EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2023

MARKS: 150

TIME: 3 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.

2. Answer **ALL** the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will **NOT** necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.

8. Diagrams are **NOT** necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

Seven scores for a test out of 50 marks were recorded. However, when recording the data the statistician had some difficulty with his computer. He listed the following conditions that he had remembered.

Scores	a	b	c	d	e	f	g

➢ Maximum Value is 42
➢ Range is 35
➢ Median is 23
➢ The difference between the median and the upper quartile is 14
➢ The Inter – Quartile Range is 22
➢ $e = 2c$
➢ The mean is 25.

Determine the values of the scores. (Show all working)   (10)

[10]

QUESTION 2

The class teacher of a grade 10 class collected the following data for her learners. She recorded the time taken by the learners to complete different tasks assigned.

Time in minutes	No. of learners			
$0 < t \leq 10$	5			
$10 < t \leq 20$	8			
$20 < t \leq 30$	18			
$30 < t \leq 40$	7			
$40 < t \leq 50$	2			

2.1 Calculate the estimated mean time.   (3)

2.2 Draw a frequency polygon (Ogive) curve on the system of axes provided.   (5)

2.3 Calculate how many learners took more than 60 % of the time to complete the tasks assigned to them.   (3)

[11]

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QUESTION 3

In the sketch below, A(-8;6), B(8 ; 10), C and D(−2;0) are the vertices of a trapezium having AD ∥ BC. T is the midpoint of DB. From the vertex B, BM is drawn perpendicular to the x – axis and intersects DC in $F\left(8 ; 3\frac{1}{3}\right)$.
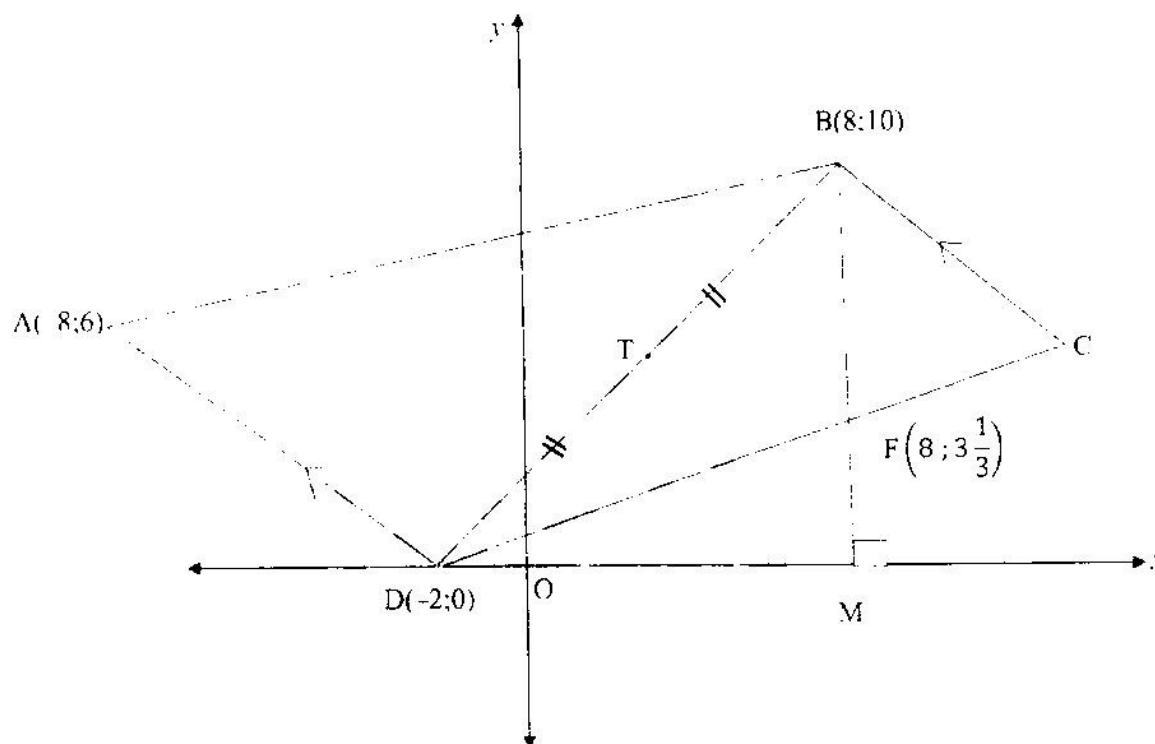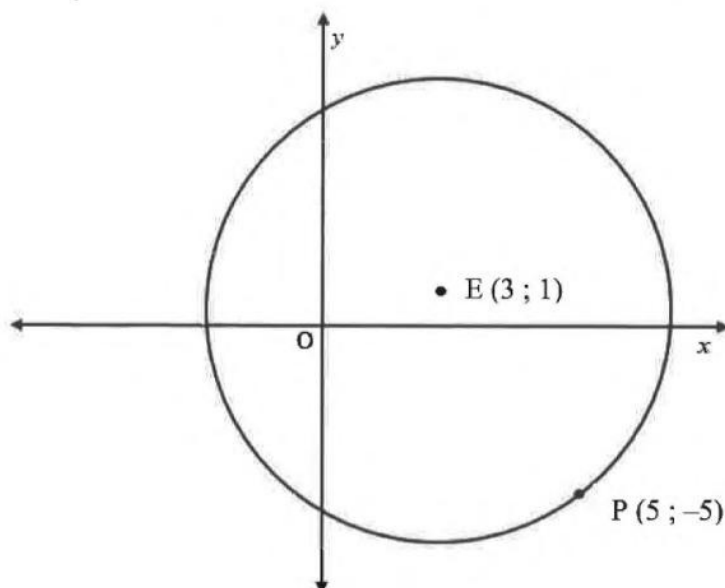

3.1	Calculate the gradient of AD.	(2)
3.2	Determine the equation of BC in the form $y = mx + c$.	(3)
3.3	Prove that BD ⊥ AD.	(3)
3.4	Calculate the size of BD̂M.	(2)
3.5	If TC ⊥ DM and points T and C are symmetrical about the line BM. Calculate the coordinates of C.	(3)
3.6	Calculate the area of ABDF	(5)
		[18]

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QUESTION 4

In the diagram below, the circle centred at E (3 ; 1) passes through the point, P (5 ; –5).

4.1 Determine the equation of :

    4.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)

    4.1.2 The tangent to the circle at P (5 ; –5) in the form $y = mx + c$. (5)

4.2 A smaller circle is drawn inside the circle. Line EP is the diameter of the small circle.

    Determine the:

    4.2.1 Coordinates of the centre of the smaller circle. (4)

    4.2.2 Length of the radius. (2)

4.3 Hence, or otherwise, determine whether the point C(9 ; 3) lies inside, outside or on the circle centre, E. (4)

[19]

QUESTION 5

5.1 If $\tan 12° = q$, then determine the value of the following, without using a calculator:

     5.1.1 $\cos 192°$ (3)

     5.1.2 $\cos 24°$ (3)

     5.1.3 $1 - 2\sin^2 6°$ (2)

5.2 Evaluate the following trigonometric expression without using a calculator:

$$\frac{2\sin^2(x - 180°)\cos(180° - x)}{\cos(90° + x)\sin x - \cos(x - 90)\sin(720° - x)}$$ (7)

5.3 Given: $(1 - \tan A)\left(\dfrac{\cos A}{\cos 2A}\right) = \dfrac{1}{\cos A + \operatorname{Sin} A}$

     5.3.1 Prove the identity. (3)

     5.3.2 Write down the maximum value of $\dfrac{1}{\cos A + \operatorname{Sin} A}$ (2)

5.4 Calculate the value of

$$\sum_{38^0}^{52°}\cos^2 A$$ (4)

[24]

QUESTION 6

6.1 Sketch the graphs of $f(x) = \sin\frac{1}{2}x$ and $g(x) = \cos(x + 60°)$ for $x \in [-180°; 180°]$ on the grid given at the back. (6)

6.2 Use your graphs to answer the following questions:

     6.2.1 Write down the range of the graph of f. (2)

     6.2.2 State the period of g. (1)

     6.2.3 Determine the values of x for which $f(x) = g(x)$. (4)

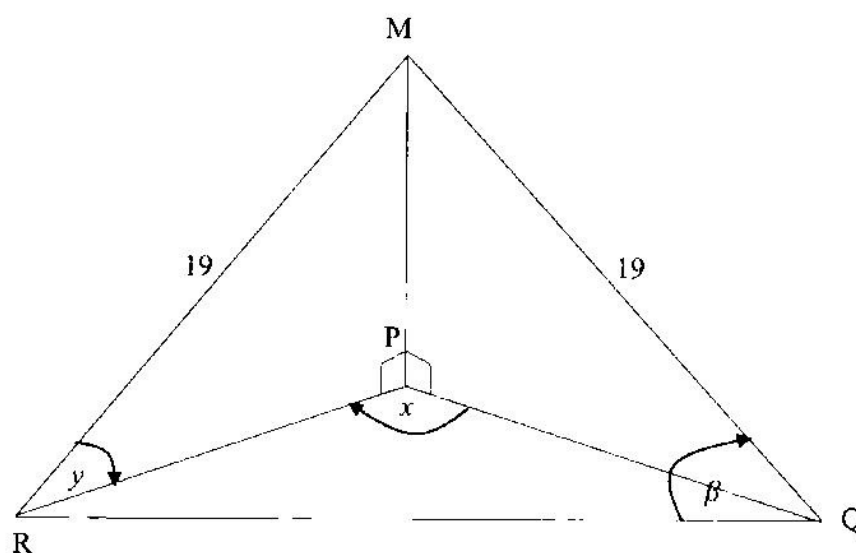   6.2.4 If $h(x) = g(x + 30°)$, write down the equation of h. (3)

[16]

QUESTION 7

In the diagram below, MP is a vertical tower. MQ and MR are wire ropes used to stabilize
MP and are each 19 metres in length. R, P and Q are in the same horizontal plane.
The angle of elevation of M from R is y.
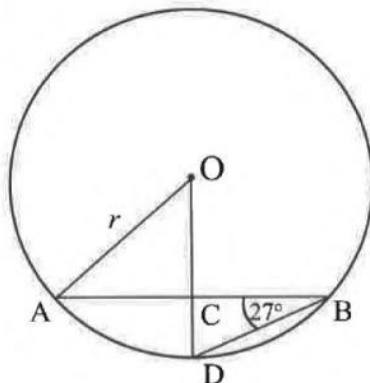$R\hat{P}Q = x$ and $M\hat{Q}R = \beta$

7.1   Prove that the Area of $\triangle PQR = \dfrac{361 \sin x \cos^2 y}{2}$ (5)

7.2   Show that, $RQ = 38 \cos \beta$ (6)

[11]

QUESTION 8

8.1   O is the centre of the circle, radius r, and chord AB = $\sqrt{128}$ cm. OCD $\perp$ AB and
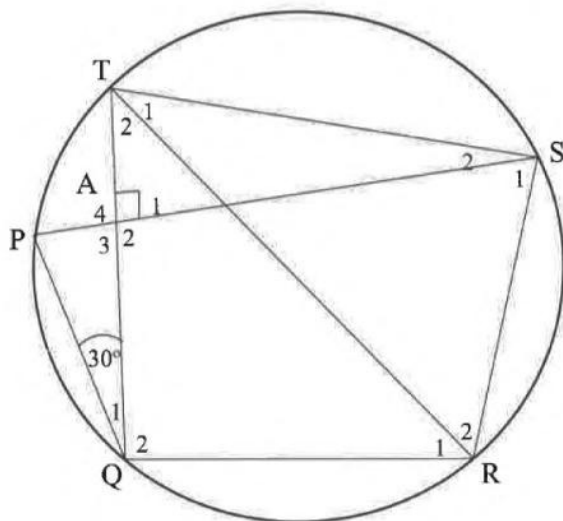OC : CD = 3 : 2. $A\hat{B}D = 27°$

8.1.1   Calculate the length of the radius of the circle, without using a calculator.   (5)

8.1.2   Calculate, with reason and $A\hat{O}D$.   (2)

8.2   Complete the following statement:

    (2)

8.2.1   The exterior angle of a cyclic quadrilateral is ……………………………..

8.2.2   TQ is a chord of the circle PQRST. QAT $\perp$ PAS. $\hat{Q}_1 = 30°$ and $\hat{P} = \hat{S}_1$.
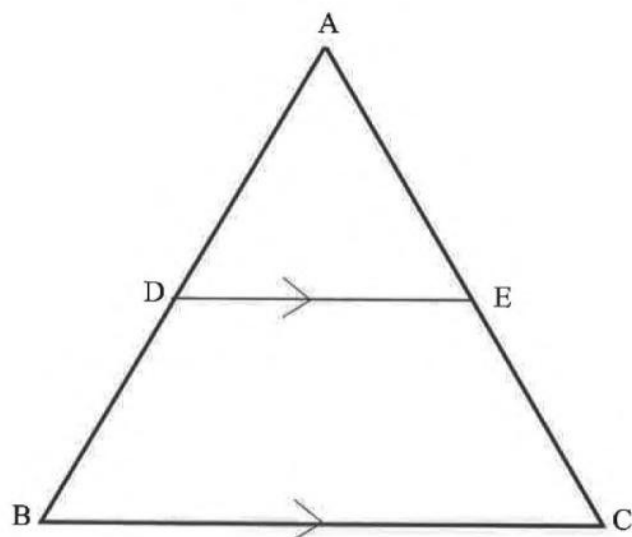

a)   Name 3 angles each equal to 60°.   (3)

b)   Calculate the size of $Q\hat{R}S$.   (2)

c)   Prove that PS || QR.   (2)

d)   Prove that TR is a diameter of the circle .   (2)

    [18]

QUESTION 9
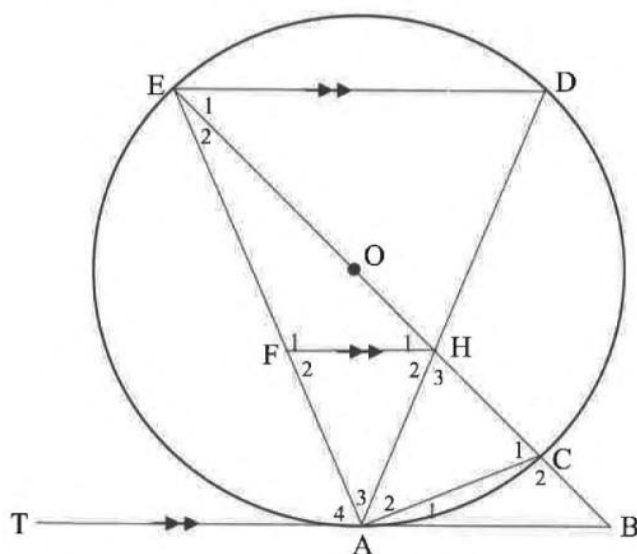
9.1   In Δ ABC, DE ∥ BC. Prove :

$$\frac{AD}{DB} = \frac{AE}{EC}$$

(6)

9.2   In the diagram, O is the centre of the circle ACDE. EC is produced to B. TAB is a tangent to the circle at A. EC and AD intersect at H. F is a point on EA. ED ∥ FH ∥ TAB. EOHCB is a straight line. AH : HD = 5: 7.

9.2.1   Prove ΔABC ∥∥ ΔEBA   (4)

9.2.2   Calculate r, the radius of the circle, if AB = 5 metres and BC $= \frac{2r}{3}$ metres.   (4)

9.2.3   Write down, with reason the value of AF : FE.   (2)

9.2.4   Determine $\frac{\text{Area of } \triangle AFH}{\text{Area of } \triangle AED}$.   (4)

9.2.5   Calculate the length of OH.   (3)

[23]

TOTAL MARKS: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum f.x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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DIAGRAM SHEET

QUESTION 2

Time in minutes	No. of learners	
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QUESTION 6.1