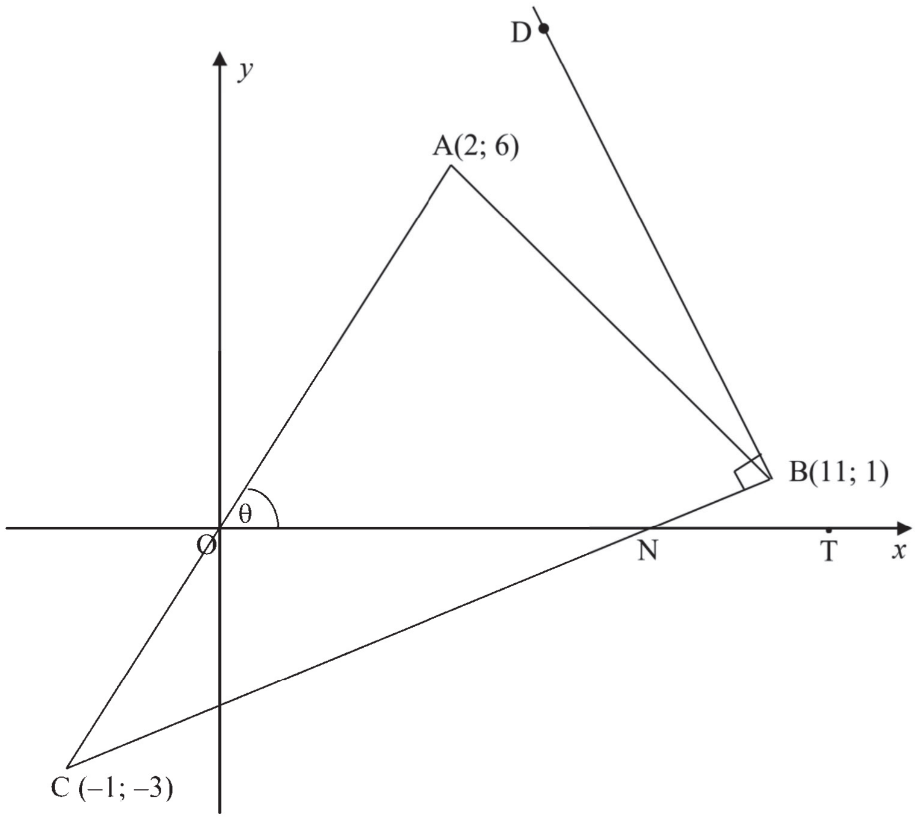
71	83	88	91	92	92	95	97	104	108	109	110	111	115	129

1.1	$\frac{1495}{15}=99,67$	✓✓ $99,67$ (2)
1.2	$\sigma=14,06$	✓SD (1)
1.3	$99,67-14,06=85,61$ $\frac{2}{15}\times100=13,33\%$	✓boundary ✓ $\frac{2}{15}$ ✓answer (3)
1.4.1	$96,67$	✓answer (1)
1.4.2	$14,06$	✓answer (1)
		[8]

QUESTION/*VRAAG* 2

2.1.1	True	✓ answer (1)
2.1.2	Positively skewed / Skewed to the right	✓ answer (1)
2.1.3	Range for company A $= 800 - 200 = 600$ Range for company B $= 600 - 100 = 500$ Biggest range : Company A	✓ both ranges ✓Company A (2)
2.1.4	$20 \times 75\% = 15$ workers	✓75% ✓ 15 workers (2)
2.2.1	$y = 5965,51 - 2,93x$	✓ 5329,84 ✓ $-2,61$ ✓ equation (3)
2.2.2	$r = -0,49$	✓ (1)
2.2.3	$y = 5965,51 - 2,93(2018)$ $= 52,77$ The correlation coefficient is moderate thus the value is fairly reliably predicted.	✓ prediction ✓ answer + reason (2)
		[12]

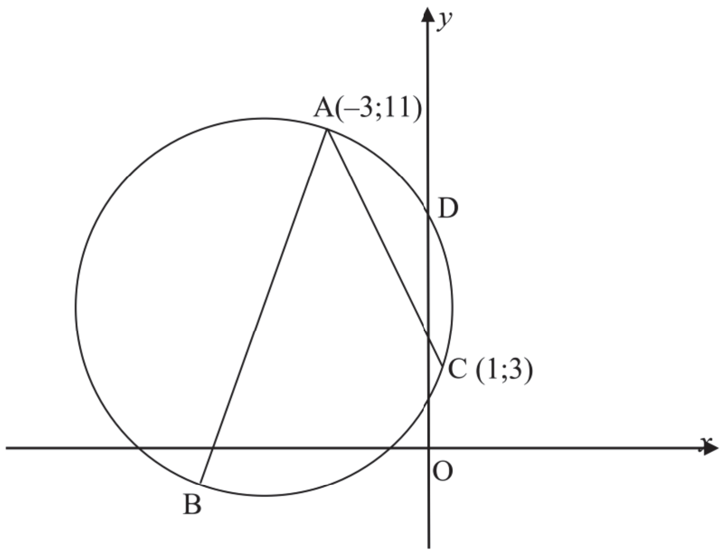
QUESTION/*VRAAG* 3



3.1.1	$N:(x\,;0)$ $m_{NC}=\dfrac{0-(-3)}{x-(-1)}=\dfrac{1}{3}$ $x+1=9$ $x=8$ OR $y-(-3)=\dfrac{1}{3}(x-(-1))$ $y=\dfrac{1}{3}x-\dfrac{8}{3}$ $For\ y=0:x=8$	✓ subst into gradient formula ✓ answer (2) ✓ subst into equation ✓ answer

3.1.2	$\tan\theta = 3$ $\theta = 71,565...^\circ$ $\tan B\hat{N}T = \frac{1}{3}$ $B\hat{N}T = 18,434...^\circ$ $\hat{C} = 71,565...^\circ - 18,434...^\circ$ $= 53,13^\circ$ Accept $\hat{C} = 53,14^\circ$	✓substitution ✓θ ✓substitution ✓ $B\hat{N}T$ ✓ answer (5)
3.2	$y - 6 = 3(x-2)$ $y = 3x$	✓substitution ✓equation (2)
3.3	$m_\perp = -3$ $y - 1 = -3(x - 11)$ $y = -3x + 34$ $3x = -3x + 34$ $6x = 34$ $x = \frac{17}{3} = 5,67$ $y = 3\left(\frac{17}{3}\right) = 17$	✓ $m_\perp = -3$ ✓equation ✓setting equations equal ✓value of x ✓value of y (5)
		[14]

QUESTION/*VRAAG* 4

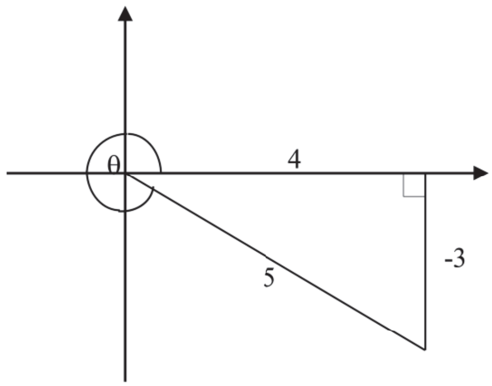


4.1	$m_{AC} = \frac{11-3}{-3-1} = \frac{8}{-4} = -2$ $mdpt_{AC} = (-1;7)$ $y-7 = \frac{1}{2}(x-(-1))$ $y = \frac{1}{2}x + \frac{15}{2}$	✓gradient of AC. ✓Midpt AC ✓⊥gradient and subst ✓ equation (4)
4.2	$\frac{1}{2}x + \frac{15}{2} = 3x + 20$ $-2,5x = 12,5$ $x = -5$ $y = 3(-5) + 20 = 5$ $\therefore$ centre$(-5;5)$	✓ set equations equal ✓solve x ✓solve y (3)
4.3	radius $= \sqrt{(-5-(-3))^2 + (5-11)^2}$ $= \sqrt{40} = 2\sqrt{10}$ diameter $= 4\sqrt{10}$ units $= 12,65$	✓substitution in formula ✓radius ✓diameter (3)
4.4	$(x+5)^2 + (y-5)^2 = 40$	✓ $(x+5)$, $(y-5)$ ✓ 40 (2)

4.5	$m_{rad} = \frac{11-5}{-3-(-5)} = 3$ $m_{\tan} = -\frac{1}{3}$ $y-11 = -\frac{1}{3}(x-(-3))$ $y = -\frac{1}{3}x+10$ $(0;p)$: $p = -\frac{1}{3}(0)+10$ $p = 10$	✓gradient ✓subst in form ✓equation ✓ value of p (4)
4.6	$(x+2)^2 + (y-7)^2 = (\sqrt{10})^2$ $(x+2)^2 + (y-7)^2 = 10$	✓$(x+2)$ ✓$(y-7)$ ✓10 (3)
4.7	Centres: (2; 3) and $(-5; 5)$ Distance between centres = $\sqrt{(2-(-5))^2+(3-5)^2}$ $= \sqrt{53} = 7,28$ Sum of radii = $2 + 2\sqrt{10}$ = 8,32 Distance between centres < Sum of radii $\therefore$ They will intesect.	✓subst of centres ✓distance ✓Sum of radii ✓conclusion (4)
		[23]

QUESTION 5

5.1

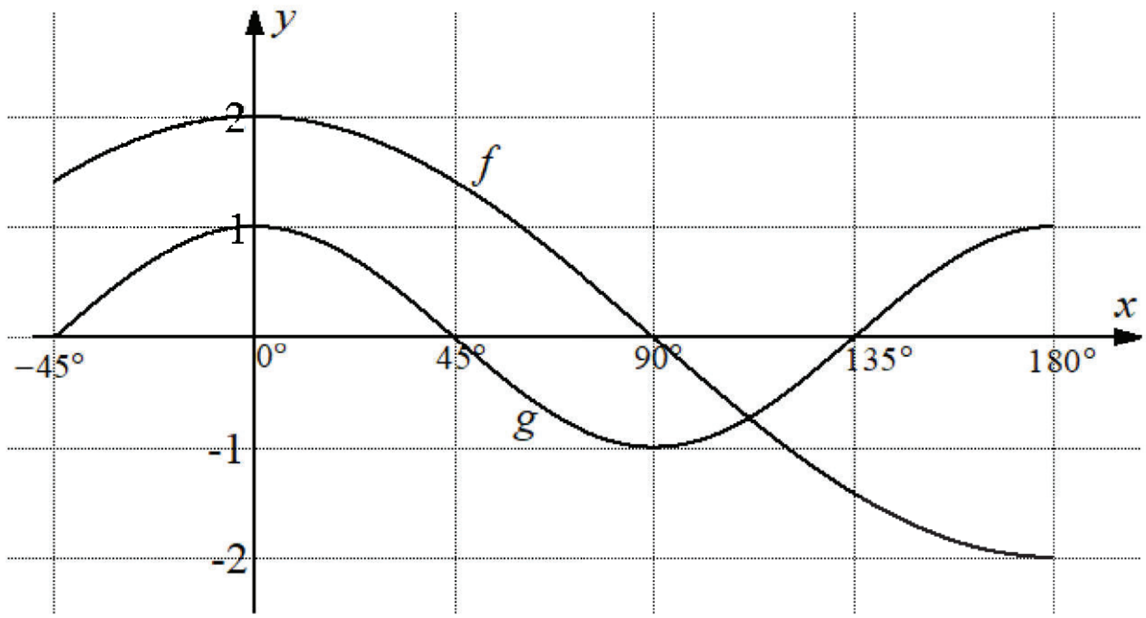


5.1.1	$(-3)^2+4^2=r^2$ $r^2=25$ $r=5$ $\sin\theta=\frac{-3}{5}$	✓value of r ✓ answer (2)
5.1.2	$\cos 2\theta=\cos^2\theta-\sin^2\theta$ $=\left(\frac{4}{5}\right)^2-\left(\frac{-3}{5}\right)^2$ $=\frac{16}{25}-\frac{9}{25}$ $=\frac{7}{25}$	✓expansion ✓substitution ✓ answer (3)
5.1.3	$\cos(\theta+30°)=\cos\theta\cos 30°-\sin\theta\sin 30°$ $=\left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{-3}{5}\right)\left(\frac{1}{2}\right)$ $=\frac{4\sqrt{3}+3}{10}$	✓expansion ✓substitution ✓ answer (3)
5.2	$(4\sin\alpha)^2+(4\cos\alpha)^2$ $=16\sin^2\alpha+16\cos^2\alpha$ $=16(\sin^2\alpha+\cos^2\alpha)$ $=16(1)=16$	✓simplification ✓answer (2)

5.3	$\sin(900°-x).\cos(-x)-\sin(x-180°).\sin(90°+x)$ $=\sin(180°-x).\cos(360°-x)-\sin(180°+x).\sin(90°+x)$ $=\sin x\cos x-(-\sin x)\cos x$ $=\sin x\cos x+\sin x\cos x$ $=2\sin x\cos x$ $=\sin 2x$	✓ $\sin x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $\cos x$ ✓simplification ✓ double angle identity (6)
5.4.1	$\frac{\sin 7x+\sin x}{2\cos 3x}=\sin 4x$ $\text{LHS}=\frac{\sin(4x+3x)+\sin(4x-3x)}{2\cos 3x}$ $=\frac{\sin 4x\cos 3x+\cos 4x\sin 3x+\sin 4x\cos 3x-\cos 4x\sin 3x}{2\cos 3x}$ $=\frac{\sin 4x\cos 3x+\sin 4x\cos 3x}{2\cos 3x}$ $=\frac{2\sin 4x\cos 3x}{2\cos 3x}$ $=\sin 4x=\text{RHS}$	✓both brackets ✓ $\sin(4x+3x)$ expansion ✓ $\sin(4x-3x)$ expansion ✓simplification (4)
5.4.2	$2\cos 3x=0$ $\cos 3x=0$ $3x=\begin{cases}90°+360°k\\-90°+360°k\end{cases}$ for $k\in z$ $x=\begin{cases}30°+120°k\\-30°+120°k\end{cases}$ OR $2\cos 3x=0$ $\cos 3x=0$ $3x=\begin{cases}90°+360°k\\270°+360°k\end{cases}$ for $k\in z$ $x=\begin{cases}30°+120°k\\90°+120°k\end{cases}$	✓ $2\cos 3x=0$ ✓ $3x=\begin{cases}90°+360°k\\-90°+360°k\end{cases}$ for $k\in z$ ✓ $x=\begin{cases}30°+120°k\\-30°+120°k\end{cases}$ (3) OR ✓ $2\cos 3x=0$ ✓ $3x=\begin{cases}90°+360°k\\270°+360°k\end{cases}$ for $k\in z$ ✓ $x=\begin{cases}30°+120°k\\90°+120°k\end{cases}$

5.5	$\sin(3x+20°)=\cos x$ $\sin(3x+20°)=\sin(90°-x)$ $3x+20°=90°-x+360°k \ \ k\in z$ $4x=70°+360°k$ $x=17,5°+90°k$ *OR* $3x+20°=180°-(90°-x)+360°k \ \ k\in z$ $3x+20°=180°-90°+x+360°k$ $2x=70°+360°k$ $x=35°+180°k$ OR $\sin(3x+20°)=\cos x$ $\cos[90°-(3x+20°)]=\cos x$ $-3x+70°=x+360°k \ \ k\in z$ $-4x=-70°+360°k$ $x=17,5°+90°k$ *OR* $-3x+70°=-x+360°k \ \ k\in z$ $-2x=-70°+360°k$ $x=35°+180°k$	✓ $\sin(90-x)$ ✓ $3x+20°=90°-x+360°k \ \ k\in z$ ✓ $x=17,5°+90°k$ ✓ $3x+20°=180°-(90°-x)+360°k$ ✓ $2x=70°+360°k$ ✓ $x=35°+180°k$ (6) ✓ $\cos[90°-(3x+20°)]$ ✓ $-4x=-70°+360°k$ ✓ $x=17,5°+90°k$ ✓ $-3x+70°=-x+360°k \ \ k\in z$ ✓ $-2x=-70°+360°k$ ✓ $x=35°+180°k$
		[29]

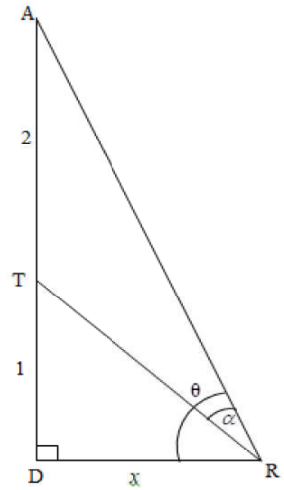
QUESTION 6



6.1	180°	✓answer (1)
6.2		✓critical values ✓notation (2)
6.3	Range of g: $y \in [-1; 1]$ Range of $3g$: $y \in [-3; 3]$ Range of $y = 3g(x) - 1$: $-4 \leq y \leq 2$ or $y \in [-4; 2]$	✓critical values ✓notation (2)
6.4	$2\cos x = \frac{1}{2}$ $\cos x = \frac{1}{4}$ $x = 75{,}5°$ $-45° \leq x \leq 75{,}5°$	✓equation ✓ $x = 75{,}5°$ ✓critical values ✓notation (4)
6.5	$\frac{1}{2}\cos^2 x - \frac{1}{4}$ $= \frac{1}{4}\left(2\cos^2 x - 1\right)$ $= \frac{1}{4}\left(\cos 2x\right)$ $= \frac{1}{4}(-1)$ $= -\frac{1}{4}$	✓factor ✓double angle ✓min of cos2x ✓answer (4)

		[13]

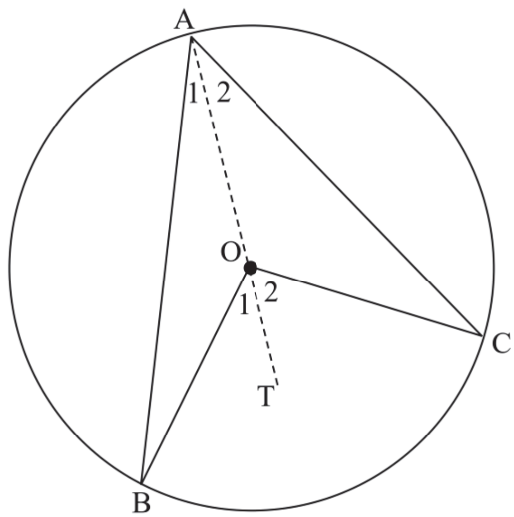
QUESTION 7



7.1	$T\hat{R}D = \theta - \alpha$ $\cos(\theta - \alpha) = \frac{x}{TR}$ $TR = \frac{x}{\cos(\theta - \alpha)}$	✓ $T\hat{R}D$ ✓cos definition (2)
7.2	$\hat{A} = 90° - \theta$ $\frac{\text{TR}}{\sin(90° - \theta)} = \frac{\text{AT}}{\sin\alpha}$ $\frac{TR}{\cos\theta} = \frac{2}{\sin\alpha}$ $TR = \frac{2\cos\theta}{\sin\alpha}$ $\frac{x}{\cos(\theta - \alpha)} = \frac{2\cos\theta}{\sin\alpha}$ $x = \frac{2\cos\theta\cos(\theta - \alpha)}{\sin\alpha}$	✓ x ✓ TR ✓ Subst in area-rule ✓answer (4)

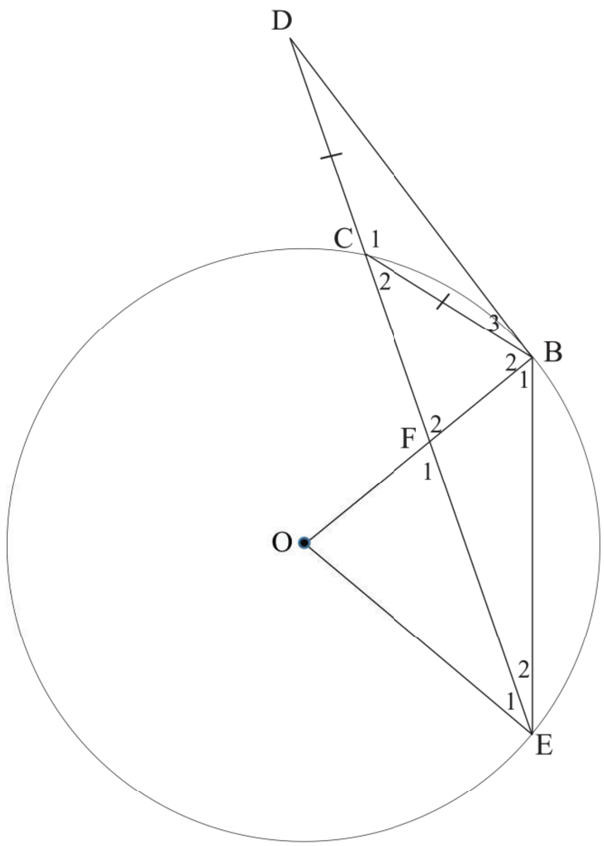
7.3	$x=\dfrac{2\cos 68,33\cos(68,33-28°)}{\sin 28°}$ $=1,1992...$ $TR=\dfrac{1,1992}{\cos(\theta-\alpha)}$ $=1,5730...$ $\hat{A}=21,67°$ $A\hat{T}R=130,33°$ $Area\,\Delta ATR=\dfrac{1}{2}(2)(1,5730)\sin 130,33°$ $=1,1992...$ $=1,20\,units^2$	✓ x ✓ TR ✓ subst in area formula ✓answer (4)
		[10]

QUESTION 8



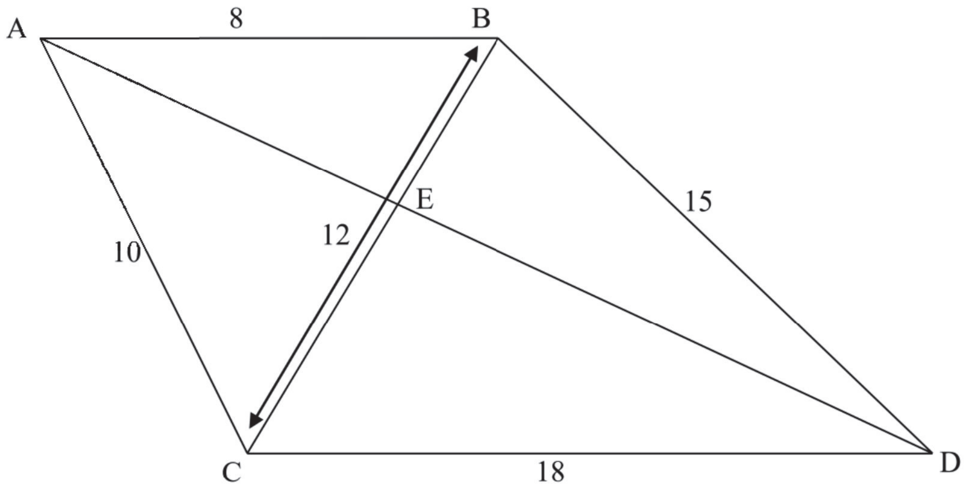
8.1	Draw line from A through O to T let $\hat{A}_1 = x$ and $\hat{A}_2 = y$ $\hat{A}_1 = \hat{B} = x$ ($\angle$'s opp = sides) $\hat{O}_1 = 2x$ (*ext* $\angle$ of Δ) similarly $\hat{O}_2 = 2y$ $B\hat{O}C = 2x + 2y = 2(x + y)$ $\therefore B\hat{O}C = 2 \times B\hat{A}C$	✓construction ✓S/R ✓ S/R ✓S ✓deduction (5)

8.2



8.2.1	$\hat{D} = x$ $(\angle\ opp = sides)$ $\hat{E}_2 = x$ (tan-chord theorem)	✓S ✓R ✓S ✓R (4)
8.2.2	$\hat{C}_2 = 2x$ $(\text{ext}\angle\ of\ \Delta)$ $2\hat{C}_2 = E\hat{O}B = 4x$ $(\angle \text{at centre} = 2\times\angle \text{ at circumf})$	✓S/R ✓S ✓R (3)
8.2.3	$O\hat{B}D = 90°$ $(\tan \perp rad)$ $\hat{B}_2 = 90° - x$	✓S ✓R (2)
8.2.4	$O\hat{B}D = 90°$ $(\tan \perp rad)$ $\hat{F}_2 = 180° - x - O\hat{B}D$ $(\text{sum}\angle s\ \Delta)$ $\hat{F}_2 = 90° - x$ $\therefore BC = FC$ (opp $\angle$'s =) $\therefore DC = BC = FC$	✓S ✓S ✓S/R (3)
		[17]

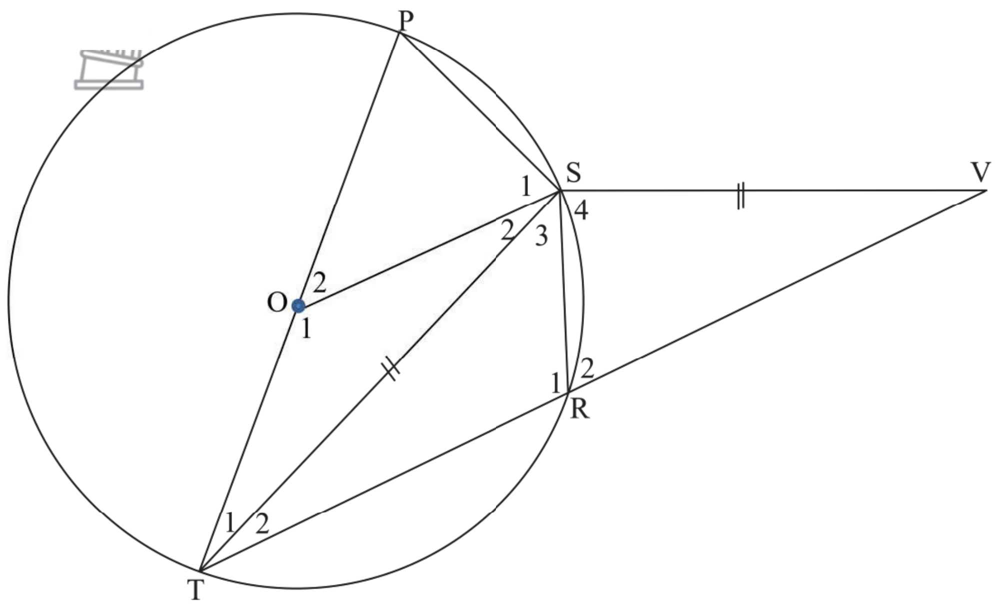
QUESTION 9



9.1	*In* ΔBCA *and* ΔCDB $\frac{BC}{CD}=\frac{12}{18}=\frac{2}{3}$ $\frac{AB}{BC}=\frac{8}{12}=\frac{2}{3}$ $\frac{AC}{DB}=\frac{10}{15}=\frac{2}{3}$ $\Delta BCA \mathbin{\vert\vert\vert} \Delta CDB$ (sides in proportion)	✓ S ✓S ✓ S ✓ S/R (4)
9.2	$A\hat{B}C = B\hat{C}D$ ($\Delta BCA \mathbin{\vert\vert\vert} \Delta CDB$) $\therefore AB \parallel CD$ (alt $\angle$s =)	✓S ✓R (2)
9.3	In ΔABE and ΔDCE: $A\hat{B}C = B\hat{C}D$ (proven) $A\hat{E}B = C\hat{E}D$ (vertically opp $\angle$) $\Delta ABE \mathbin{\vert\vert\vert} \Delta DCE$ ($\angle\angle\angle$) $\frac{AB}{DC}=\frac{BE}{CE}$ ($\vert\vert\vert\Delta$'s) $\frac{8}{18}=\frac{x}{12-x}$ $8(12-x)=18x$ $96-8x=18x$ $96=26x$ $x=3,69$ $CE=8,31$	✓ $\Delta ABE \mathbin{\vert\vert\vert} \Delta DCE$ ✓ $\frac{AB}{DC}=\frac{BE}{CE}$ ✓ $\frac{8}{18}=\frac{x}{12-x}$ ✓ $CE=8,31$ (4)

			[10]

QUESTION 10



10.1	$P\hat{S}T = 90°$ (∠in semi-circle)	✓ S ✓ R (2)
10.2	$\hat{T}_1 = \hat{T}_2$ (ST is a bisector) $\hat{T}_2 = \hat{V}$ (∠opp = sides) $\hat{T}_1 = \hat{V}$ $\hat{R}_2 = \hat{P}$ (ext ∠ of cyclic quad) $\hat{S}_4 = P\hat{S}T = 90°$ (sum ∠ of Δ)	✓ S ✓ S ✓ S ✓ R ✓ S + R (5)
10.3	ΔTSO ||| ΔTVS $\hat{T}_1 = \hat{T}_2$ (ST is a bisector) $\hat{S}_2 = \hat{V} =$ (both $= \hat{T}_1$) $\hat{O}_1 = T\hat{S}V$ (3rd ∠ Δ) ΔTSO ||| ΔTVS (∠∠∠)	✓ S ✓ S ✓ S/ R (3)

10.4	$\frac{TS}{TV}=\frac{OS}{VS}$ $\Delta TSO\ \lvert\lvert\lvert\ \Delta TVS\ (\angle\angle\angle)$ $TS.VS = OS.TV$ *but* $TS = VS$ *given* *and* $OS = \frac{1}{2}PT$ *both radii* $\therefore VS.VS = \frac{1}{2}PT.TV$ $\therefore 2VS^2 = PT.TV$	✓ S ✓ S ✓S+R ✓substitution (4)
		[14]

TOTAL: 150